Meson-baryon scattering
some aspects of

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MM, Bruns, Kubis, Meißner (2009)

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- *How?*
- *Results*
  - Scattering lengths
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  - Low-energy theorems
- *Summary*

Why?

Why hadrons?

- strong force $\xrightarrow{\text{Standard Model}}$ Quantum Chromodynamics
- low energy $\Rightarrow$ large coupling constant
- quarks confined in “composite objects“ $=$ hadrons

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Why meson-baryon(pion-nucleon) scattering?

- fundamental part of various processes
- large amount of data up to quite high energies

  $\hookrightarrow$ GWU: 30K data points for $\pi N \to \pi N$
- simplicity of the process

How?

- on-shell $\xrightarrow{\text{Lorentz invariance}}$ $T_{\phi B} = T_1 + \not{p} T_2$
- threshold $\xrightarrow{\text{s-wave}}$ $T_{\phi B}(s_{thr}) = T_1(s_{thr}) + \sqrt{s} T_2(s_{thr})$
- isospin invariance $\xrightarrow{\text{Wigner-Eckart theorem}}$ 26 lin. ind. channels
- scattering length: $a_{\phi B} = \frac{m_B}{4\pi(m_B+M_\phi)} T_{\phi B}(s_{thr})$

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Weinberg (1979), Gasser and Leutwyler (1983)

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- χPT in $SU(3)$ formulation:
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  - 1st chiral order: $\mathcal{L}^{(1)}_{\phi B} \longrightarrow$ WT and Born-graphs.
  - 2nd chiral order: $\mathcal{L}^{(2)}_{\phi B} \longrightarrow$ contact terms (11 LEC's $\longleftarrow$ FIT)

How?

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  $\mathcal{L}_{\phi B}^{(3)} \longrightarrow$ contact terms (13 LEC's $\leftarrow$ neglected)

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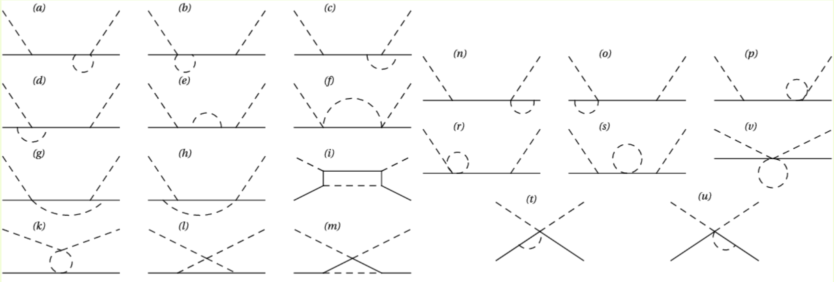
  $\mathcal{L}^{(3)}_{\phi B} \longrightarrow$ contact terms (13 LEC's $\leftarrow$ neglected)

  $\mathcal{L}^{(1)}_{\phi B}$, $\mathcal{L}^{(2)}_{\phi}$, $\mathcal{L}^{(4)}_{\phi} \longrightarrow$ wave function renormalization

  $\mathcal{L}^{(1)}_{\phi B} \longrightarrow$ one-loop diagrams (+crossed):



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- baryons carry intrinsic scale $m_0 \sim 1$ GeV (even if $m_{u,d,s} = 0$)

e.g.

$$H(p^2, M^2, m_0^2) = \frac{1}{i} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - M^2)((k-p)^2 - m_0^2)}$$

$$= \frac{\Gamma(2 - d/2)}{(4\pi)^{d/2}} \int_0^1 {\Delta_z}^{\frac{d}{2}-2} dz$$

$$\Delta_z = m_0^2 z^2 - 2m_0 M \frac{p^2 - M^2 - m_0^2}{2m_0 M} z(1-z) + M^2(1-z)^2$$

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- UV-divergences $\longmapsto \frac{1}{d-4}$ (*WI, unitarity, causality preserved*)

't Hooft, Veltman (1979)

How? ... infrared regularization

- IR-divergences $(M \to 0) \longmapsto$ small z

$$\underbrace{\int_0^1 (...) dz}_{\mathbf{H}} = \overbrace{\underbrace{\int_0^\infty (...) dz}_{\mathbf{I}}}^{M^{d-3}\left(c_0 + c_1 M + c_2 M^2 + ...\right)} - \overbrace{\underbrace{\int_1^\infty (...) dz}_{\mathbf{R}}}^{\left(d_0 + d_1 M + d_2 M^2 + ...\right)}$$

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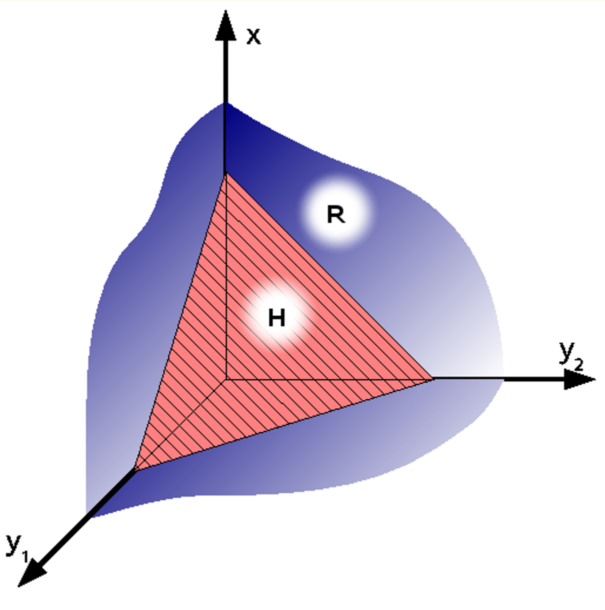
- (well separated chiral orders) + (manifest Lorentz invariance)
  $\rightsquigarrow$ *"infrared regularization"*

Becher, Leutwyler (1999)

How? ... infrared regularization



Scattering lengths

- renormalization scale 0.938 GeV$< \mu <$1.314 GeV
- F_ϕ, M_ϕ, D, F: physical values, $m_0 = 1.15$ GeV
- $\mathcal{O}(q^2)$ LECs:

$$\{b_0, b_D, b_F\} \text{ fit to } \{\sigma_{\pi N}, m_N, m_\Sigma, m_\Lambda, m_\Xi\}$$

Ellis, Torikoshi (2000), Bernard, Kaiser, Meißner (1993)

$$\{b_1, .., b_4, b_8, .., b_{11}\} \text{ fit to } \{a^+_{\pi N}, a^{(1)}_{KN}, a^{(0)}_{KN}\}/d_0$$

Schroeder[πN] (2001), Martin[KN] (1980)

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Channel	=	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)_{IR[HB]}$	$+\mathcal{O}(q^3)_{IR[HB]}$	$\sum_{IR[HB]}$	
$a^{(3/2)}_{\pi N}$	=	-0.12	$+0.05[+0.05]$	$+0.04[-0.06]$	$-0.04^{+0.07}_{-0.07}[-0.13^{+0.03}_{-0.03}]$	-0.13 ± 0.01
$a^{(1/2)}_{\pi N}$	=	$+0.21$	$+0.05[+0.05]$	$-0.19[+0.00]$	$+0.07^{+0.07}_{-0.07}[+0.26^{+0.03}_{-0.03}]$	-0.25 ± 0.03
$a^{(3/2)}_{\pi \Xi}$	=	-0.12	$+0.04[+0.04]$	$+0.10[-0.09]$	$+0.02^{+0.06}_{-0.07}[-0.17^{+0.03}_{-0.03}]$	
$a^{(1/2)}_{\pi \Xi}$	=	$+0.23$	$+0.04[+0.04]$	$-0.24[-0.03]$	$+0.02^{+0.08}_{-0.10}[+0.23^{+0.03}_{-0.03}]$	
$a^{(2)}_{\pi \Sigma}$	=	-0.24	$+0.10[+0.07]$	$+0.15[-0.07]$	$+0.01^{+0.04}_{-0.04}[-0.24^{+0.01}_{-0.01}]$	
$a^{(1)}_{\pi \Sigma}$	=	$+0.22$	$+0.09[+0.11]$	$-0.21[+0.00]$	$+0.10^{+0.16}_{-0.17}[+0.33^{+0.06}_{-0.06}]$	
$a^{(0)}_{\pi \Sigma}$	=	$+0.46$	$+0.11[-0.01]$	$-0.47[+0.04]$	$+0.10^{+0.17}_{-0.19}[+0.49^{+0.07}_{-0.08}]$	
$a^{(1/2)}_{\pi \Lambda}$	=	-0.01	$+0.03[+0.03]$	$-0.03[-0.11]$	$-0.01^{+0.04}_{-0.04}[-0.09^{+0.01}_{-0.01}]$	

Scattering lengths

Channel	=	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)_{IR}$	$+\mathcal{O}(q^3)_{IR}$	$\sum_{IR}$	
$a^{(1)}_{KN}$	=	-0.45	$+0.60$	-0.48	$-0.33^{+0.32}_{-0.32}$	-0.33
$a^{(0)}_{KN}$	=	$+0.04$	-0.15	$+0.13$	$+0.02^{+0.64}_{-0.64}$	$+0.02$
$a^{(1)}_{\bar{K}N}$	=	$+0.20$	$+0.22$	$-0.26 + 0.18i$	$+0.16^{+0.39}_{-0.44} + 0.18i$	$+0.37 + 0.60i$
$a^{(0)}_{\bar{K}N}$	=	$+0.53$	$+0.97$	$-0.40 + 0.22i$	$+1.11^{+0.47}_{-0.59} + 0.22i$	$-1.70 + 0.68i$
$a^{(3/2)}_{K\Sigma}$	=	-0.31	$+0.33$	$-0.30 + 0.12i$	$-0.28^{+0.52}_{-0.49} + 0.12i$	
$a^{(1/2)}_{K\Sigma}$	=	$+0.47$	$+0.19$	$+0.20 + 0.01i$	$+0.87^{+0.55}_{-0.64} + 0.01i$	
$a^{(3/2)}_{\bar{K}\Sigma}$	=	-0.22	$+0.24$	$-0.35 + 0.08i$	$-0.33^{+0.44}_{-0.47} + 0.08i$	
$a^{(1/2)}_{\bar{K}\Sigma}$	=	$+0.34$	$+0.38$	$+0.27 + 0.01i$	$+0.98^{+0.59}_{-0.59} + 0.01i$	
$a^{(1)}_{K\Xi}$	=	$+0.15$	$+0.34$	$-0.02 + 0.17i$	$+0.48^{+0.43}_{-0.43} + 0.17i$	
$a^{(0)}_{K\Xi}$	=	$+0.66$	$+0.98$	$-0.62 + 0.14i$	$+1.02^{+0.51}_{-0.68} + 0.14i$	
$a^{(1)}_{\bar{K}\Xi}$	=	-0.50	$+0.66$	-0.42	$-0.26^{+0.34}_{-0.34}$	
$a^{(0)}_{\bar{K}\Xi}$	=	-0.15	$+0.02$	$+0.13$	$+0.00^{+0.78}_{-0.68}$	
$a^{(1/2)}_{K\Lambda}$	=	-0.04	$+0.50$	$-0.27 + 0.14i$	$+0.19^{+0.55}_{-0.56} + 0.14i$	
$a^{(1/2)}_{\bar{K}\Lambda}$	=	-0.05	$+0.5$	$-0.40 + 0.18i$	$+0.04^{+0.55}_{-0.56} + 0.18i$	
$a^{(1/2)}_{\eta N}$	=	-0.01	$+0.26$	$-0.13 + 0.19i$	$+0.13^{+0.60}_{-0.65} + 0.19\ i$	$+0.62 + 0.30i$
$a^{(1/2)}_{\eta\Xi}$	=	-0.09	$+0.84$	$-0.49 + 0.17i$	$+0.25^{+0.74}_{-0.73} + 0.17i$	
$a^{(1)}_{\eta\Sigma}$	=	-0.04	$+0.22$	$-0.15 + 0.13i$	$+0.03^{+0.24}_{-0.24} + 0.13i$	
$a^{(0)}_{\eta\Lambda}$	=	-0.04	$+0.70$	$-0.51 + 0.38i$	$+0.15^{+0.51}_{-0.55} + 0.38i$	$+0.64 + 0.80i$

Scattering lengths ... reordering

- very slow (no) convergence

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$\rightsquigarrow$ reordering of the chiral series:

$$a = a^{(0)} + a^{(1)} + a^{(2)} + \ldots \qquad \text{and} \qquad b = b^{(0)} + b^{(1)} + b^{(2)} + \ldots$$

$$\Rightarrow \mathcal{L}_{\phi B} = \mathcal{L}_{\phi B}^{(1)}\Big|_{a \to a^{(0)}} + \left(\mathcal{L}_{\phi B}^{(1)}\Big|_{a \to a^{(1)}} + \mathcal{L}_{\phi B}^{(2)}\Big|_{b \to b^{(0)}} \right) + \ldots$$

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baryon masses:

$m_\Lambda = (767 + 529 - 0.591 + 414)\text{MeV} \rightarrow m_\Lambda = (1150 + 39 + 1 + 8)\text{MeV}$

Mojzis, Kambor (2000)

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scattering lengths:

- case by case improvement only
- reason: no leading order fitting parameter

Matching to SU(2)

Why?

- Relation zwischen SU(2) und SU(3) LECs

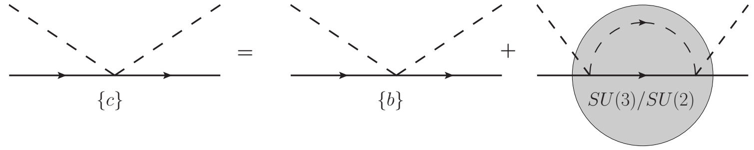
Matching to $SU(2)$

Why?

- Relation zwischen SU(2) und SU(3) LECs

How?

- integrate out strange quark
  EFT(three-flavor) $\longrightarrow$ EFT(two-flavor)



Matching to SU(2)

How?

- double-scale expansion: $m_0 \gg M_K \gg M_\pi$
  1. IR-regularized loop integrals in three-flavor formulation
  2. expand in $\{(t - 2M_\pi^2),\, M_\pi^2,\, (s - m_0)^2\}$
  3. expand in $\{M_K\}$ to first order

- four different structures:

$$\{\sigma^{\mu\nu}, \mathbb{1}\} \otimes \{(t - 2M_\pi^2),\, M_\pi^2,\, (s - m_0)^2\}$$

Result:

$$c_1 = b_0 + \frac{b_D}{2} + \frac{b_F}{2} + \frac{M_K}{256\pi F_\pi^2}\left[5D^2 - 6DF + 9F^2 + \frac{2}{3\sqrt{3}}(D - 3F)^2\right] + \mathcal{O}(M_K^2)$$

Matching to SU(2)

Result...

$$c_2 = b_8 + b_9 + b_{10} + 2b_{11} - \frac{M_K}{128\pi F_\pi^2}\left[6 + \frac{19}{3}D^4 + 4D^3F + \frac{58}{3}D^2F^2 - 12DF^3 \right.$$
$$\left. + 25F^4 - \frac{8(D-3F)^2(D+F)^2}{3\sqrt{3}}\right] + \mathcal{O}(M_K^2)$$

$$c_3 = b_1 + b_2 + b_3 + 2b_4 + \frac{M_K}{128F_\pi^2\pi}\left[5D^2 - 6DF + 9F^2 + \frac{19}{3}D^4 + 4D^3F + \frac{58}{3}D^2F^2 \right.$$
$$\left. - 12DF^3 + 25F^4 + \frac{8(D-3F)^2(D+F)^2}{3\sqrt{3}}\right] + \mathcal{O}(M_K^2)$$

$$c_4 = 4(b_5 + b_6) + \frac{M_K}{96\pi F_\pi^2}\left[D^2 - 6DF - 3F^2 - \frac{9}{2}D^4 - 10D^3F + D^2F^2 - 18DF^3 \right.$$
$$\left. - \frac{33}{2}F^4 - \frac{2(D-3F)^2(D+F)^2}{\sqrt{3}}\right] + \mathcal{O}(M_K^2)$$

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$\Rightarrow$ shifts are significant:

$\Delta c_1 = +0.2\,\text{GeV}^{-1}$, $\Delta c_2 = -2.1\,\text{GeV}^{-1}$
$\Delta c_3 = +1.6\,\text{GeV}^{-1}$, $\Delta c_4 = +2.0\,\text{GeV}^{-1}$

$\Delta(c_2 + c_3 - 2c_1) = -0.1\,\text{GeV}^{-1}$

Matching to SU(2)... hyperons

...same can be done for the pion-hyperon sector...

- double-scale expansion: $m_0 \gg M_K \gg M_\pi$
- $\mathring{m}_\Sigma - \mathring{m}_\Lambda = \mathcal{O}(M_K^2) \rightsquigarrow$ not vanishing in the $SU(2)$-chiral limit
  BUT! $m_\Sigma - m_\Lambda \approx 87$ MeV $\rightsquigarrow$ neglect for the loop-contributions

Result

- NLO constraints on $c_i^\Sigma, c_i^\Lambda, c_i^\Xi$
- shifts are significant but cancel in relevant combinations

Low-energy theorems

Why?

- Pion-nucleon LETs useful for chiral extrapolations
- $T^-_{\pi N}$ is stable to kaon-mass effects

$$T^+_{\pi N} = \frac{M_\pi^2}{F_\pi^2}\left\{-\frac{g^2}{4m_N} + 2(c_2 + c_3 - 2c_1) + \frac{3g^2 M_\pi}{64\pi F_\pi^2} + \mathcal{O}\left(M_\pi^2\right)\right\}$$

$$T^-_{\pi N} = \frac{M_\pi}{2F_\pi^2}\left\{1 + \frac{g^2 M_\pi^2}{4m_N^2} + \frac{M_\pi^2}{8\pi^2 F_\pi^2}\left(1 - 2\log\frac{M_\pi}{\mu}\right) + M_\pi^2 d^r_{\pi N}(\mu) + \mathcal{O}\left(M_\pi^4\right)\right\}$$

$d^r_{\pi N}(\mu)$ - generic $\mathcal{O}(q^3)$ counterterm

Low-energy theorems

...same in the pion-hyperon sector

- chiral extrapolations for lattice calculations
- isospin-even and -odd combinations: $T^+_{\pi\Xi}$, $T^-_{\pi\Xi}$
- $\pi\Xi$ sector very similar to πN

$$T^+_{\pi\Xi} = \frac{M_\pi^2}{F_\pi^2}\left\{-\frac{g_\Xi^2}{4m_\Xi} + 2(c_2^\Xi + c_3^\Xi - 2c_1^\Xi) + \frac{3g_\Xi^2 M_\pi}{64\pi F_\pi^2} + \mathcal{O}\big(M_\pi^2\big)\right\}$$

$$T^-_{\pi\Xi} = \frac{M_\pi}{2F_\pi^2}\left\{1 + \frac{g_\Xi^2 M_\pi^2}{4m_\Xi^2} + \frac{M_\pi^2}{8\pi^2 F_\pi^2}\left(1 - 2\log\frac{M_\pi}{\mu}\right) + M_\pi^2 d^r_{\pi\Xi}(\mu) + \mathcal{O}\big(M_\pi^4\big)\right\}$$

Low-energy theorems

$\pi\Sigma$ and $\pi\Lambda$ sector...

- isospin-even and -odd combinations: $T_{\pi\Lambda}$, $T^{-}_{\pi\Sigma}$, $T^{+}_{\pi\Sigma}$, $\tilde{T}_{\pi\Sigma}$
- additional expansion in $m_\Sigma - m_\Lambda$ counted as $\mathcal{O}(M_\pi)$:

$$\tilde{T}_{\pi\Sigma} = \frac{M_\pi^2}{F_\pi^2}\left\{ -\frac{g_\Sigma^2}{4m_\Sigma} + 4\left(2c_{2a}^\Sigma + c_{3a}^\Sigma - c_1^\Sigma\right)(\mu) + \frac{3g_{\Sigma\Lambda}^2 M_\pi}{64\pi F_\pi^2} f\left(m_\Sigma - m_\Lambda, M_\pi, \mu\right) + \mathcal{O}\left(M_\pi^2\right) \right\}$$

$$\bar{T}^{+}_{\pi\Sigma} = \frac{M_\pi^2}{F_\pi^2}\left\{ \frac{g_\Sigma^2}{4m_\Sigma} - \frac{g_{\Sigma\Lambda}^2}{2(m_\Lambda + m_\Sigma)} + 4\left(4c_{2b}^\Sigma + c_{3b}^\Sigma\right)(\mu) + \frac{3[g_\Sigma^2 - g_{\Sigma\Lambda}^2 f\left(m_\Sigma - m_\Lambda, M_\pi, \mu\right)]M_\pi}{32\pi F_\pi^2} + \mathcal{O}\left(M_\pi^2\right) \right\}$$

Low-energy theorems

...

$$\bar{T}^-_{\pi\Sigma} = \frac{2M_\pi}{F_\pi^2}\left\{1 + \frac{g_\Sigma^2 M_\pi^2}{16 m_\Sigma^2} + \frac{g_{\Sigma\Lambda}^2 M_\pi^2}{4(m_\Lambda + m_\Sigma)^2} + M_\pi^2 d^r_{\pi\Sigma}(\mu) + \frac{M_\pi^2}{8\pi^2 F_\pi^2}\left(1 - 2\log\frac{M_\pi}{\mu}\right) + \mathcal{O}\left(M_\pi^3\right)\right\}$$

$$T_{\pi\Lambda} = \frac{M_\pi^2}{F_\pi^2}\left\{-\frac{g_{\Sigma\Lambda}^2}{2(m_\Lambda + m_\Sigma)} + 2\left(c_2^\Lambda + c_3^\Lambda - 2c_1^\Lambda\right)(\mu) + \frac{3g_{\Sigma\Lambda}^2 M_\pi}{64\pi F_\pi^2} f\left(m_\Lambda - m_\Sigma, M_\pi, \mu\right) + \mathcal{O}\left(M_\pi^2\right)\right\}$$

Summary

- scattering lengths are calculated to one-loop in $SU(3) - \chi PT$
- convergence is rather slow $\rightsquigarrow$ ~~reordering~~
- matching relations on LECs between $SU(2)$ (pion-hyperon sector) and $SU(3)$ up to one-loop are calculated
- novel low-energy theorems are derived for the pion-hyperon sector

  ... and used by Torok et al. for two-flavor chiral extrapolations for $a_{\pi^+\Sigma^+}$ and $a_{\pi^+\Xi^0}$