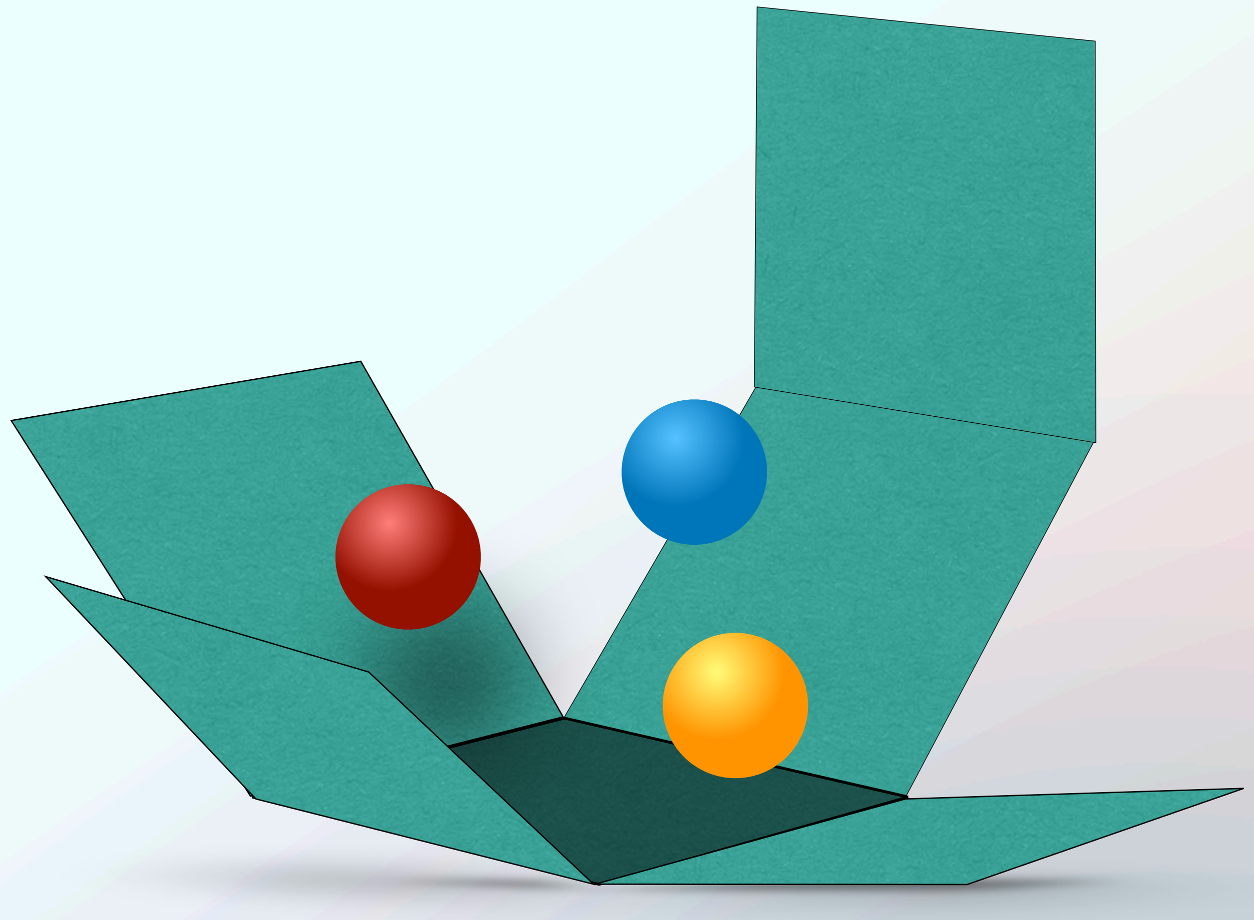
RESONANT 3-BODY SYSTEMS IN/OUT OF A BOX



Maxim Mai

with M. Döring, F.-R. Lopez, A. Rusetsky, C. Urbach, M. Garofalo, A. Alexandru, D. Sadasivan, C. Culver, R. Brett

NSF

UNIVERSITÄT BONN

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

3-BODY PROBLEM

Hadronic 3-body problem

- intricate kinematics/dynamics
- many open questions of strong interactions

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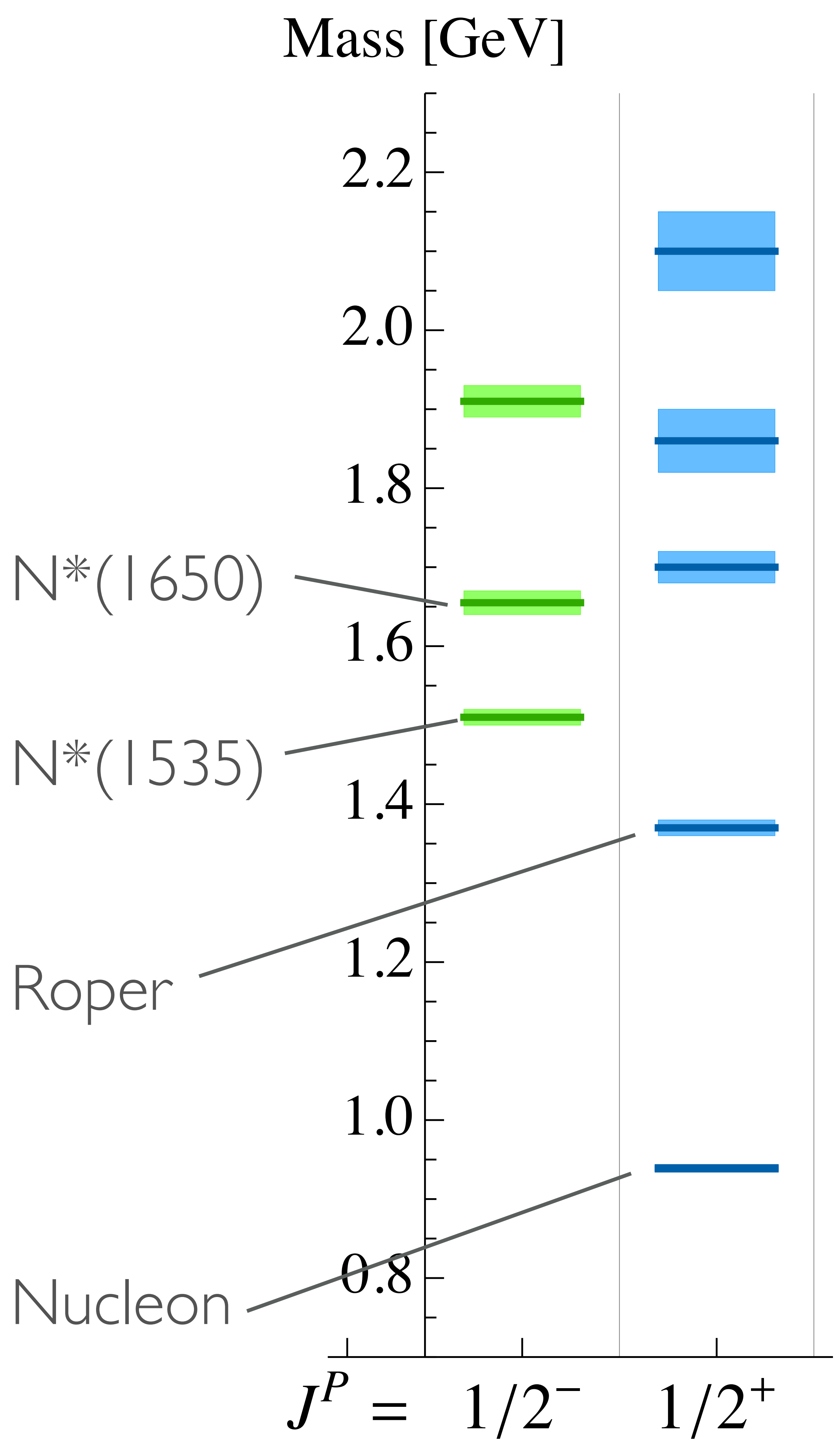
reversed pattern wrt constituent quark models[1]



Data.: Particle Data Goup (Workman et al.)

1) Loring et al.; Kapstick/Isgur; Glozman/Riska *Phys.Rept.* 268;

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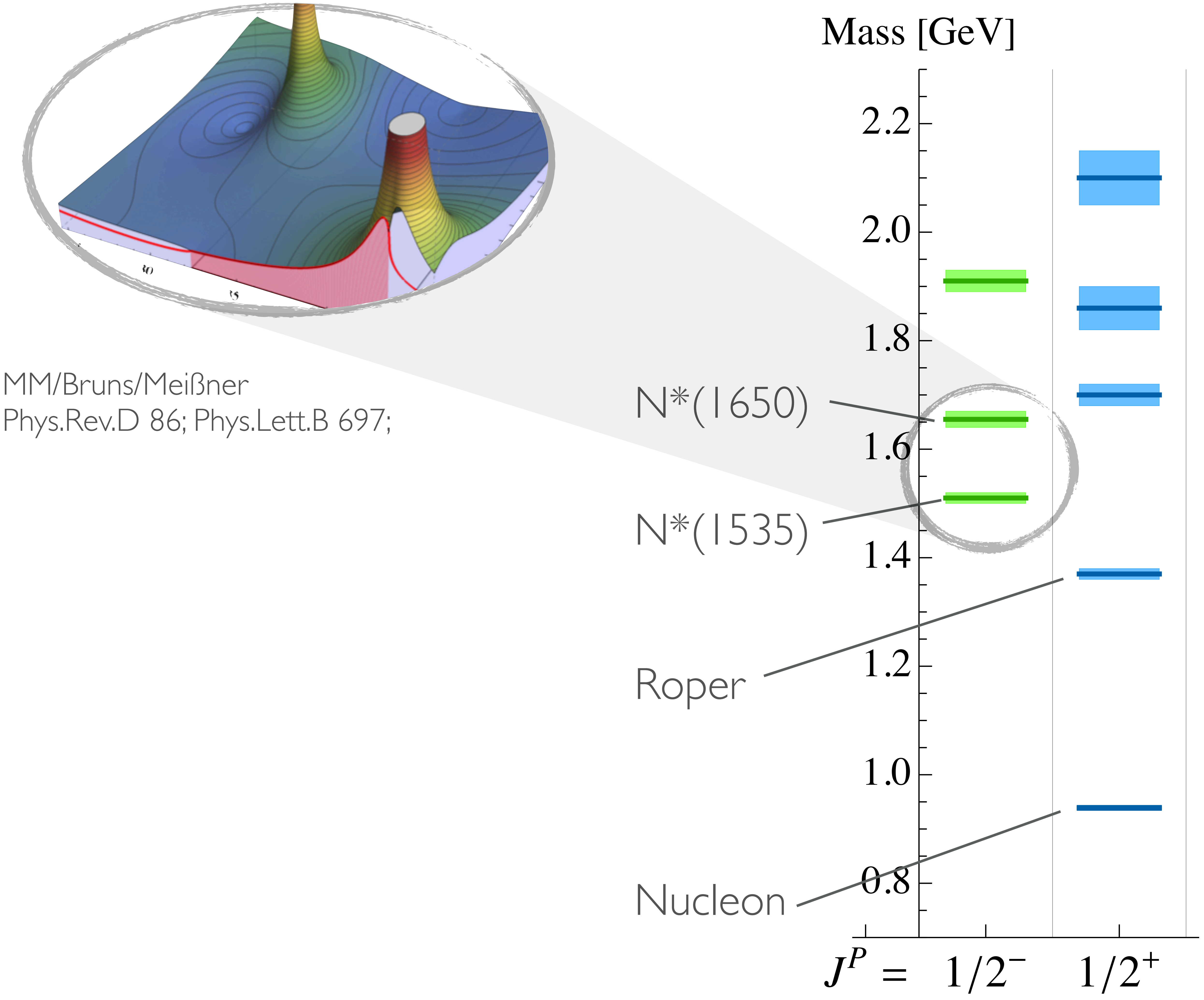

MM/Bruns/Meißner
Phys.Rev.D 86; Phys.Lett.B 697;

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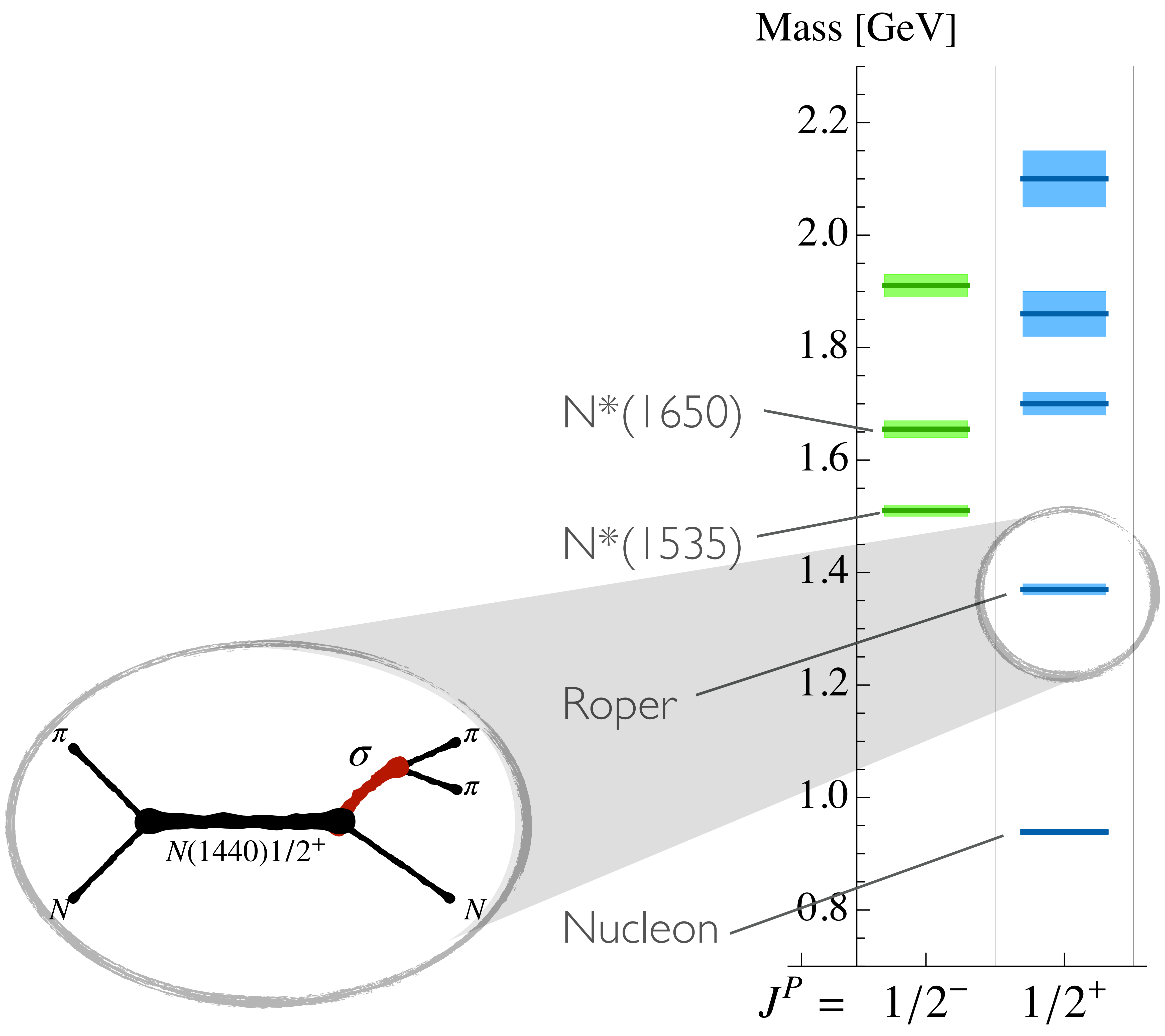

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CONCEPTS AND TECHNIQUES

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Theoretical access to observables from **transition amplitudes** (T)

I. Analytic ..

II. Unitary ..

III. Crossing symmetric ..

...functions of momentum bilinears

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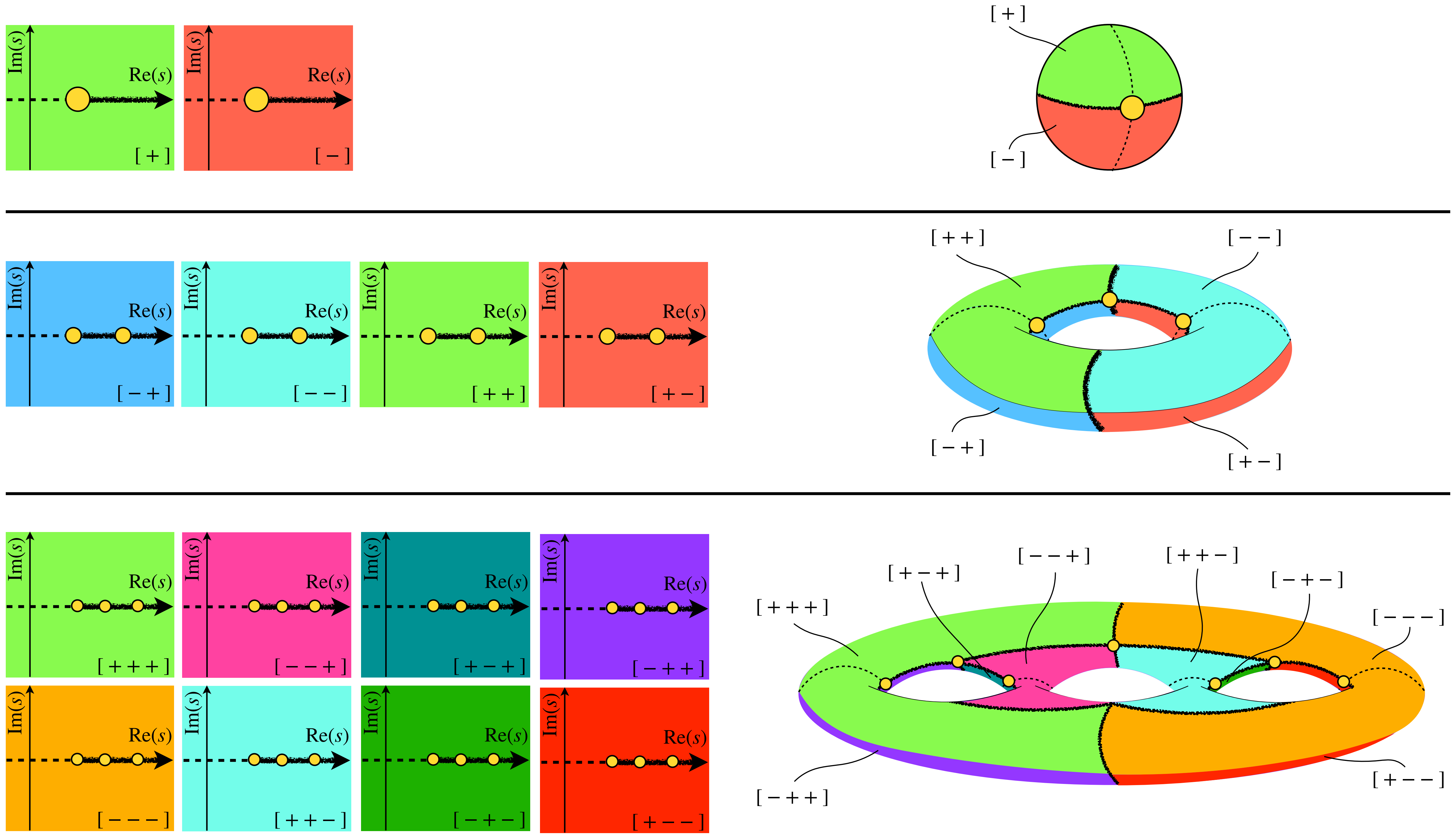

MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.

CONCEPTS AND TECHNIQUES

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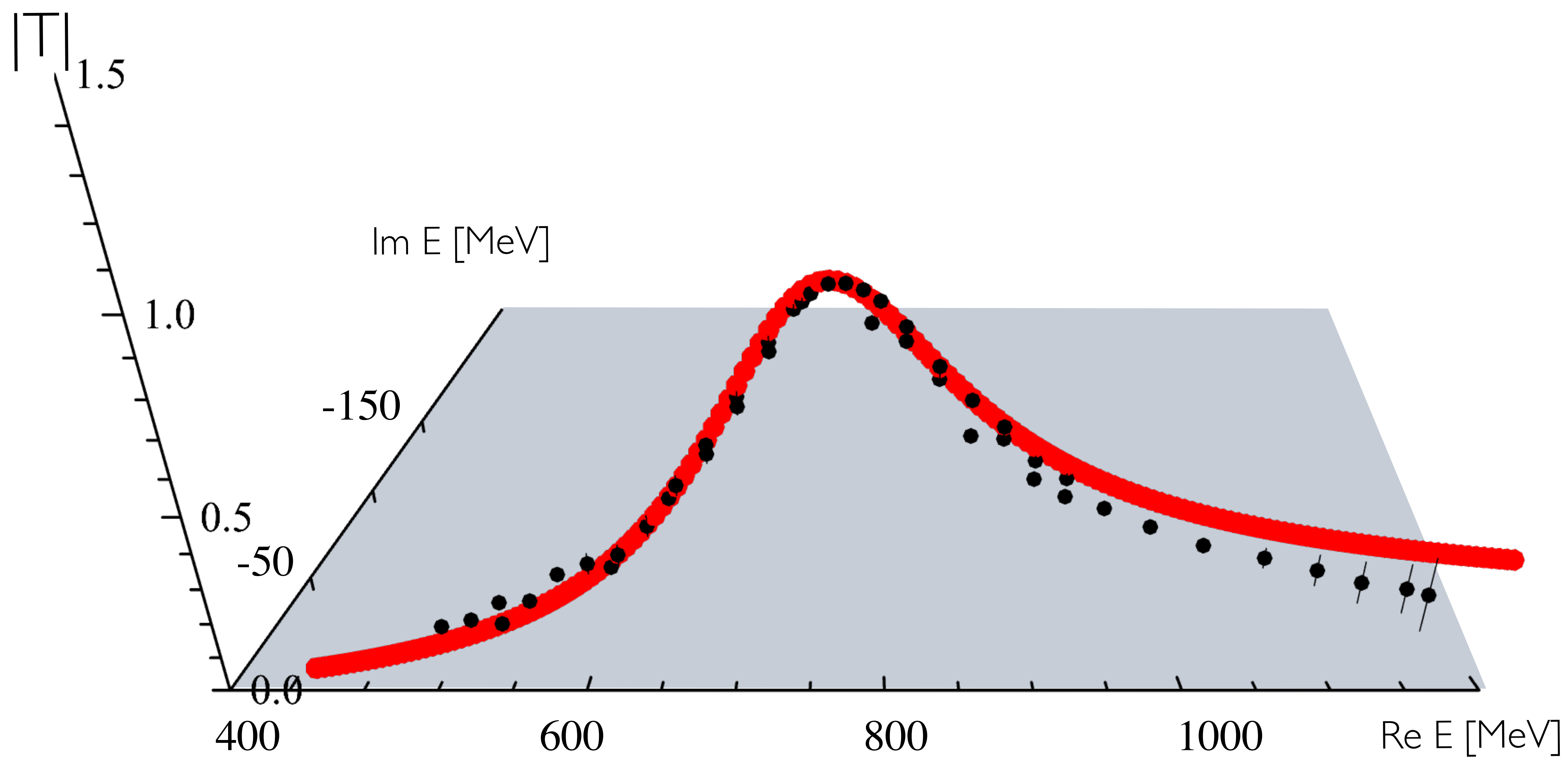
...functions of momentum bilinears

... constrained for real energies from experiment or lattice



Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

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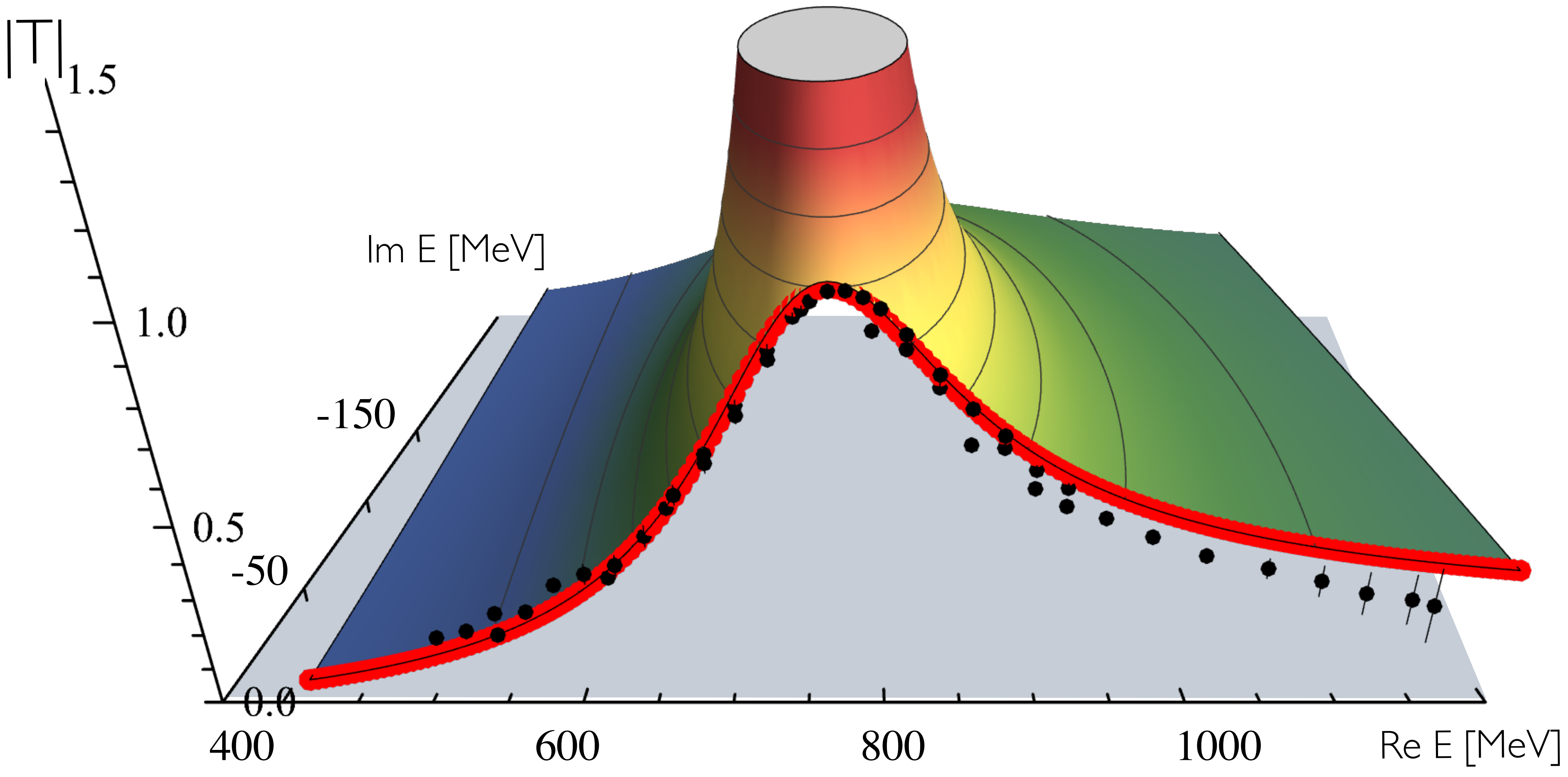
unstable states (resonances):

➥ *poles on the complex Riemann surface*



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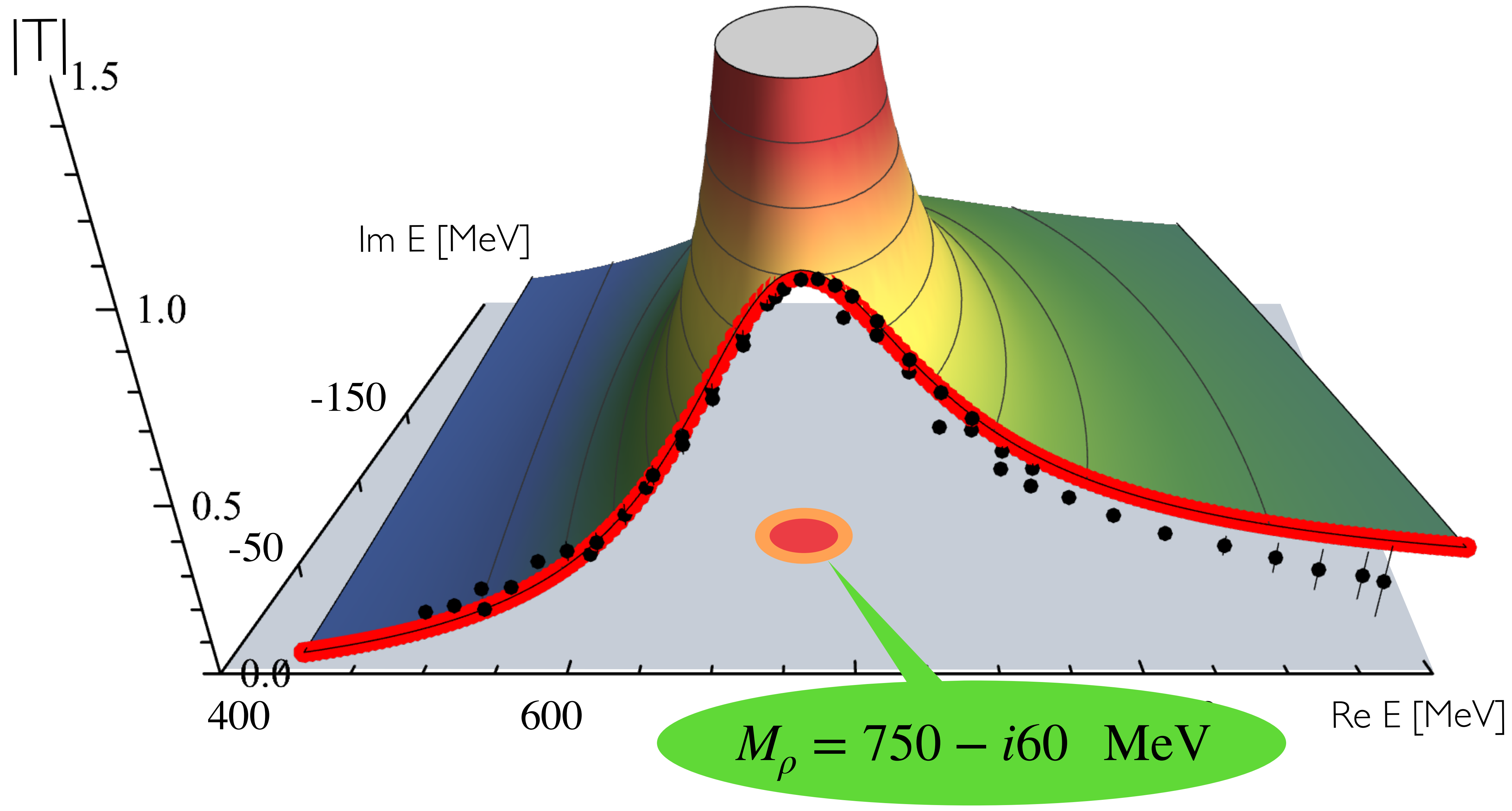
➥ *poles on the complex Riemann surface*

➥ ***universal resonance parameters***



Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

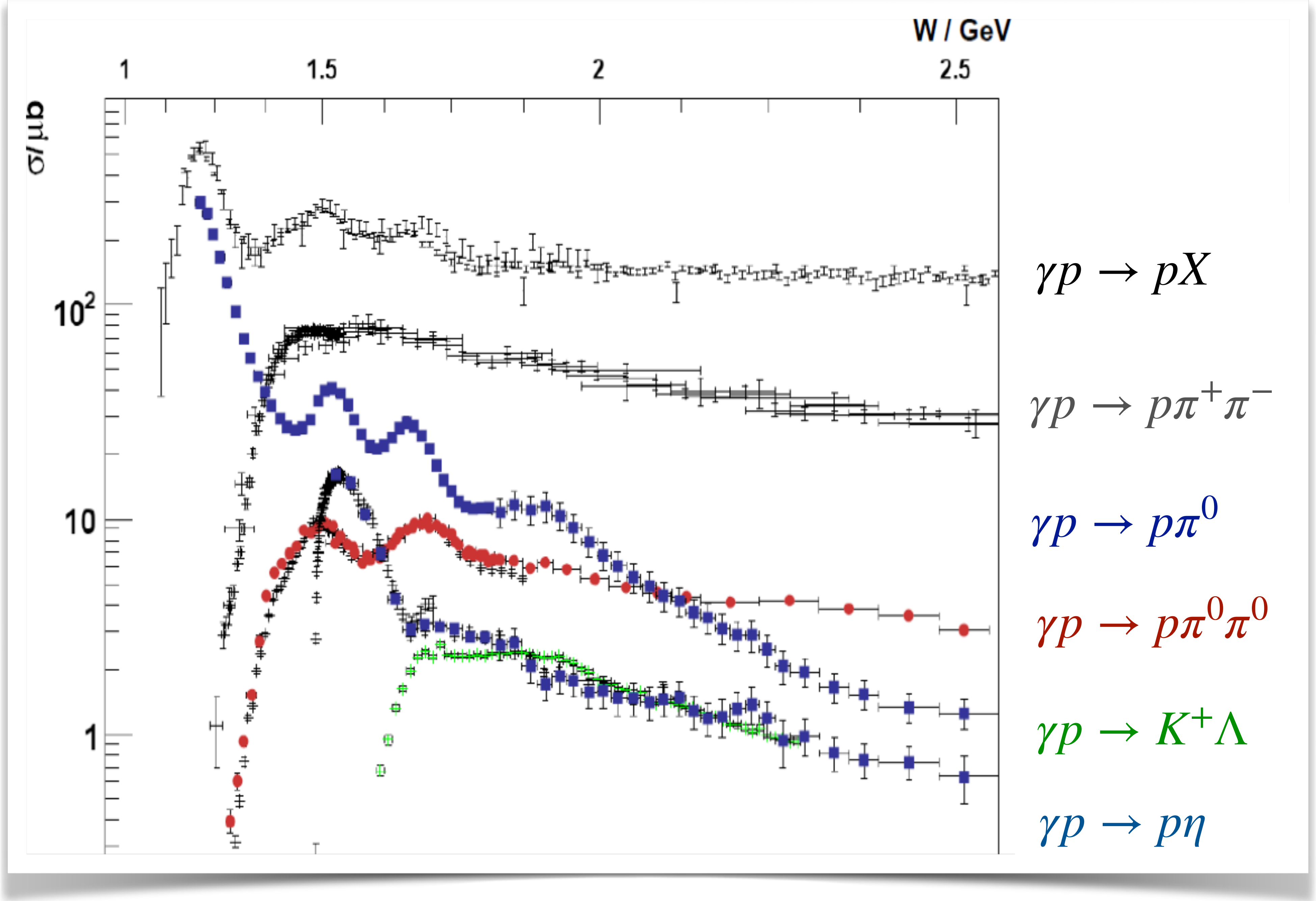
CONCEPTS AND TECHNIQUES

Recent example from phenomenology:

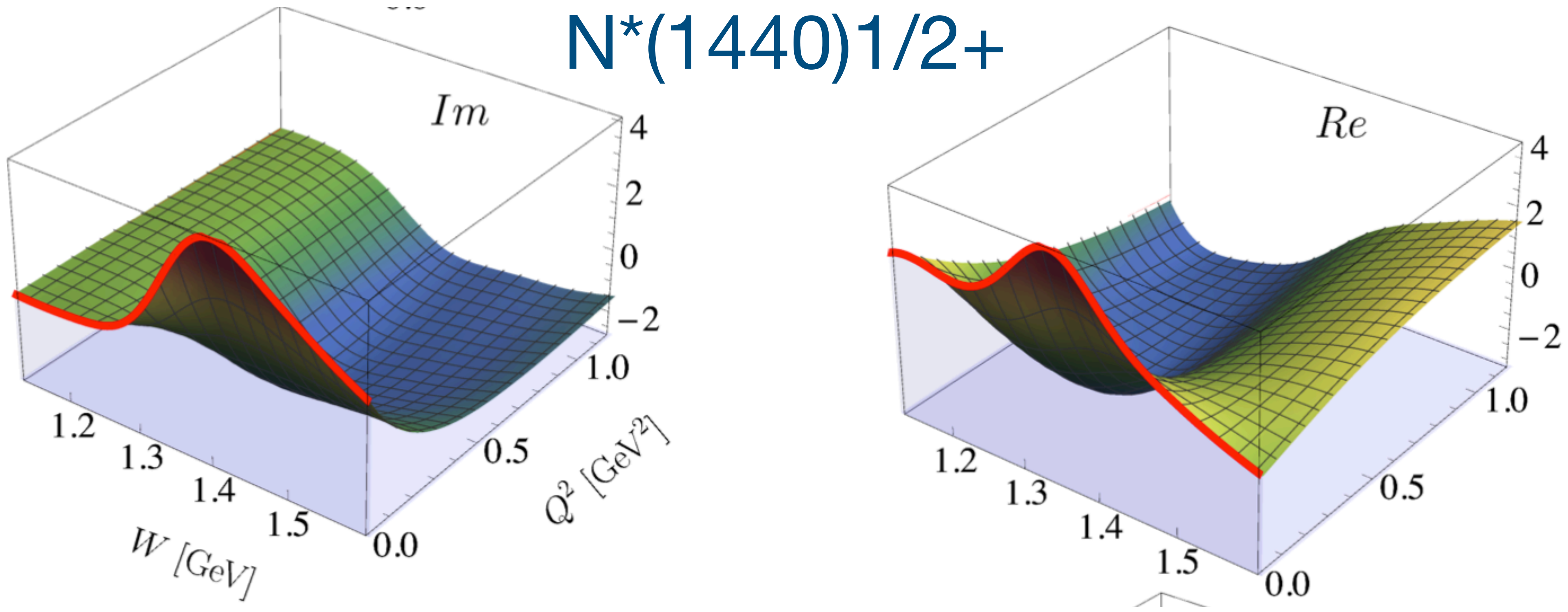
Jülich-Bonn-Washington model[1]

- dynamical model (unitarity)
- scattering and electroproduction (10^5) data
- helicity couplings of resonances

[JBW] MM et al. *Phys.Rev.C* 103 (2021) 6;
[JBW] MM et al. 2111.04774 [nucl-th]
INTERACTIVE WEB INTERFACE: https://jbw.phys.gwu.edu



N*(1440)1/2+



CONCEPTS AND TECHNIQUES

$$\mathcal{L}_{\mathrm{QCD}} = \sum \bar{q}_f^a \left(i \not{D}_{ab} - m_f \delta_{ab}\right) q_f^b - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

Quantum chromodynamics (QCD)

- SU(3) color gauge symmetry
- quark-gluon dynamics

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- SU(3) color gauge symmetry
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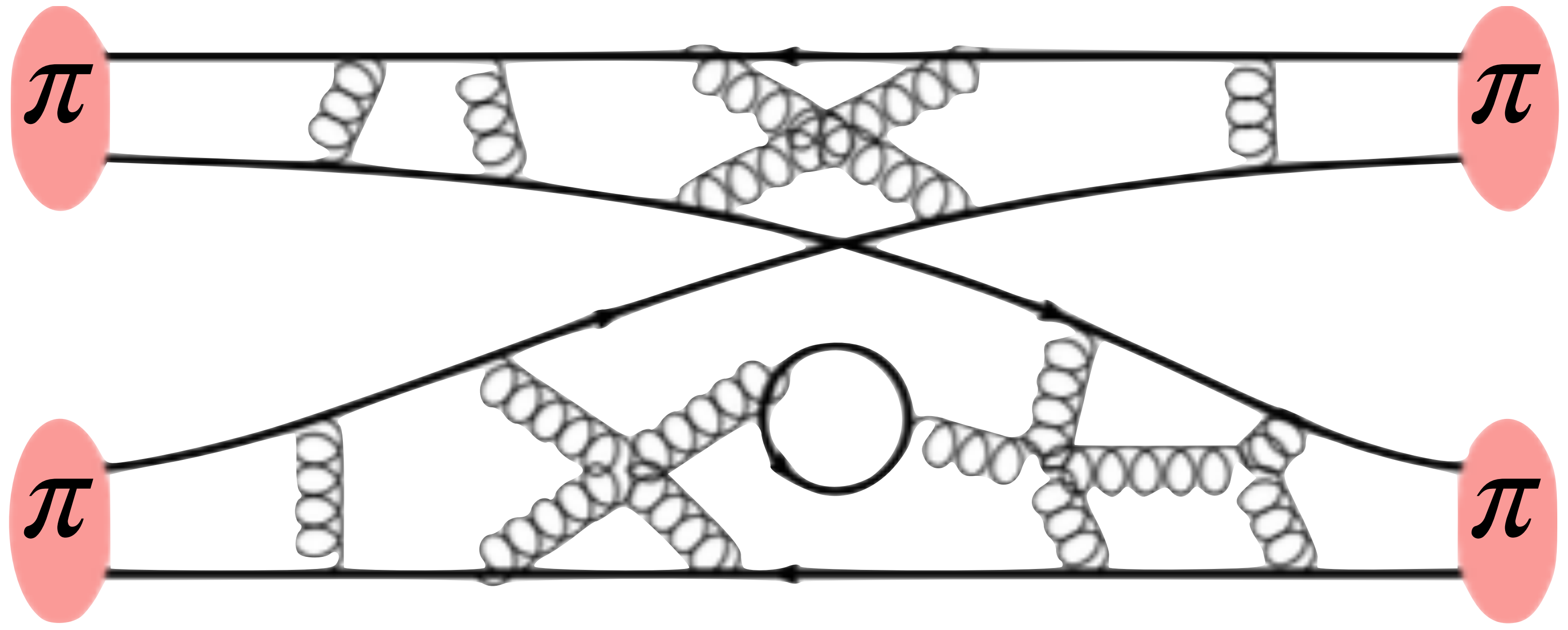
... non-perturbative at low energies



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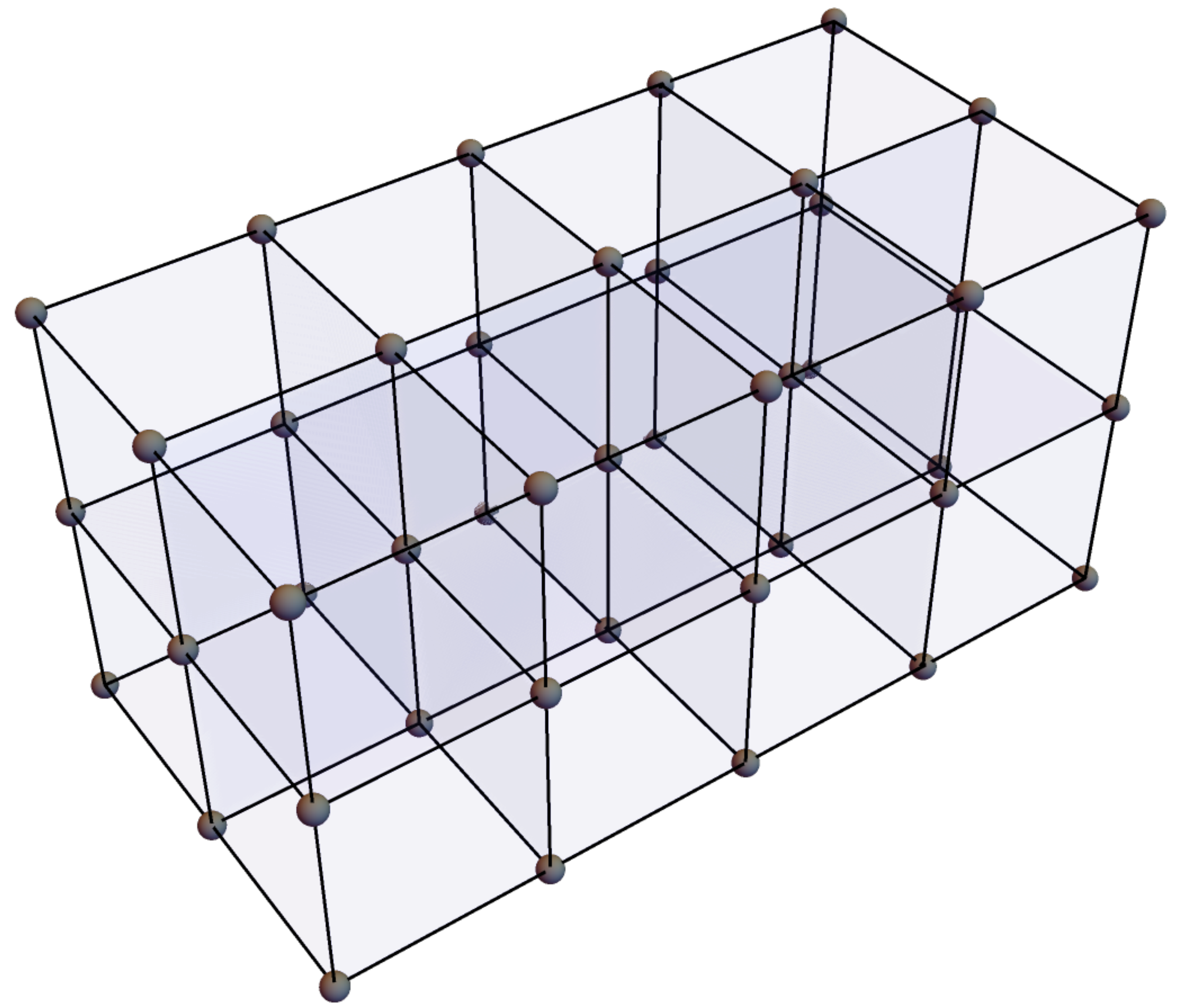
Lattice QCD

- numerical evaluation of QCD Green's func.
- discretized Euclidean space time
- in finite volume



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> mapping to the physical world:

Quantization Condition[1]

1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky, ...

Recent reviews:
Briceño et al. Rev.Mod.Phys. 90 (2018);
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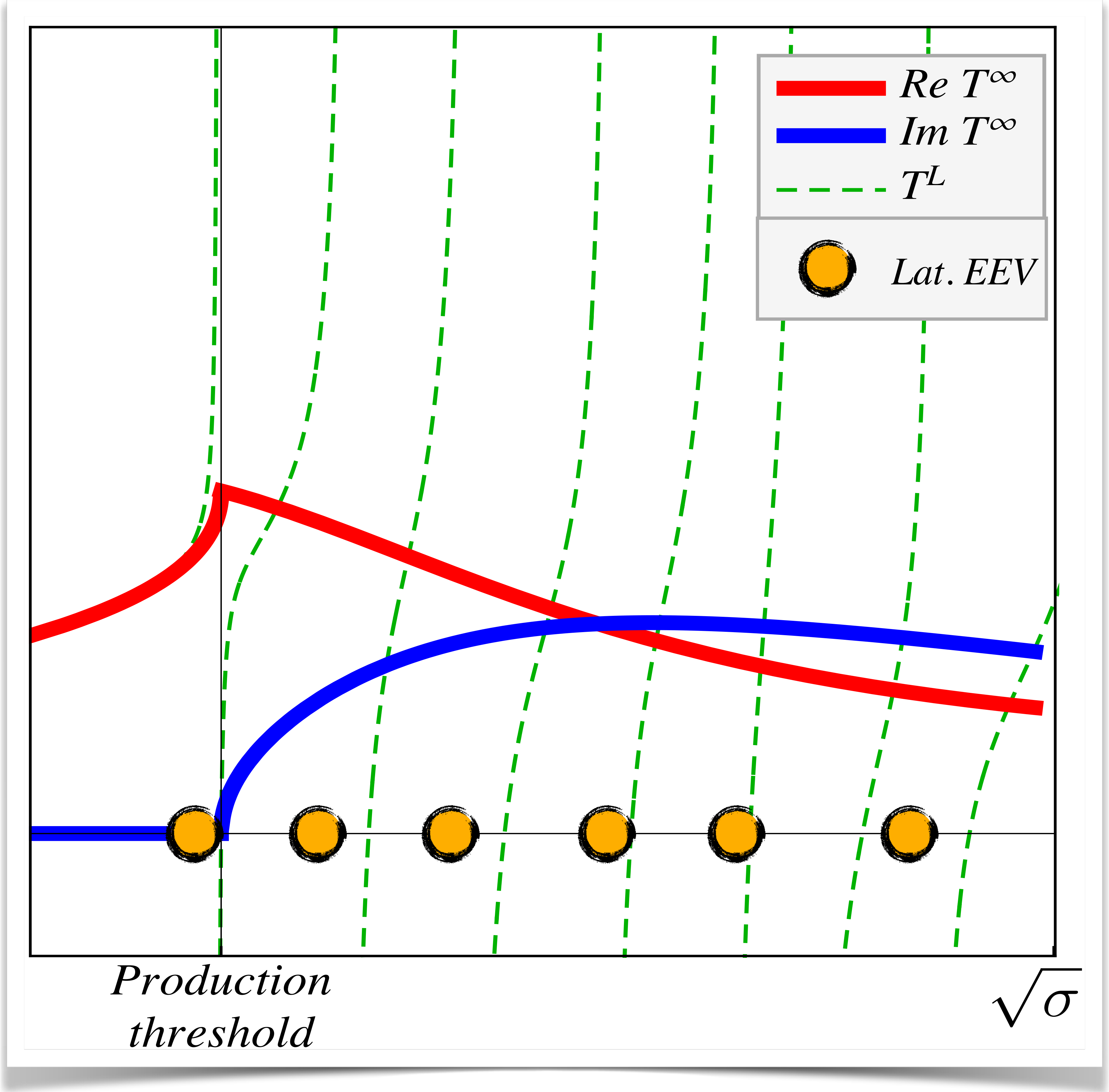

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CONCEPTS AND TECHNIQUES

Current frontier: 3-body dynamics from LQCD

➥ **3-body Quantization Conditions**[1]

➥ RFT / FVU / NREFT

➥ many perturbatively interacting systems are studied[2]

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo….
Reviews:
Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);
MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ….

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$$0 = \det\left(L^3\left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F}\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....
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"what can we learn about 3-body resonances from theory and experiment?"

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CASE 1

Explicit 3-body resonances in φ^4 theory

Garofalo, MM, Lopez, Rusetsky, Urbach [in preparation]

TOY MODEL

Complex φ^4 theory with explicit three-body state

Key questions:

- How does the avoided level crossing appear in 3-body systems?
- Can one prove RFT/FVU equivalence on the same data?

$$S = \int dx \sum_{i=0,1} \left[\frac{1}{2}\partial_\mu\varphi_i^\dagger\partial_\mu\varphi_i + \frac{1}{2}m_i\varphi_i^\dagger\varphi_i + \lambda_i(\varphi_i^\dagger\varphi_i)^2\right] + \frac{g}{2}\varphi_1^\dagger\varphi_0^3 + h.c.$$

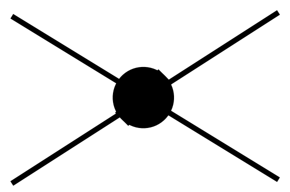
AVOIDED LEVEL CROSSING

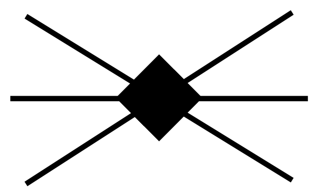
$g = 5$

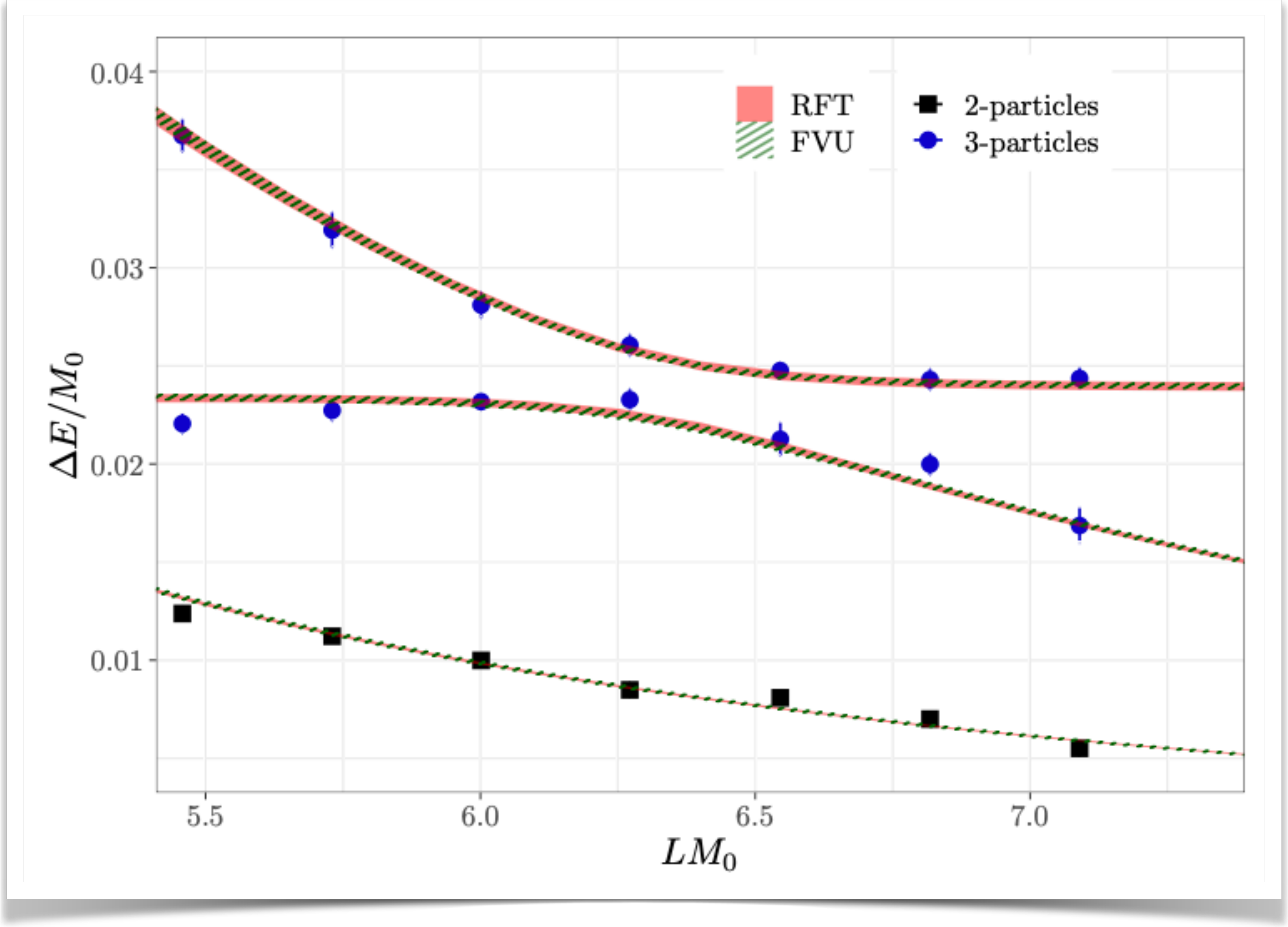
Variate g($\varphi_1 \to \varphi_0\varphi_0\varphi_0$) coupling:

- avoided level crossing becomes wider
- RFT and FVU

$$q^* \cot\delta = \frac{1}{aM_0} \qquad C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$







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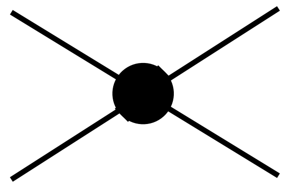
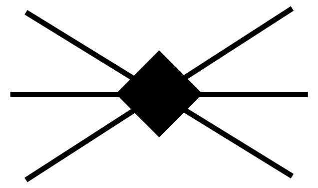
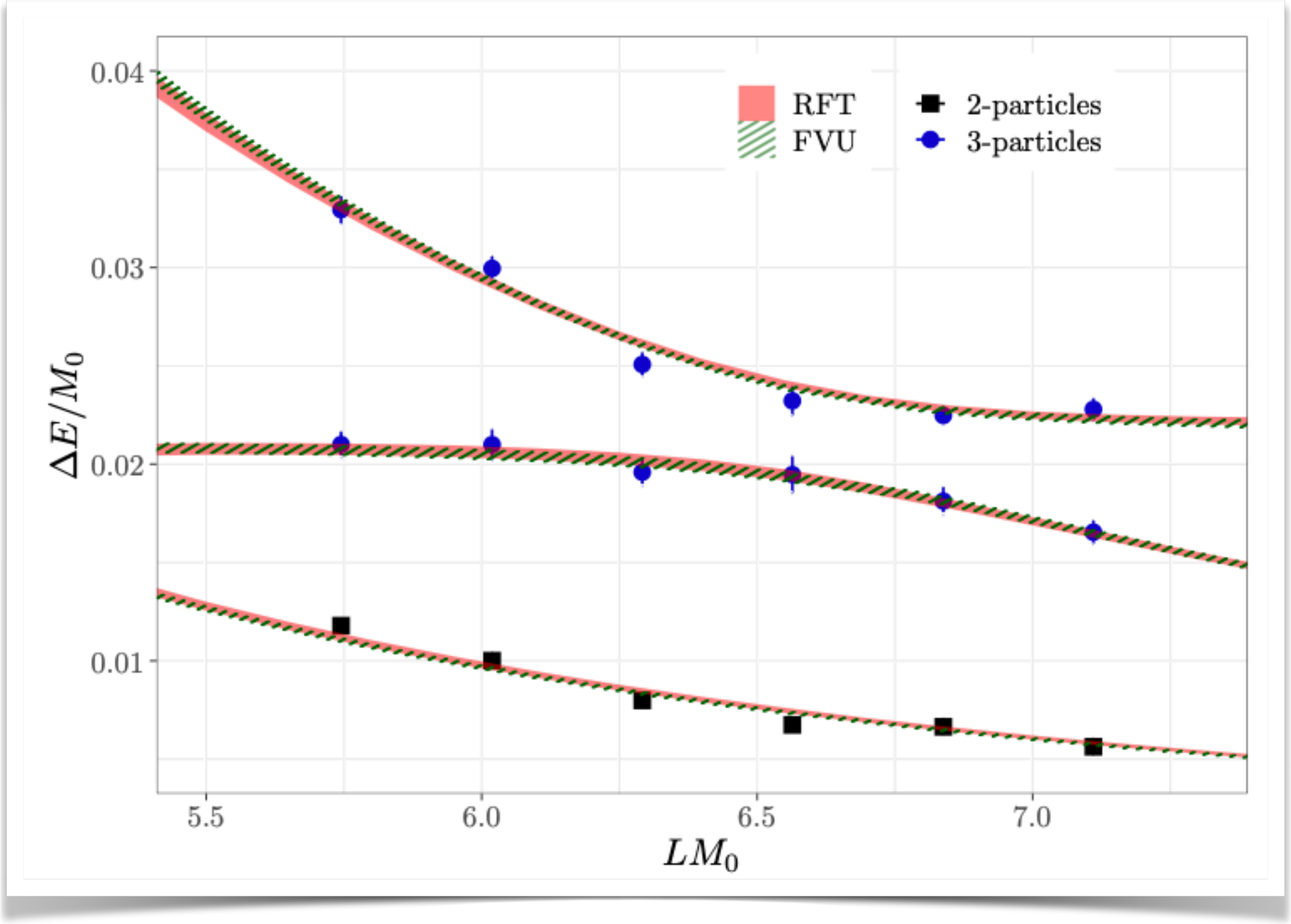


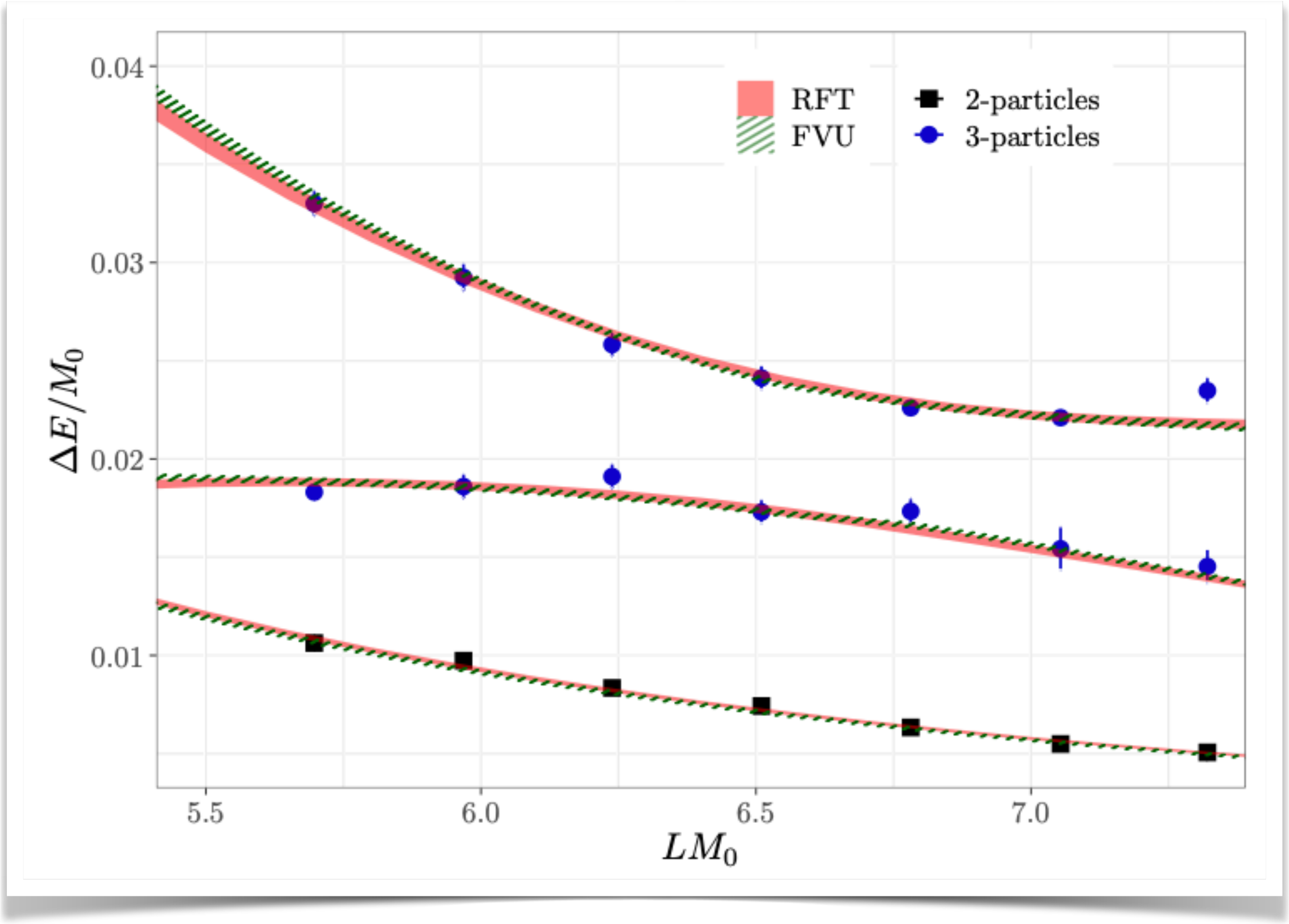


$g = 10$



AVOIDED LEVEL CROSSING

$g = 20$



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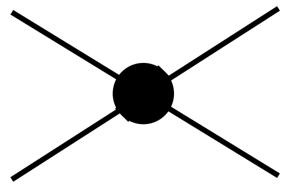

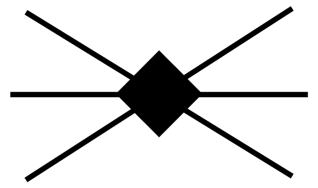

AVOIDED LEVEL CROSSING

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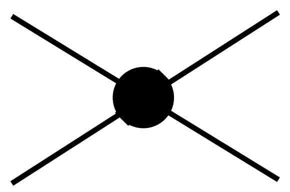
g		a	m_1	c_0	c_1	m'_1	c'_0	c'_1	$\chi^2_{\rm dof}$
5	FVU	−0.1512(9)	3.0229(1)	−0.0188(35)	–	–	–	–	2.9
	RFT	−0.1522(12)	–	–	–	3.0232(2)	31.6(8.4)	–	2.5
	FVU	−0.1569(12)	3.0233(2)	−0.0297(57)	2.29(38)	–	–	–	1.5
	RFT	−0.1571(10)	–	–	–	3.0237(2)	37.6(9.0)	2789(540)	1.5
10	FVU	−0.1521(11)	3.0205(2)	−0.0475(66)	–	–	–	–	1.7
	RFT	−0.1531(13)	–	–	–	3.0212(3)	80(14)	–	1.6
	FVU	−0.1549(16)	3.0205(2)	−0.0595(99)	0.93(41)	–	–	–	1.5
	RFT	−0.1563(27)	–	–	–	3.0213(3)	97(16)	1773(980)	1.4
20	FVU	−0.1444(11)	3.0184(2)	−0.1136(77)	–	–	–	–	1.6
	RFT	−0.1450(17)	–	–	–	3.0199(2)	178(17)	–	1.6
	FVU	−0.1464(14)	3.0183(2)	−0.1363(148)	0.84(39)	–	–	–	1.3
	RFT	−0.1484(16)	–	–	–	3.0200(2)	210(23)	2227(600)	1.2

$$q^* \cot\delta = \frac{1}{aM_0}$$



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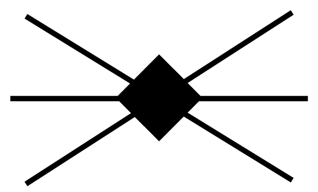


... same fit quality

... observables determined consistently

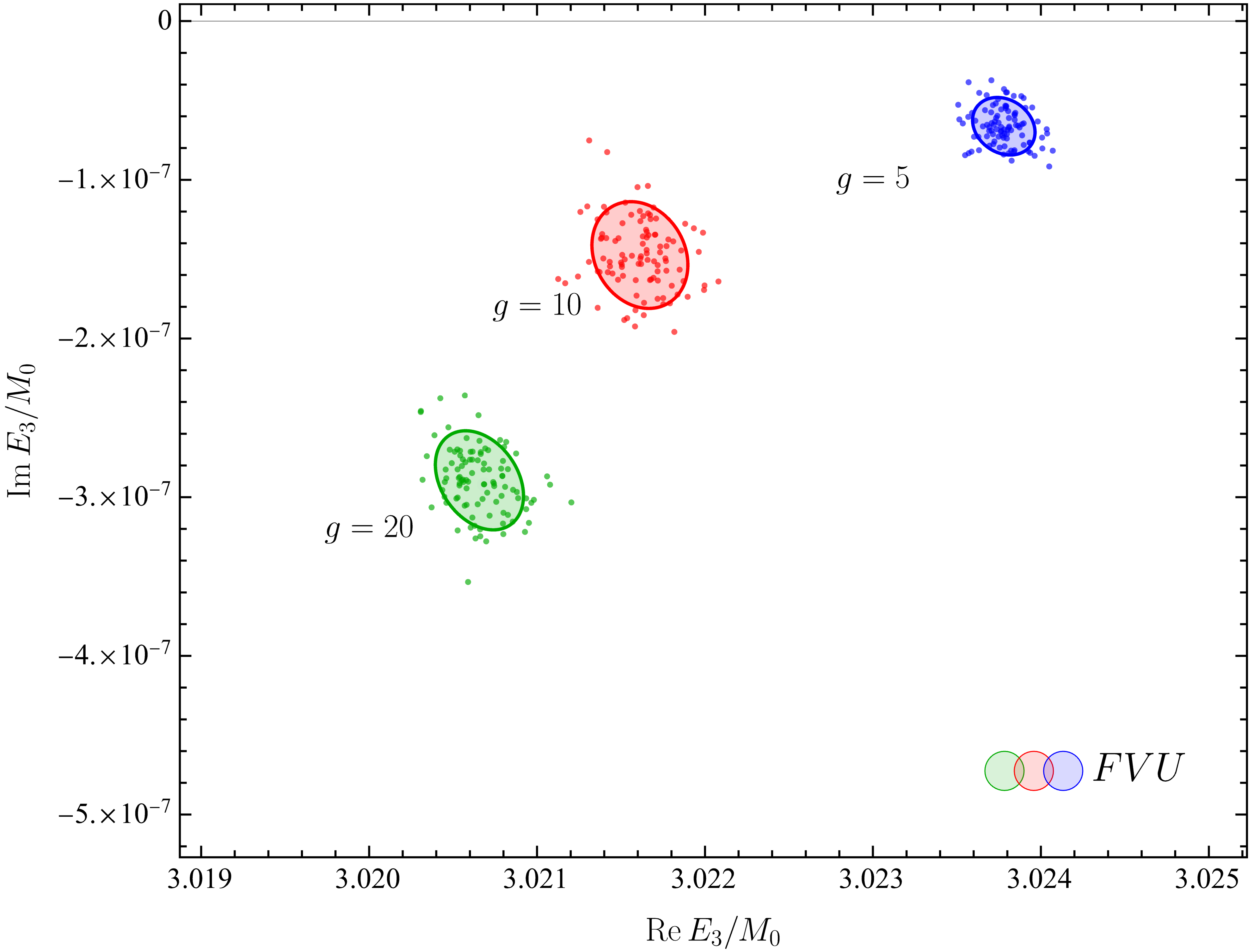
RESONANCE PARAMETERS

Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ g^2

  -- avoided level crossing gap >> width



1) Sadasivan/MM/.. *Phys.Rev.D* 101 (2020)

RESONANCE PARAMETERS

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  -- resonance width grows ~ ***g²***

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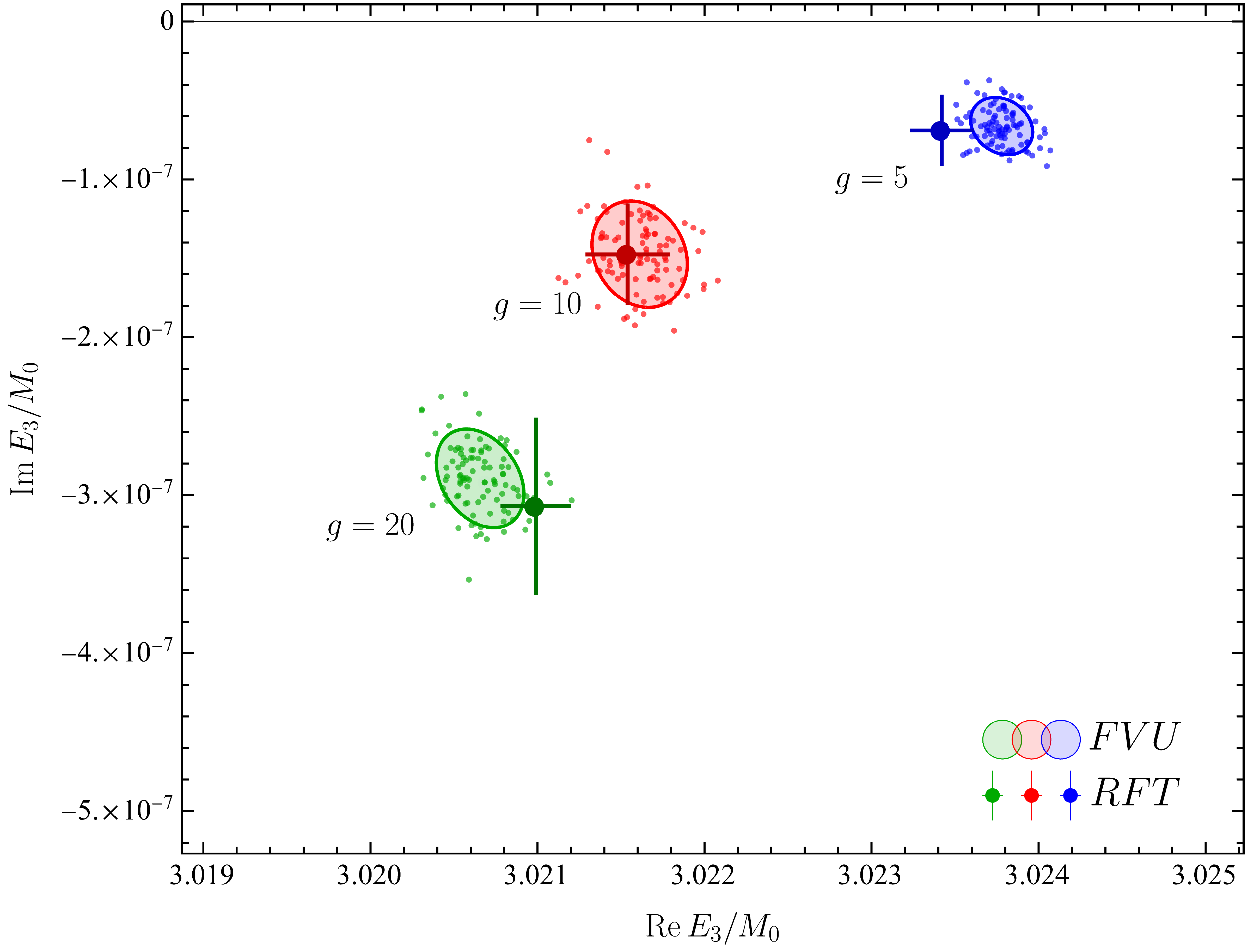
- Similarly from RFT with Breit-Wigner like approximation



1) Sadasivan/MM/.. *Phys.Rev.D* 101 (2020)

"what can we learn about 3-body resonances from theory and experiment?"

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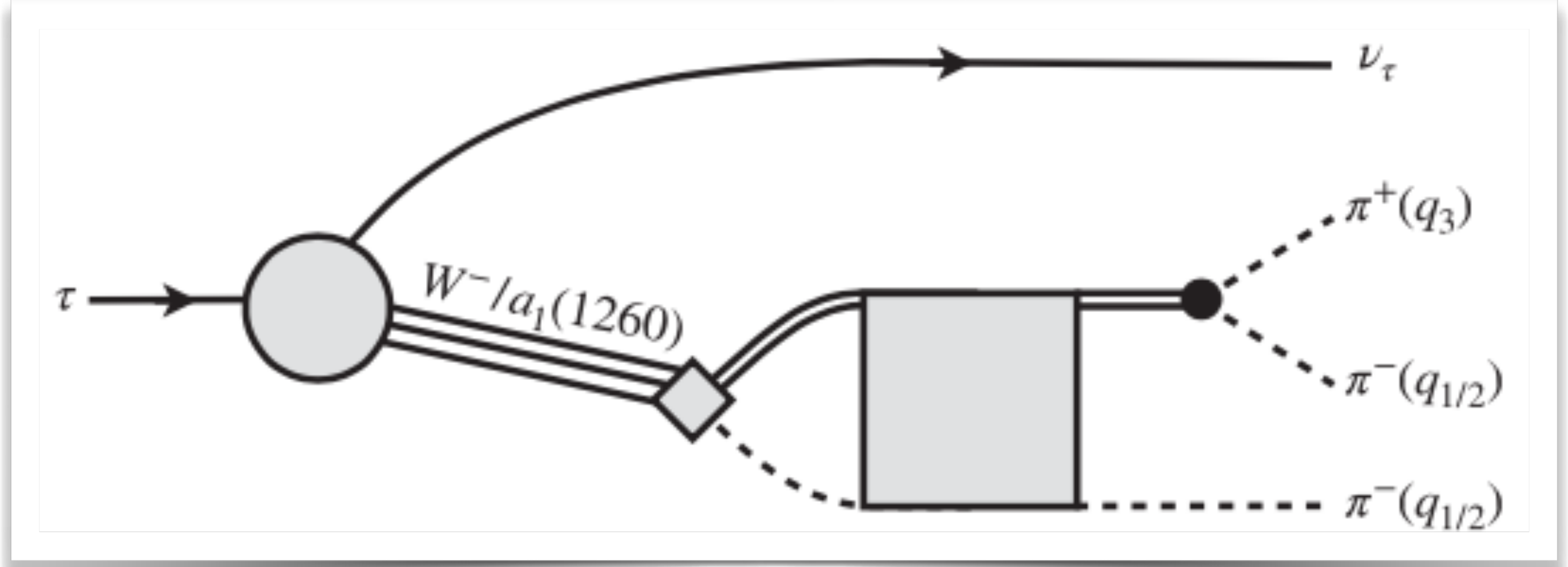
CASE 2

a_1(1260) from phenomenology and lattice QCD

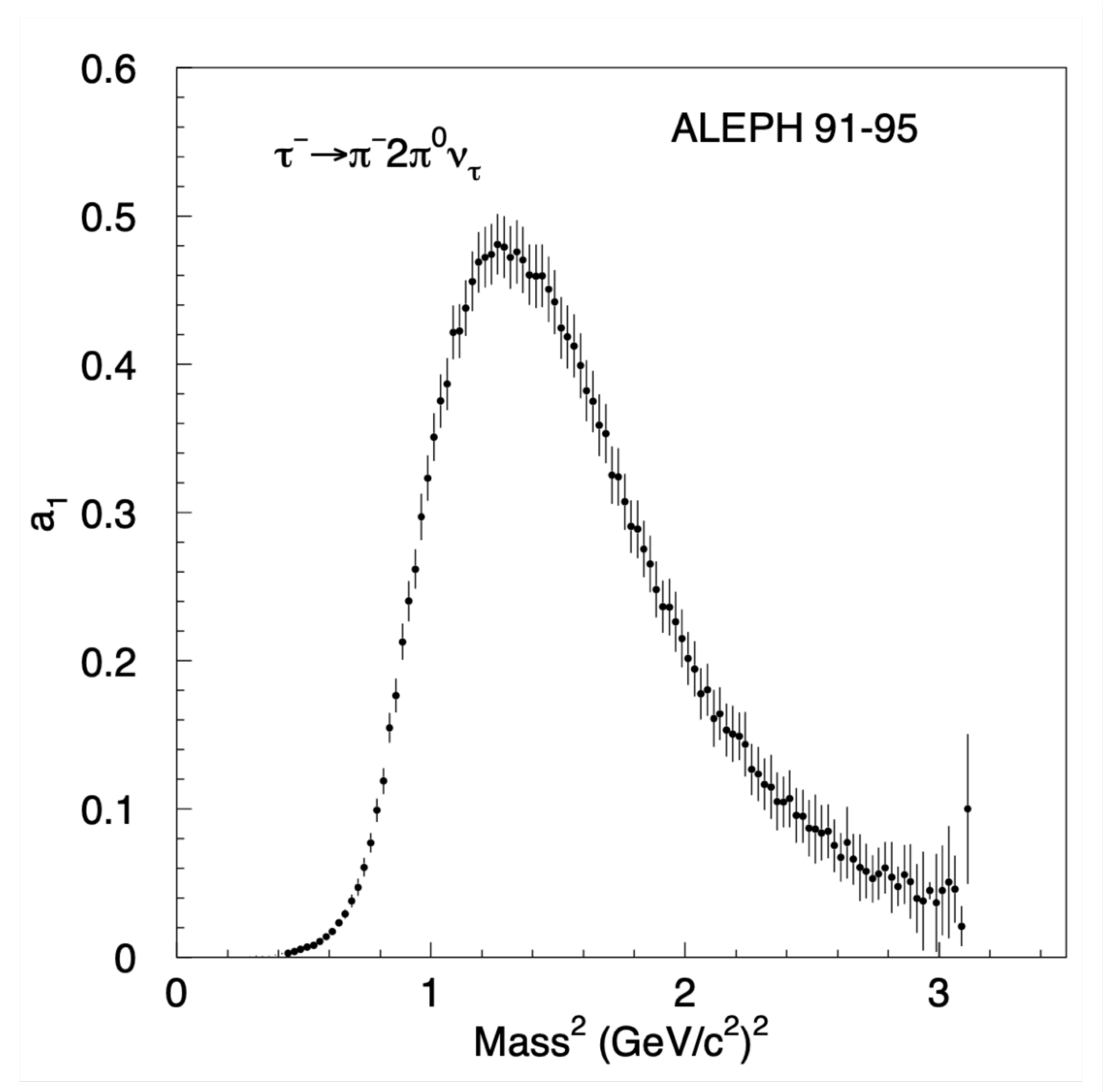
Sadasivan, MM, Akdag, Döring	Phys.Rev.D 101 (2020) 9
MM, Alexandru, Brett, Culver, Döring, Lee, Sadasivan	Phys.Rev.Lett. 127 (2021) 22
Sadasivan, Alexandru, Akdag, Amorim, Brett, Culver, Döring, Lee, MM	Phys.Rev.D 105 (2022) 5

$a_1(1260)$ PHENOMENOLOGY



Experimental data on 3π

- line-shape from tau-decays[1]
- new measurement on the way[2]

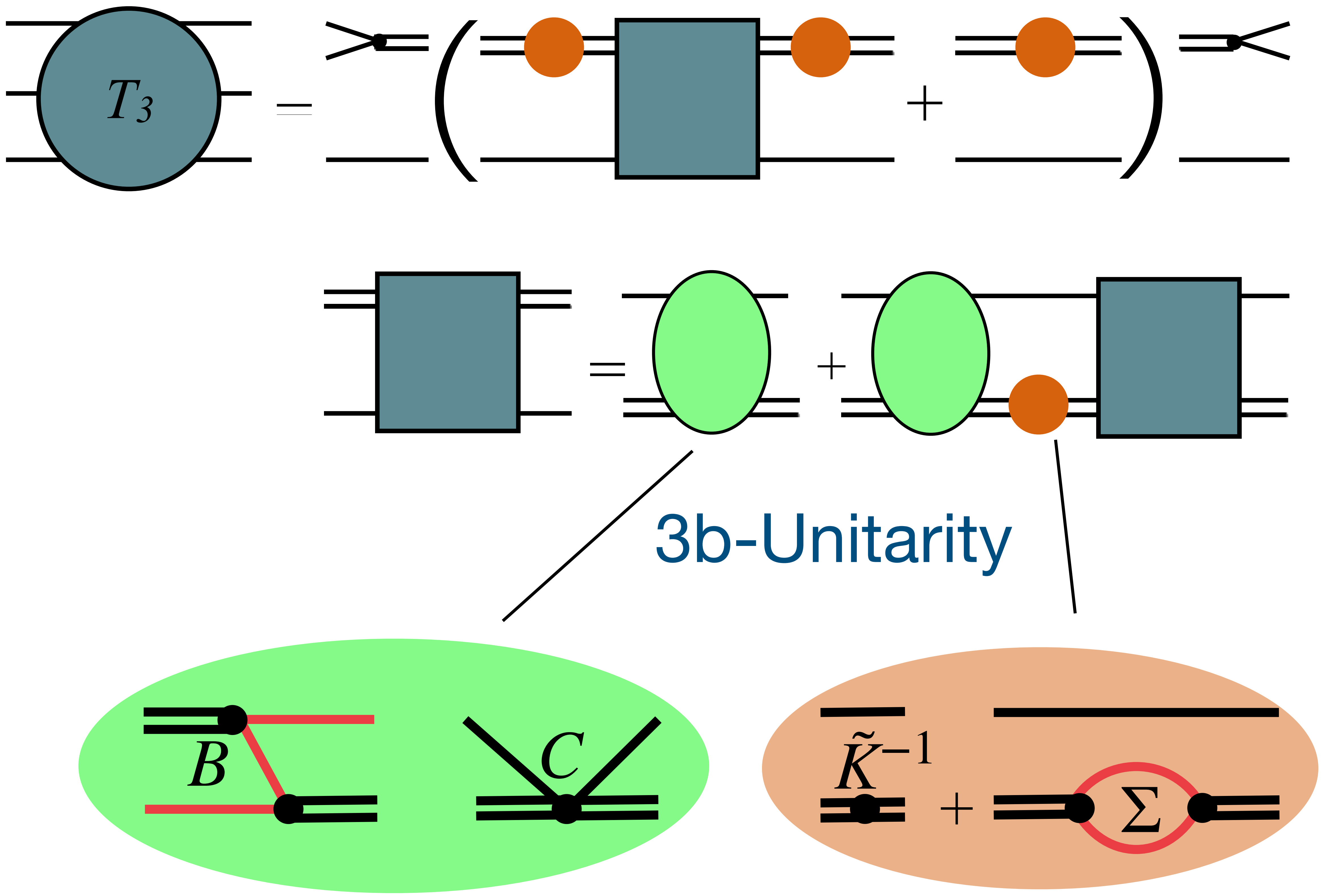


1) Schael [ALEPH] *Phys.Rept.* 421 (2005); Davier et al. [ALEPH] Eur.Phys.J.C 74
2) Priviate Communication: Stephan Paul (TUM)

a_1(1260) PHENOMENOLOGY

Three-body scattering amplitude[1,2]

- unitarity guided construction
- novel result from the S-matrix theory
- solution via complex momenta mapping



1) MM/Hu/Doring/... Eur.Phys.J.A 53 (2017)
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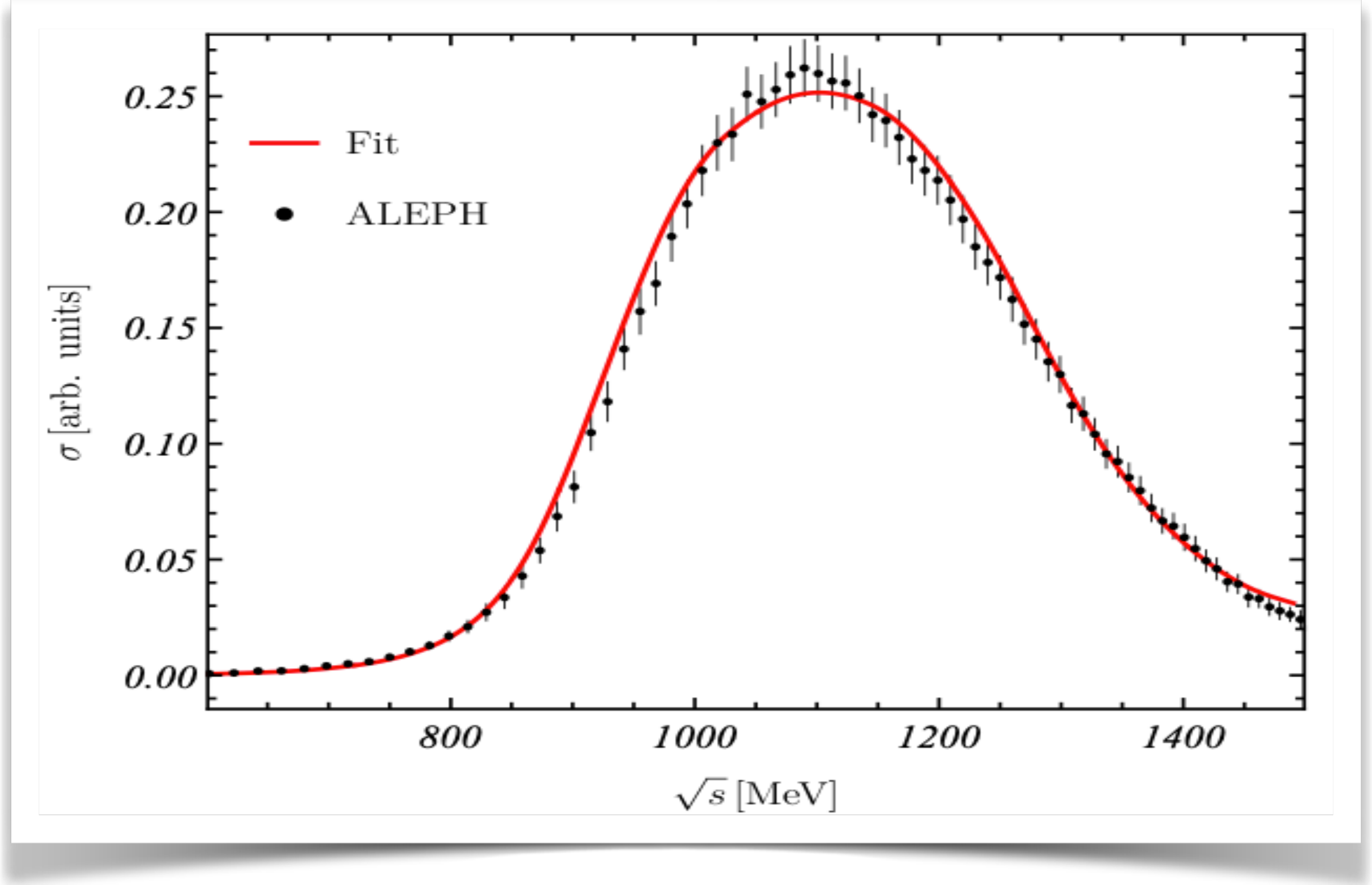

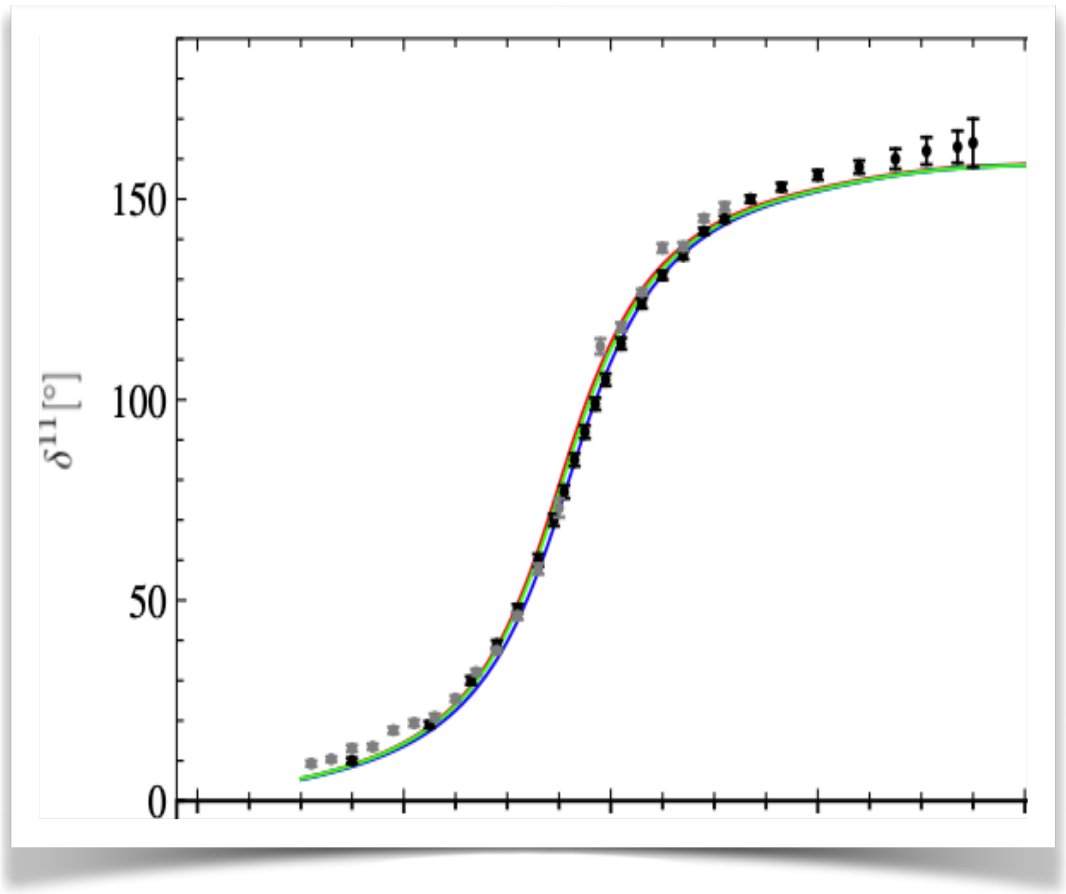

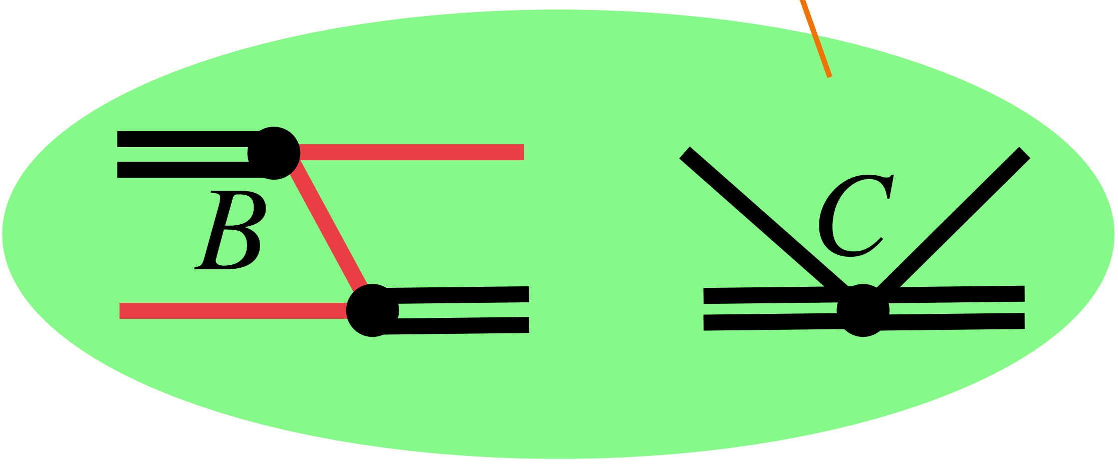

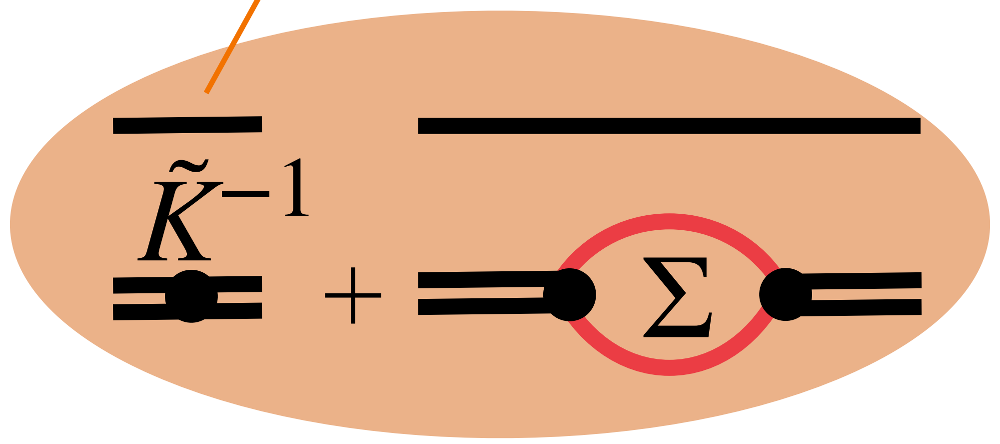

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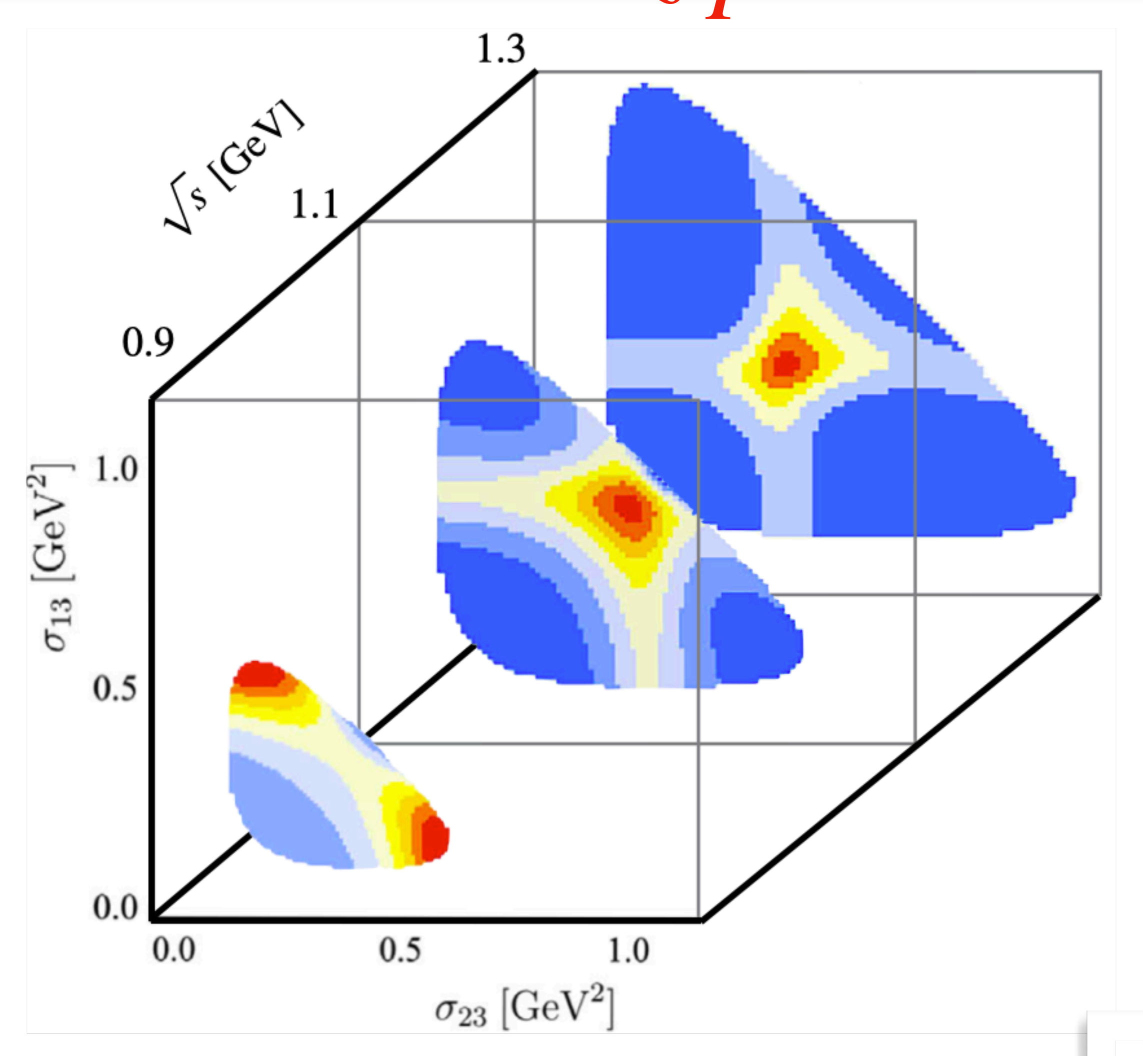
$a_1(1260)$ PHENOMENOLOGY

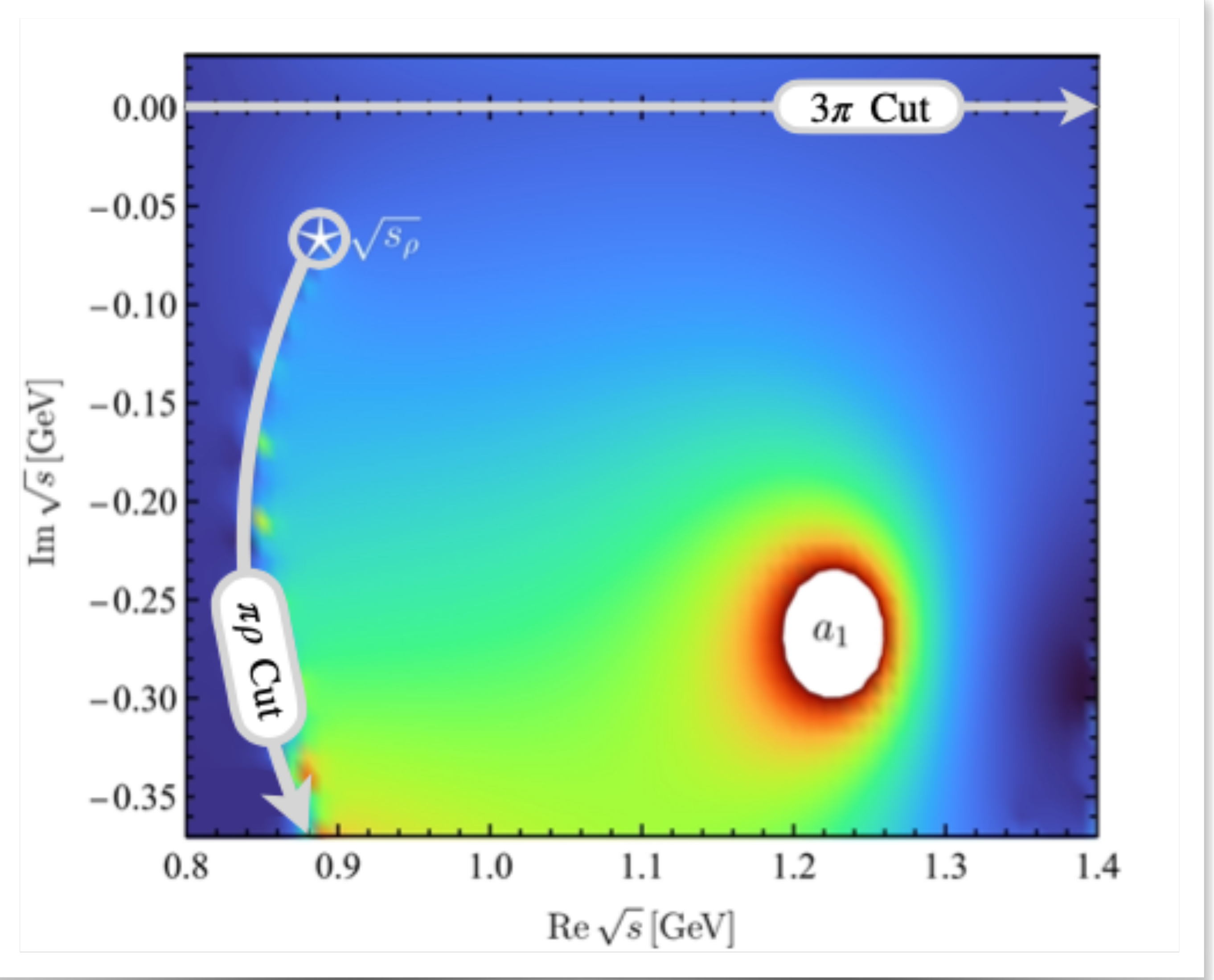
Predictions:

- other kinematics: Dalitz Plot
- complex energies:
  - > universal parameters of **$a_1(1260)$**

Dalitz plot



Universal parameters



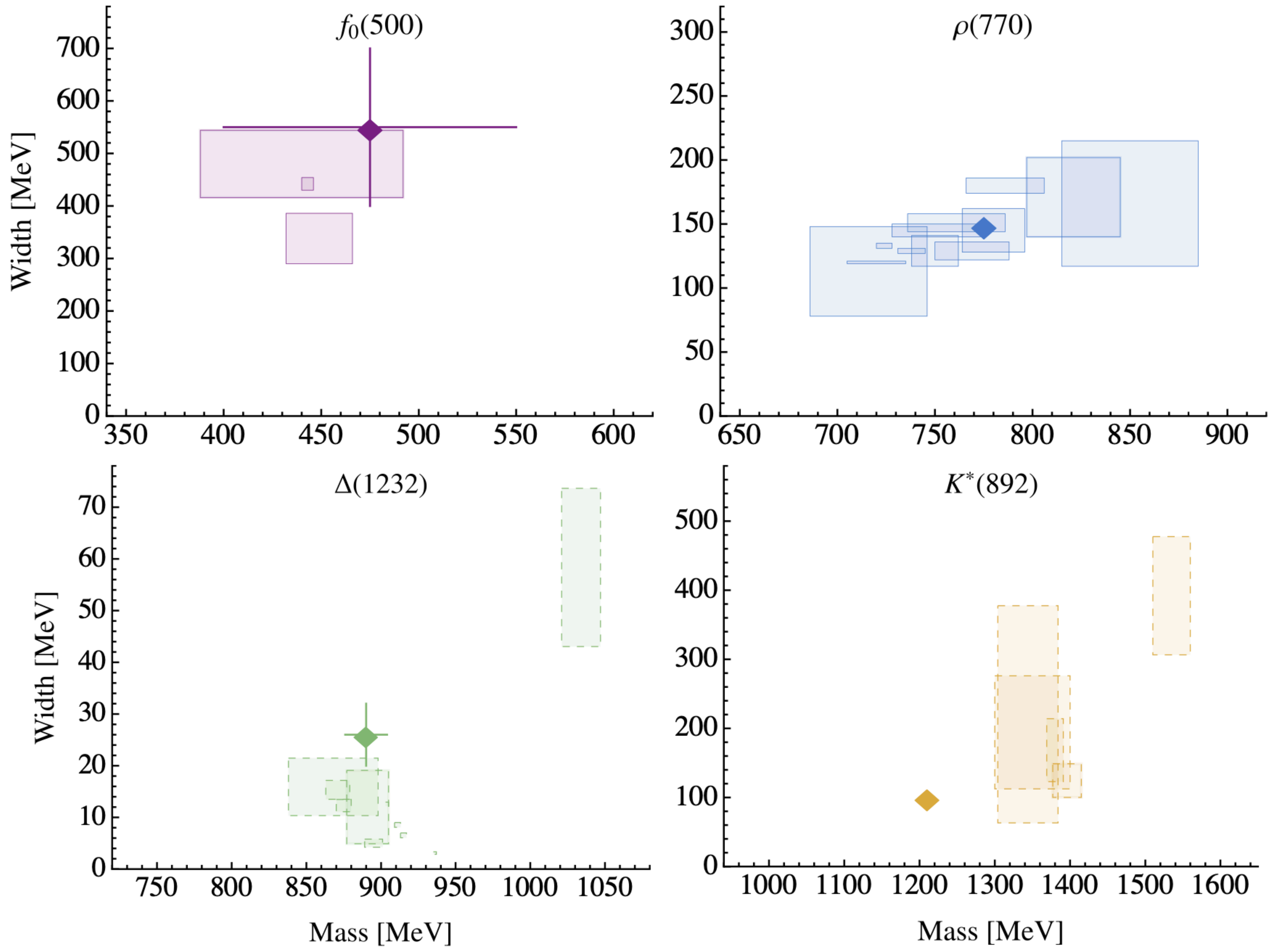
Sadasivan/MM/.. *Phys.Rev.D* 101 (2020)

$a_1(1260)$ LATTICE QCD

a_1(1260) LATTICE QCD

Previous studies

- many studies of 2-body systems[1]
- perturbative 3-body systems[2]: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...



MM/Meißner/Urbach 2206.01477 review in Phys. Rept.

1) [NPLQCD], [RQCD], [ETMC], [HadSpec], ...
Reviews: Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018)
MM/Meißner/Urbach 2206.01477 review in Phys. Rept.
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- perturbative 3-body systems[2]: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...

New

- first finite/infinite volume calculation of a resonant 3B system from lattice QCD[3]

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a_1(1260) LATTICE QCD

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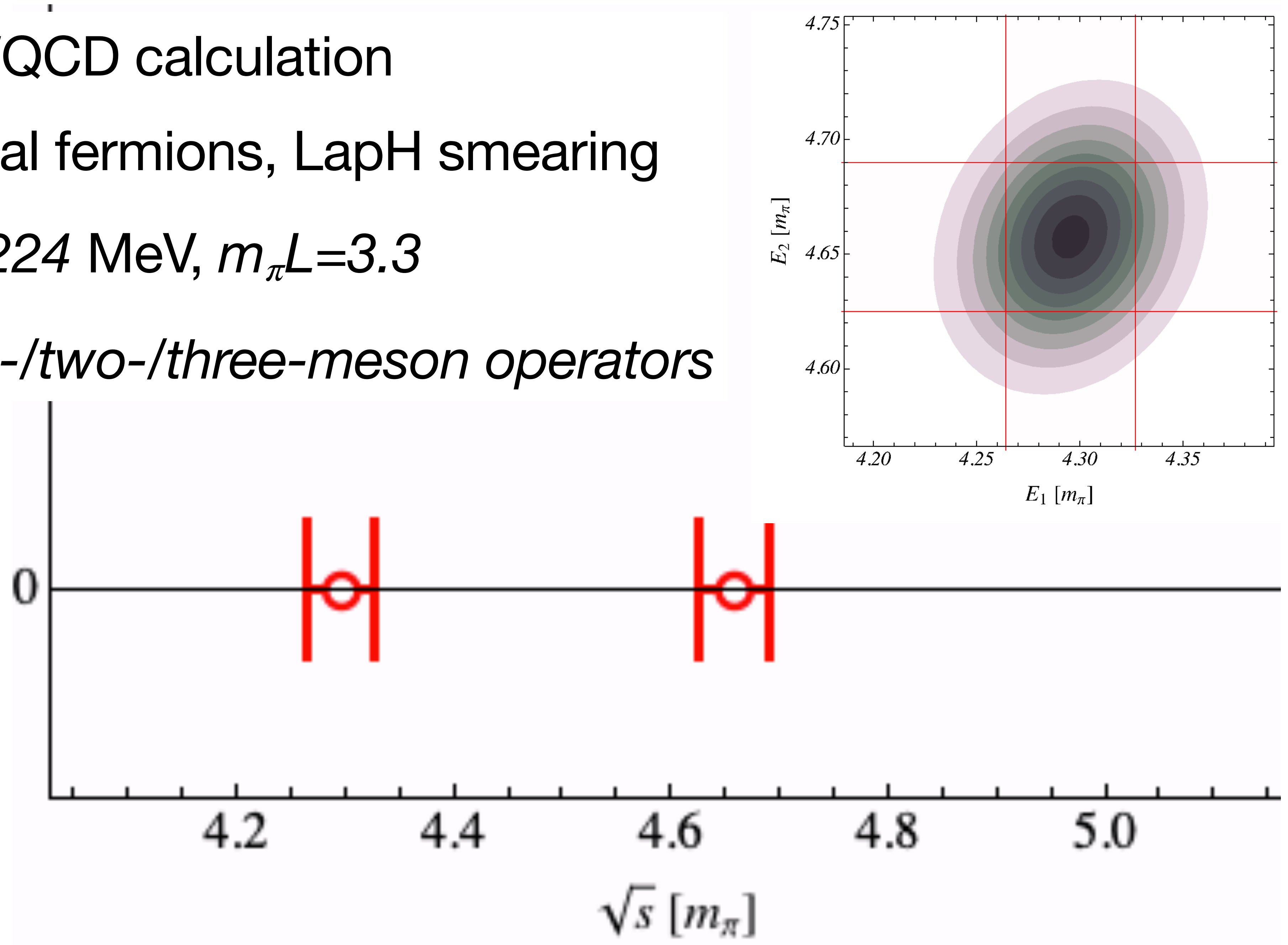
Key details of GWQCD calculation

N_f = 2 dynamical fermions, LapH smearing

$\boldsymbol{P}$=(0,0,0), m_π=224 MeV, $m_\pi L$=3.3

GEVP with one-/two-/three-meson operators



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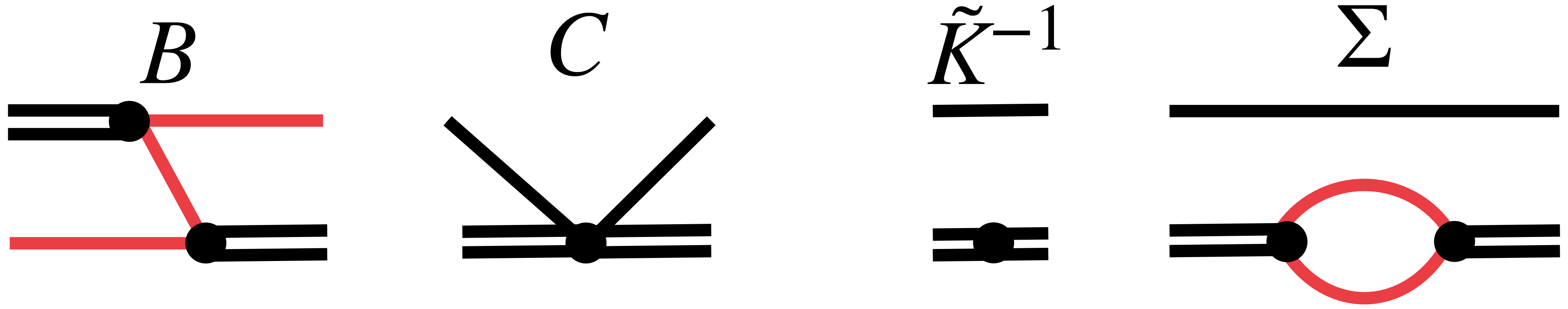
Unitarity determines 3-body scattering equation[1]

discontinuities ⇔ on-shell configurations



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a₁(1260) LATTICE QCD

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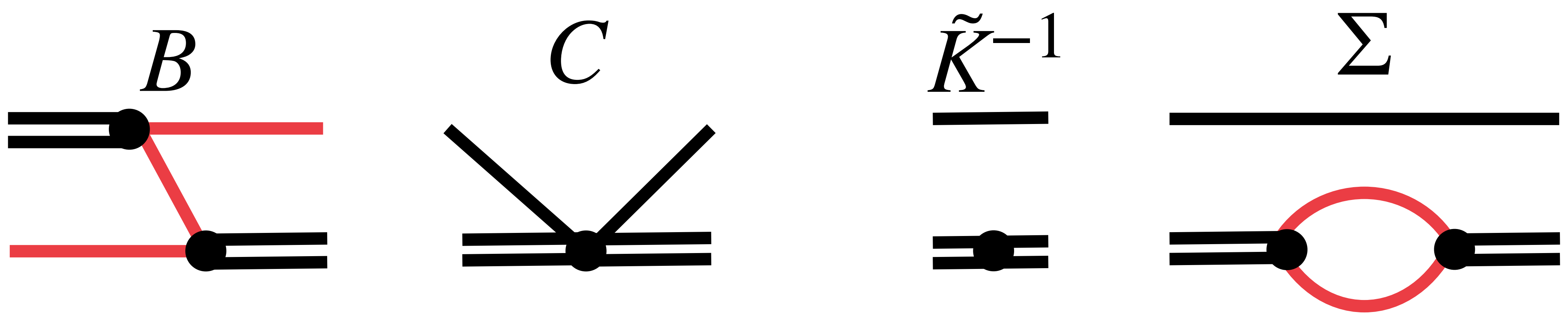

3-body Quantization Condition (FVU)[2]

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p}'\mathbf{p}}$$

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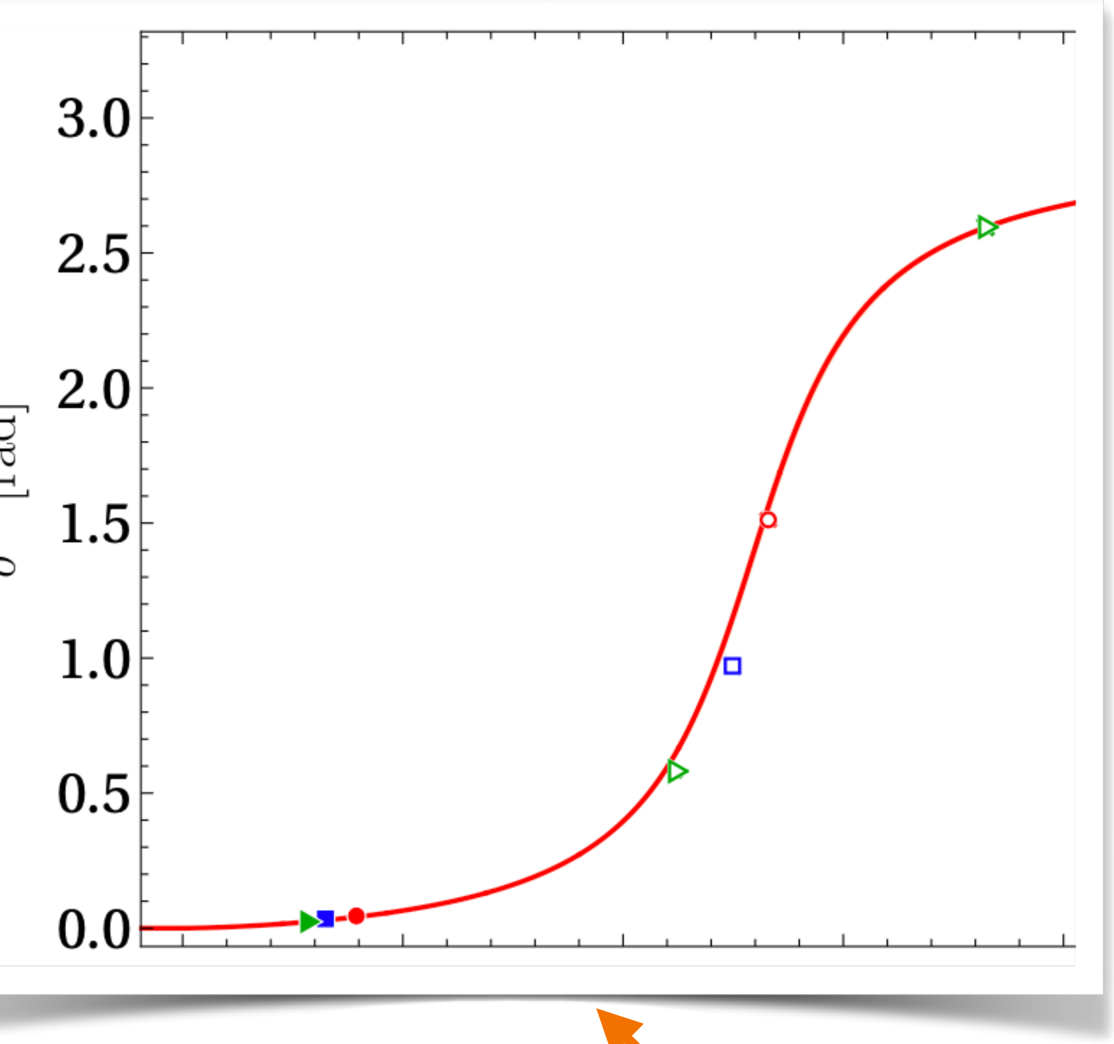
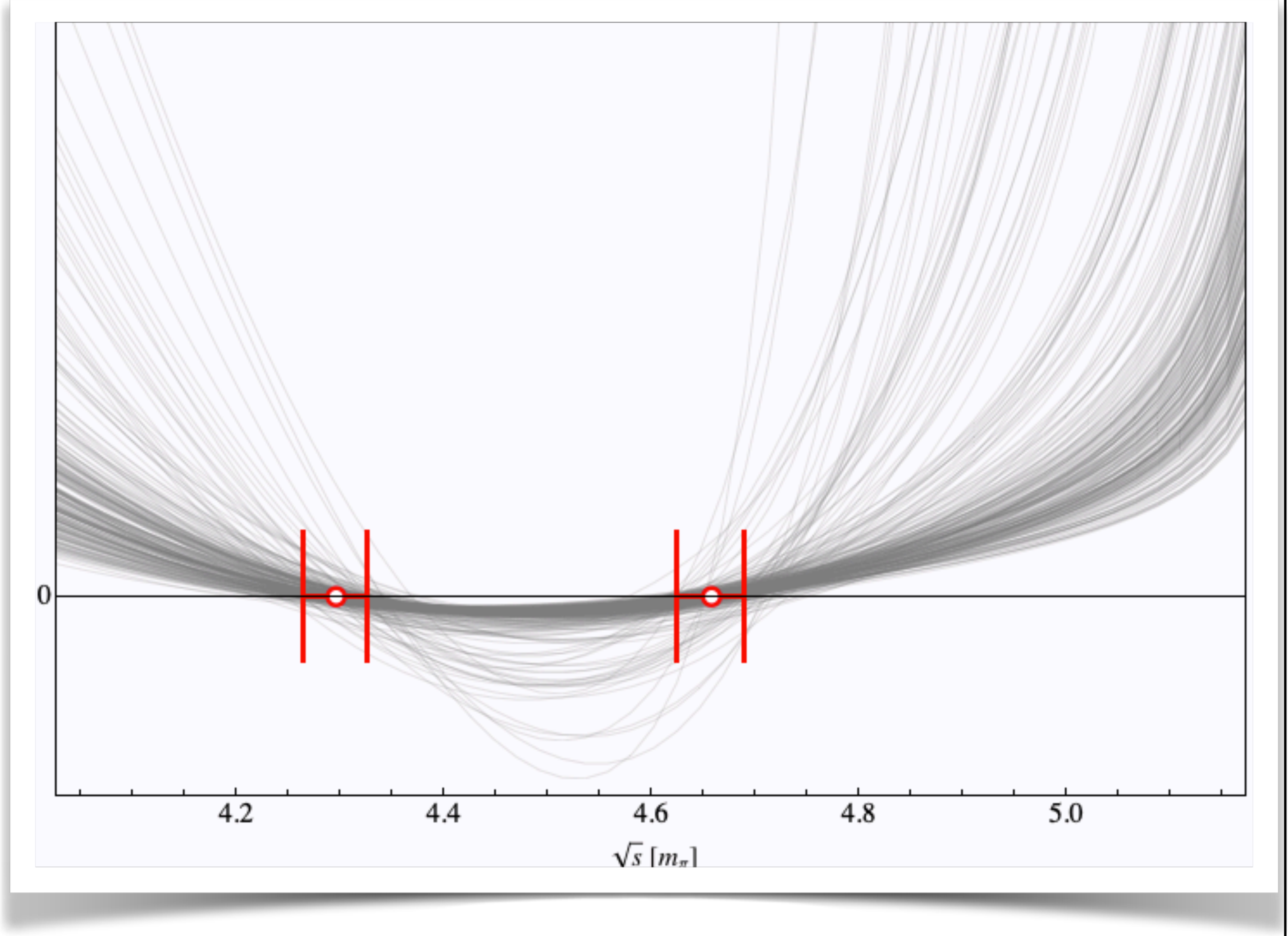
$a_1(1260)$ LATTICE QCD

[GWQCD] (2019)



MM et al. [GWQCD] Phys.Rev.Lett. 127



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$a_1(1260)$ LATTICE QCD

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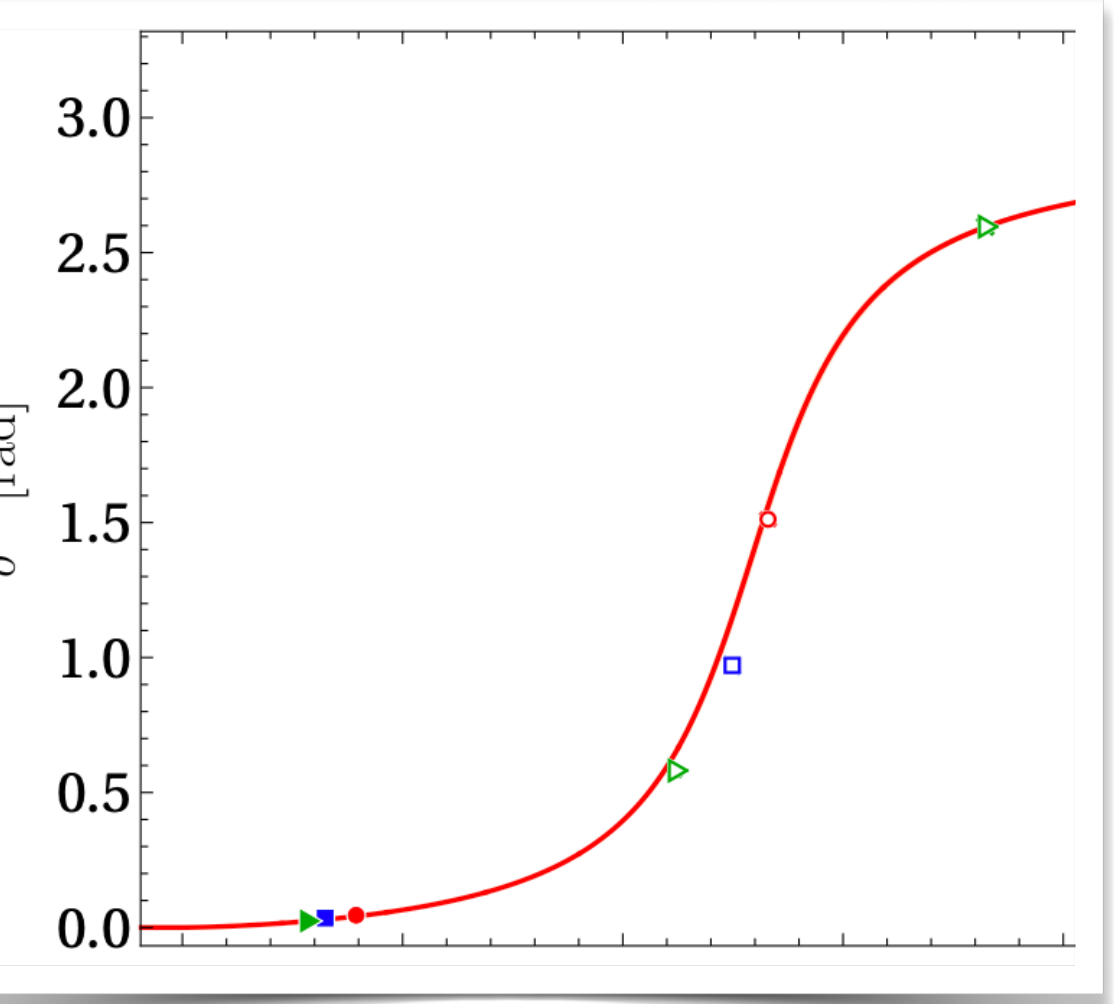

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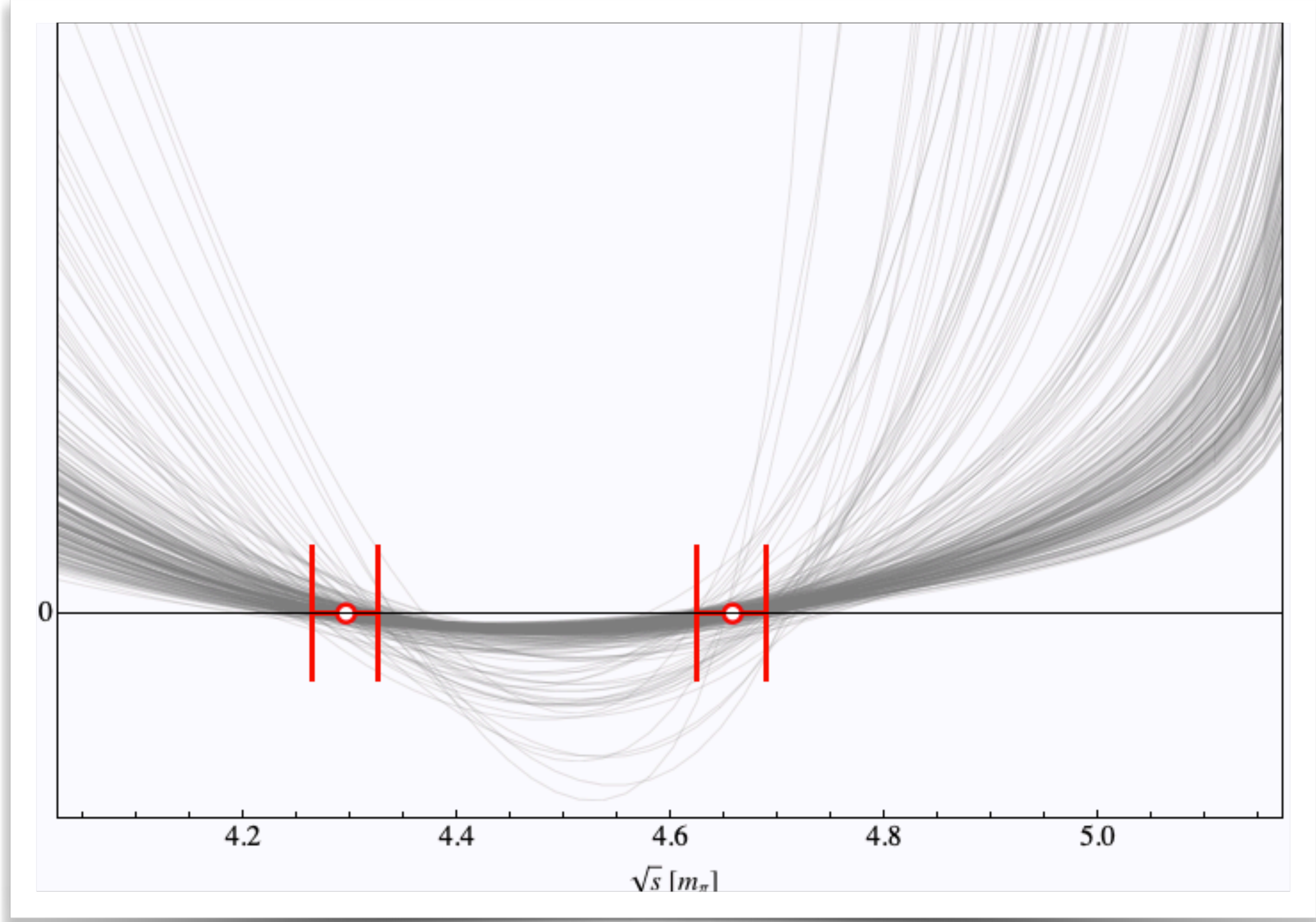
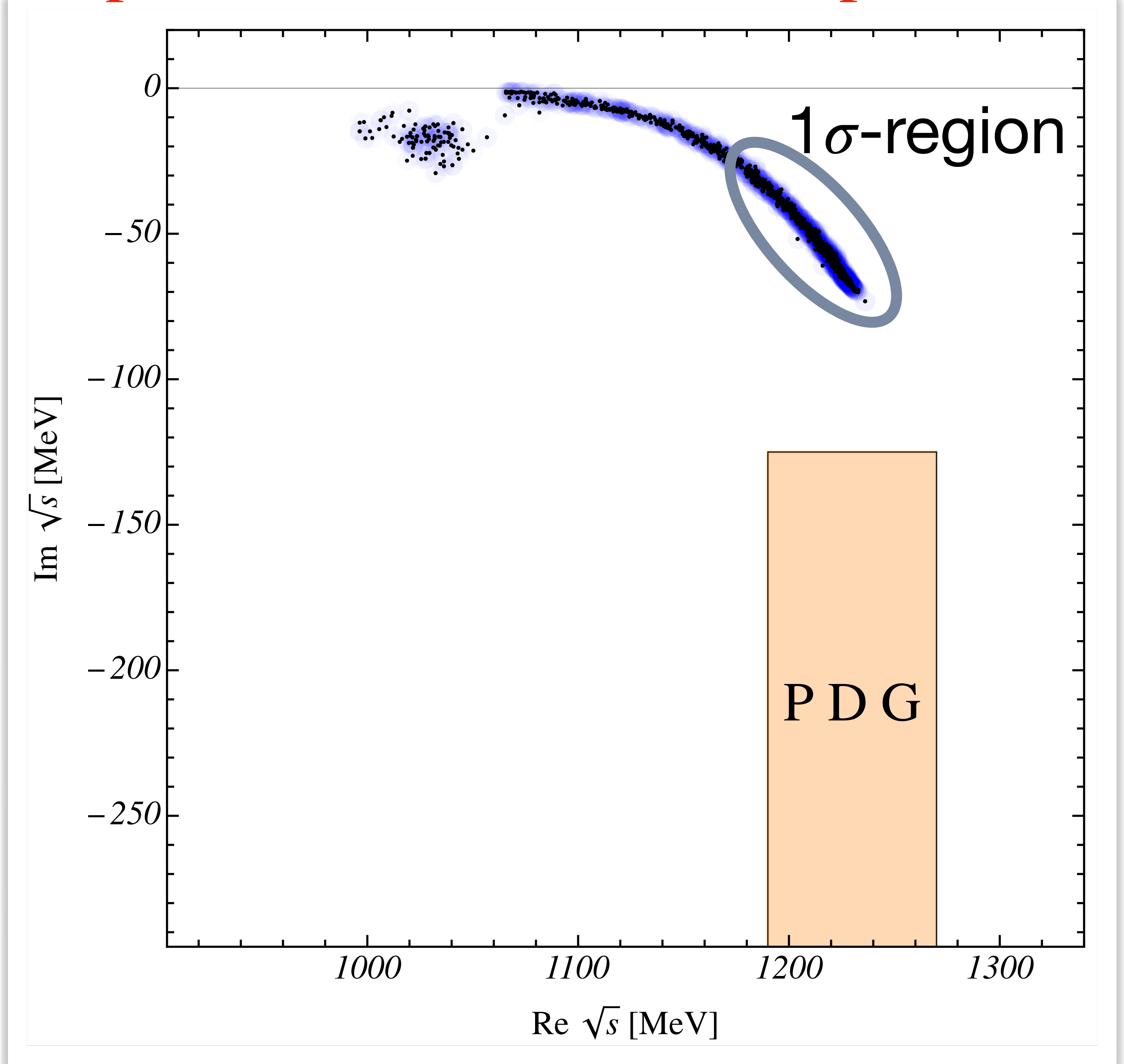


pole positions ➟ universal parameters



$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p'p}}$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1} - \Sigma_n}T^c$$

1) MM/Hu/Döring/... Eur.Phys.J.A 53 (2017)
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a_1(1260) LATTICE QCD

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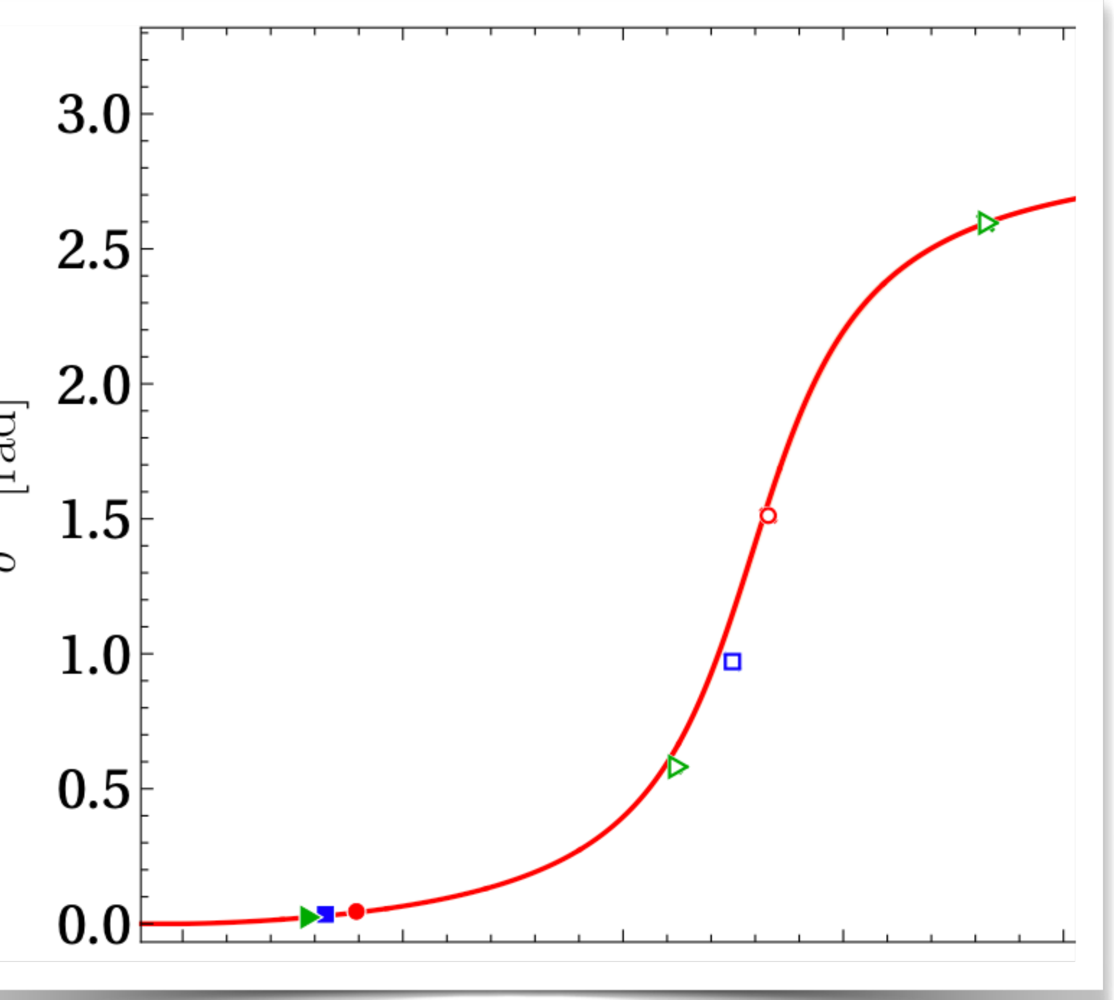

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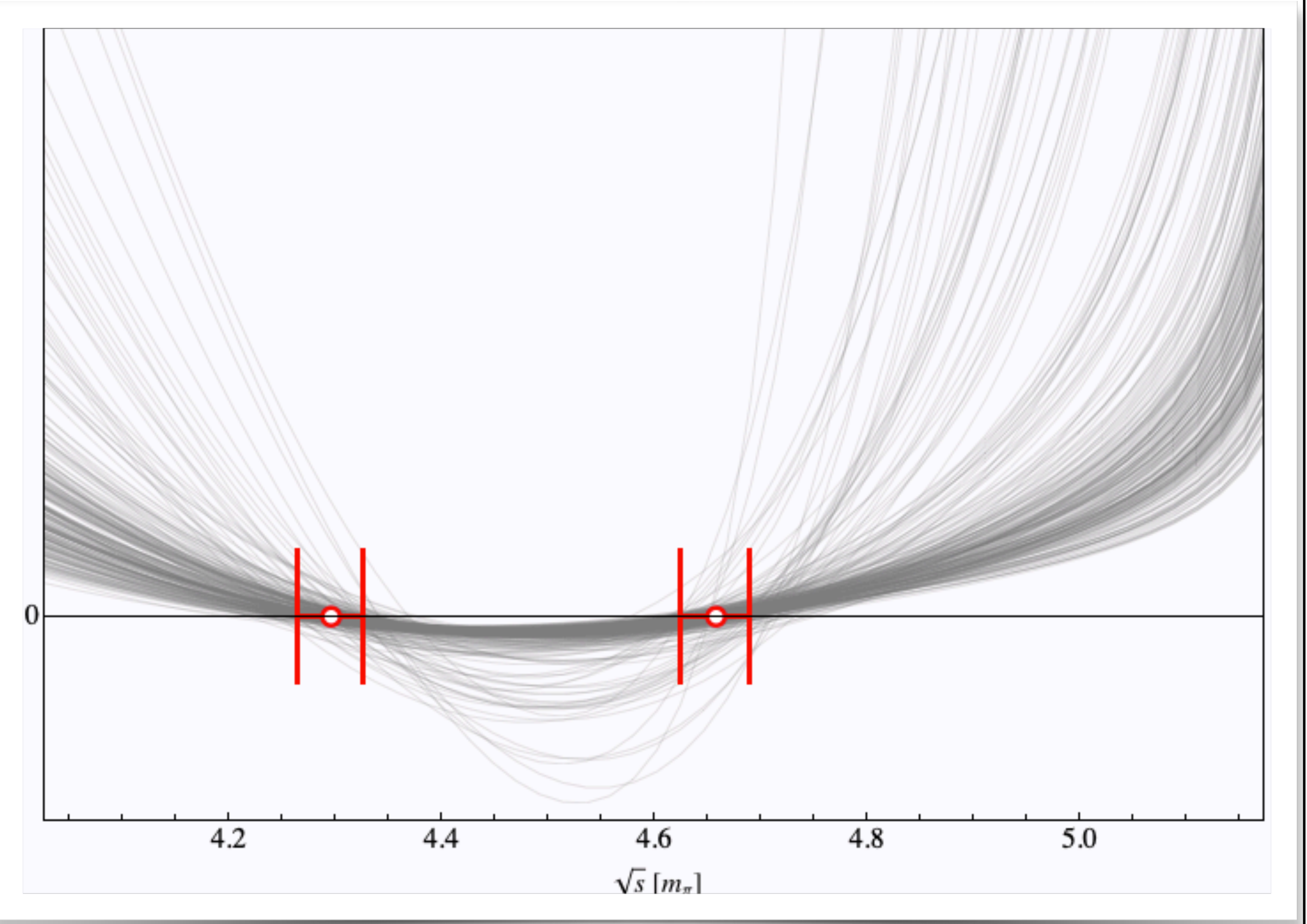

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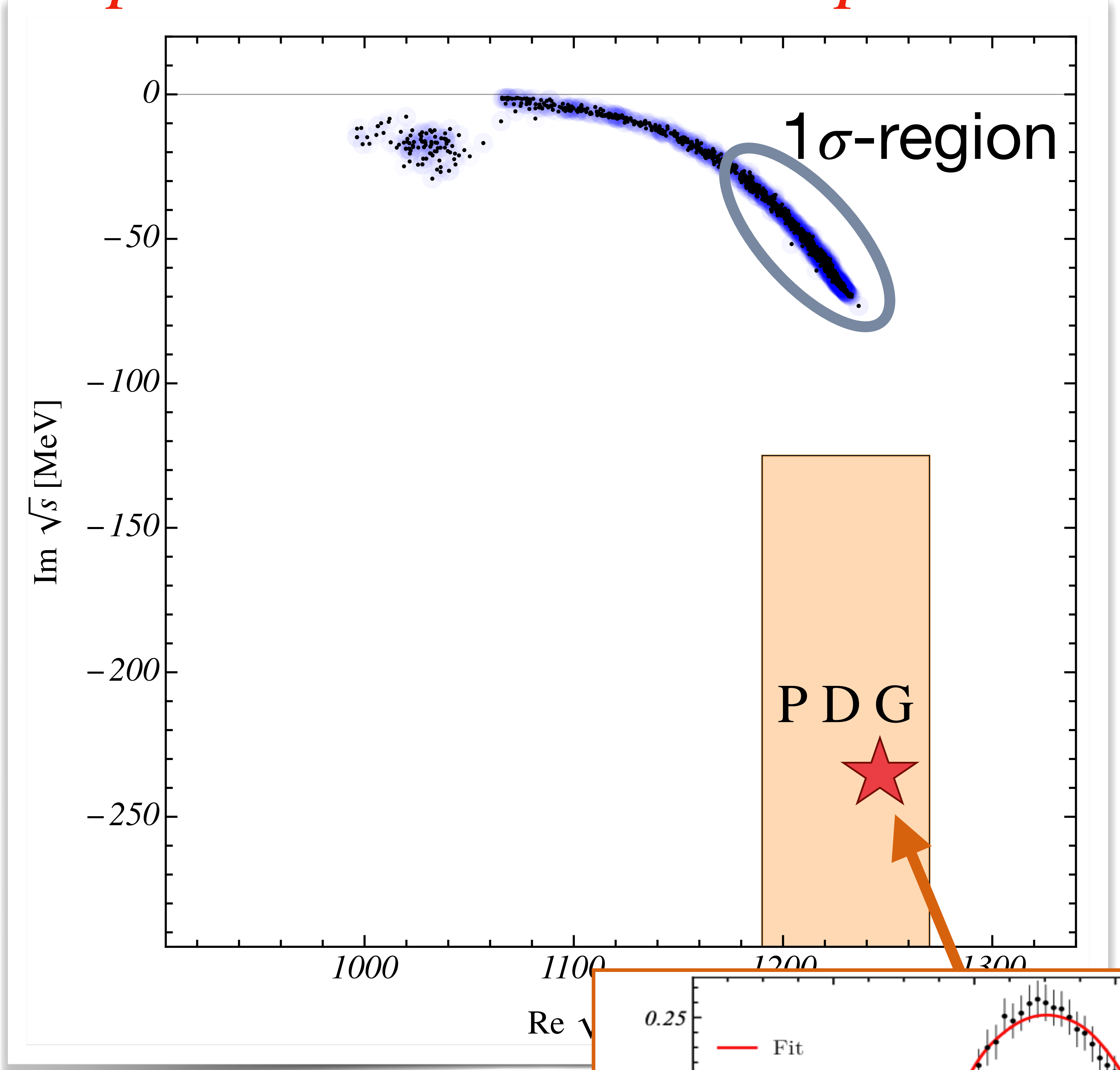
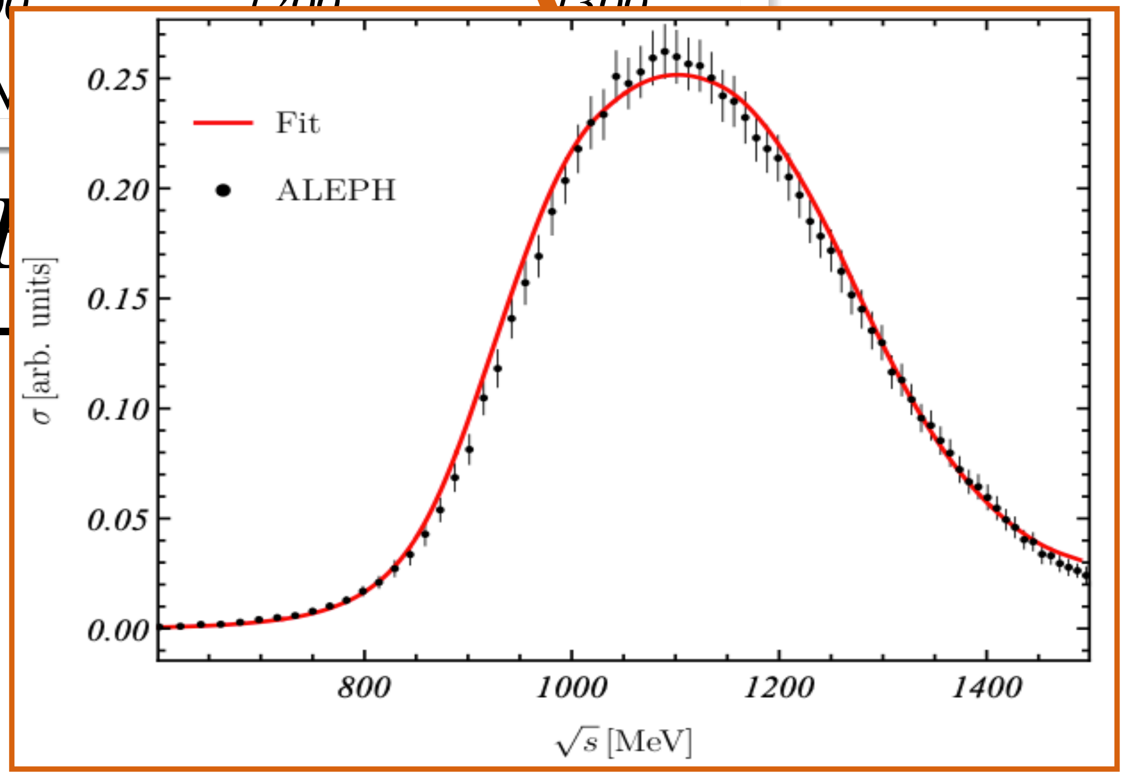


$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \ldots$$



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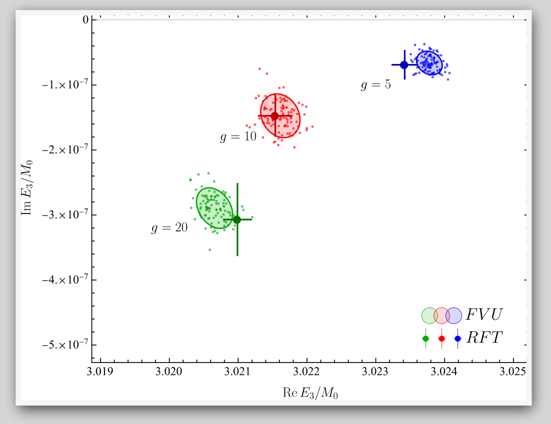
SUMMARY

"Entering new frontier in hadron spectroscopy from QCD"

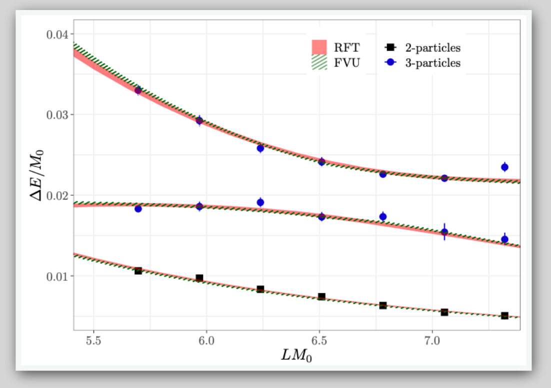
Explicit three-body resonance

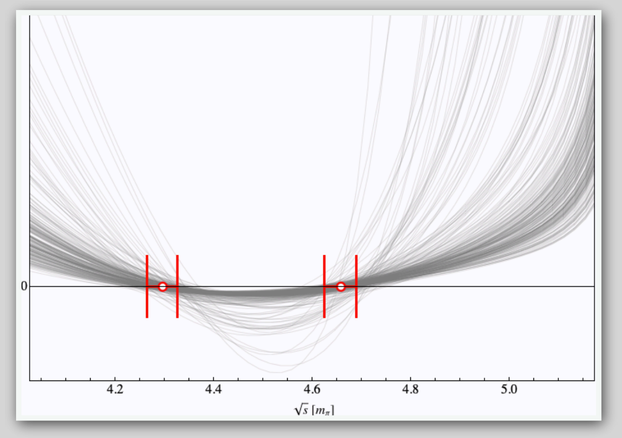


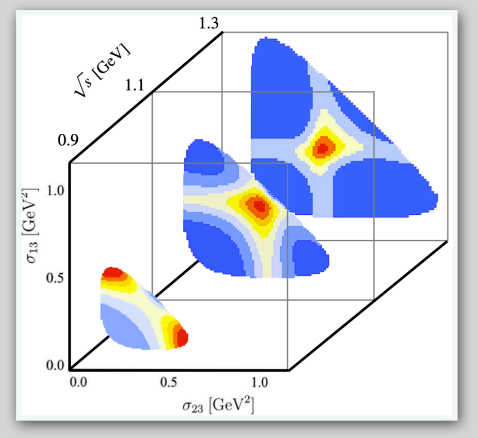
> clear example of avoided level crossing

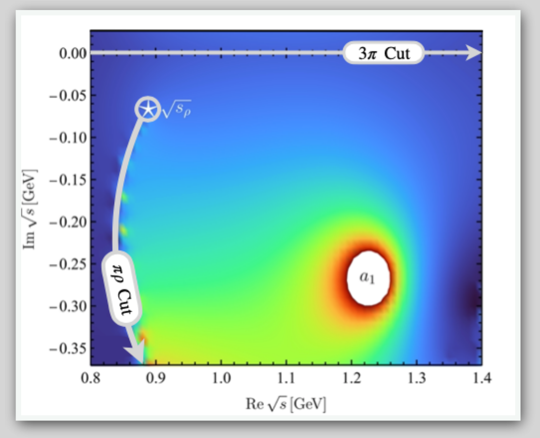
> width ~ (bare coupling)**2

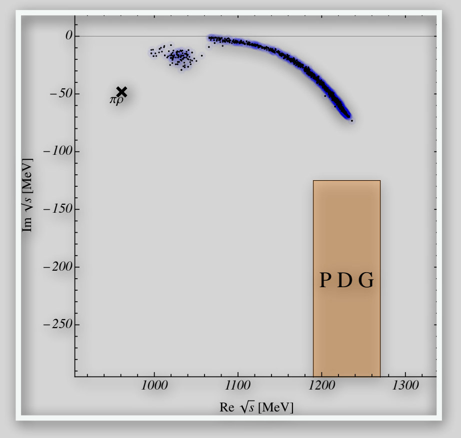
> RFT/FVU equivalence











First-ever determination of $a_1(1260)$ parameters from Lattice QCD

> Lattice QCD levels (1/2/3-meson operators)

> Dalitz plots and pole positions from experiment

> pole positions and couplings from lattice