A STRANGE-NESS LOOP

QCD

→ EFT(DYNAMICALLY GENERATED RESONANCES)

→ (LATTICE) QCD

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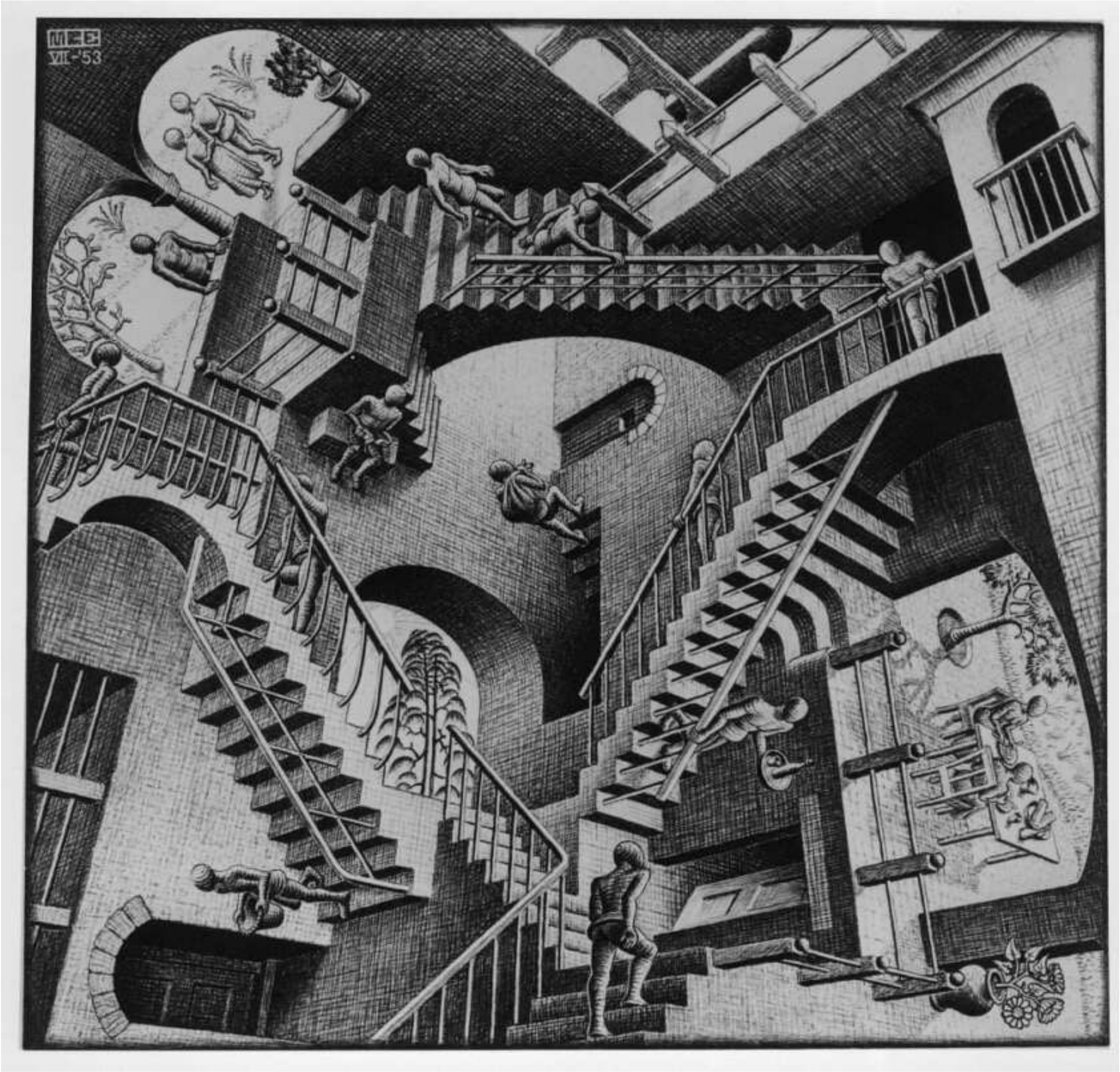
15.01.2026



Relativity (1953) – MC Escher

Heisenberg-Programm DFG

DFG: Heisenberg Programme (project number: 532635001)

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

A STRANGE-NESS LOOP

OUTLINE

1. **Motivation**

    Observations,Theory, …

2. **Dynamically Generated Resonances**

    Methodology, Examples, $\Lambda(1405)$, …

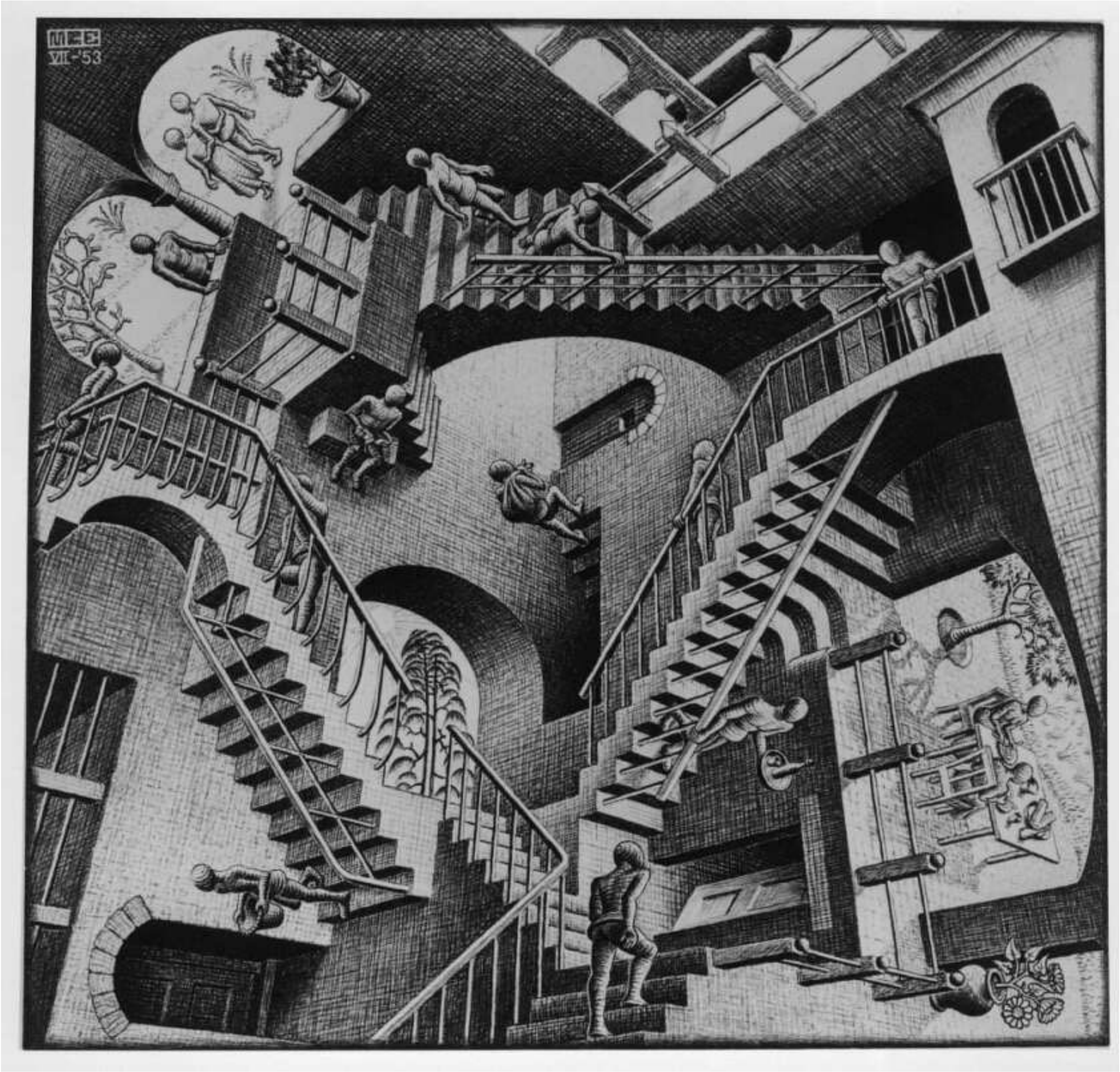
3. **Applications to lattice QCD**

    From Experiment to resonance parameters,

    Chiral trajectories,

    Quantization Conditions,…

4. **Summary/Outlook**



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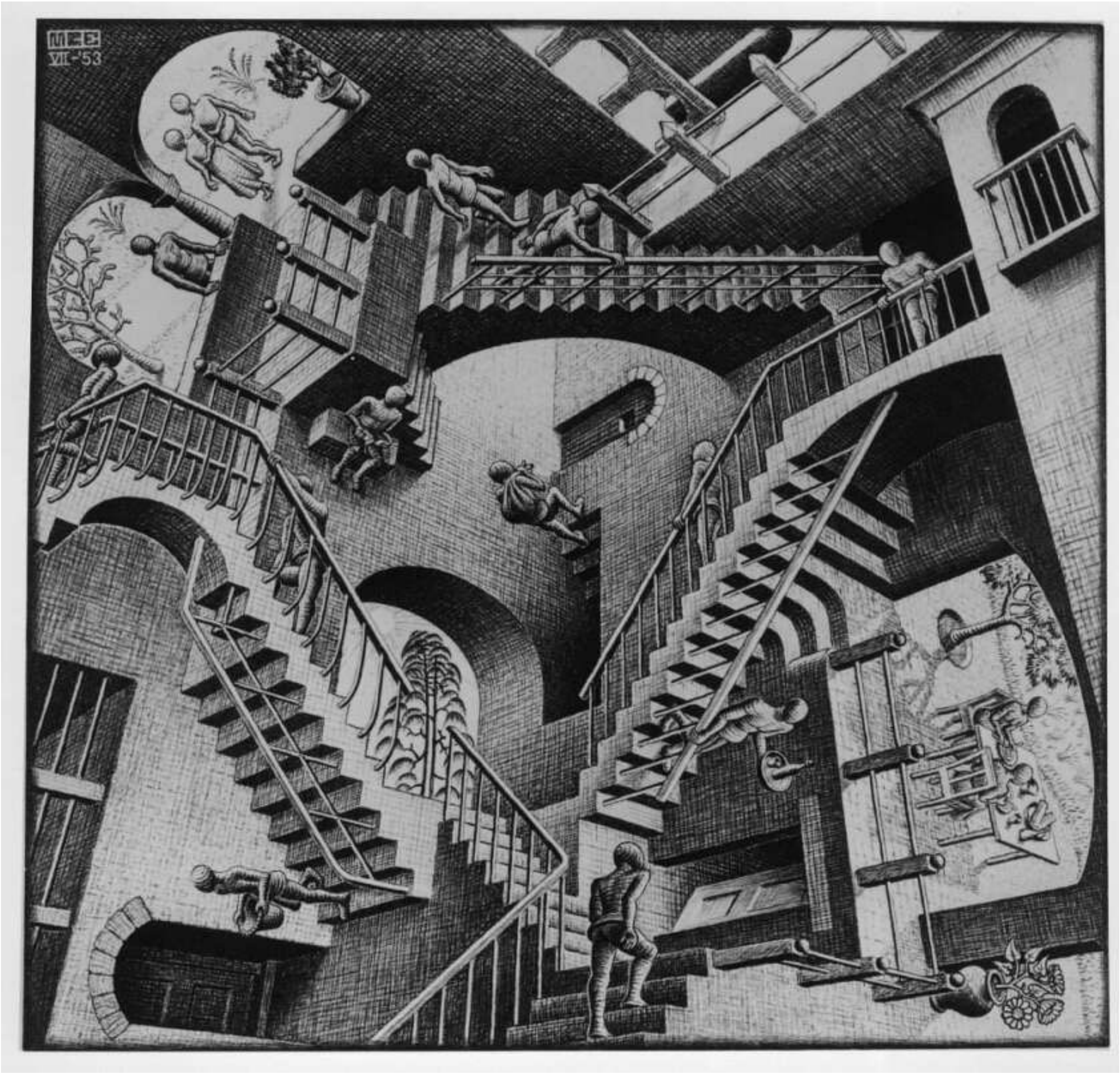


Relativity (1953) – MC Escher

HADRON SPECTRUM

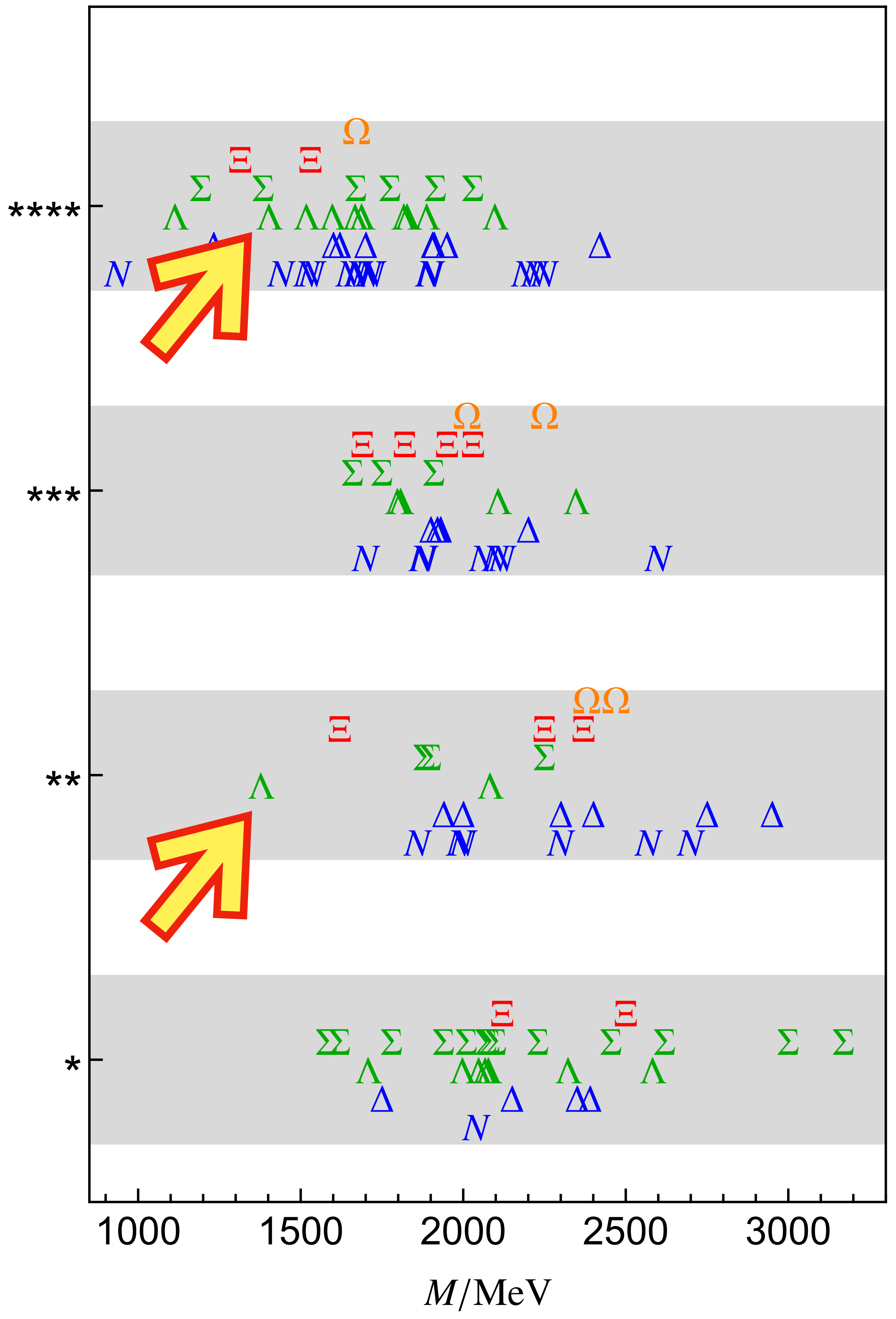
Experimental progress

- 70y research: $\Delta(1232), \rho(770), \omega(782), \dots$
- Ongoing progress, new techniques and experiments
- Many overlapping and mostly excited states

  ≈ 100 mesons + 50 baryons (****)

Theory

- QCD governs the emergence of hadron spectrum
- Strangeness enriches and complicates the picture
  - Curious case: $\Lambda(1405) + \Lambda(1380)$

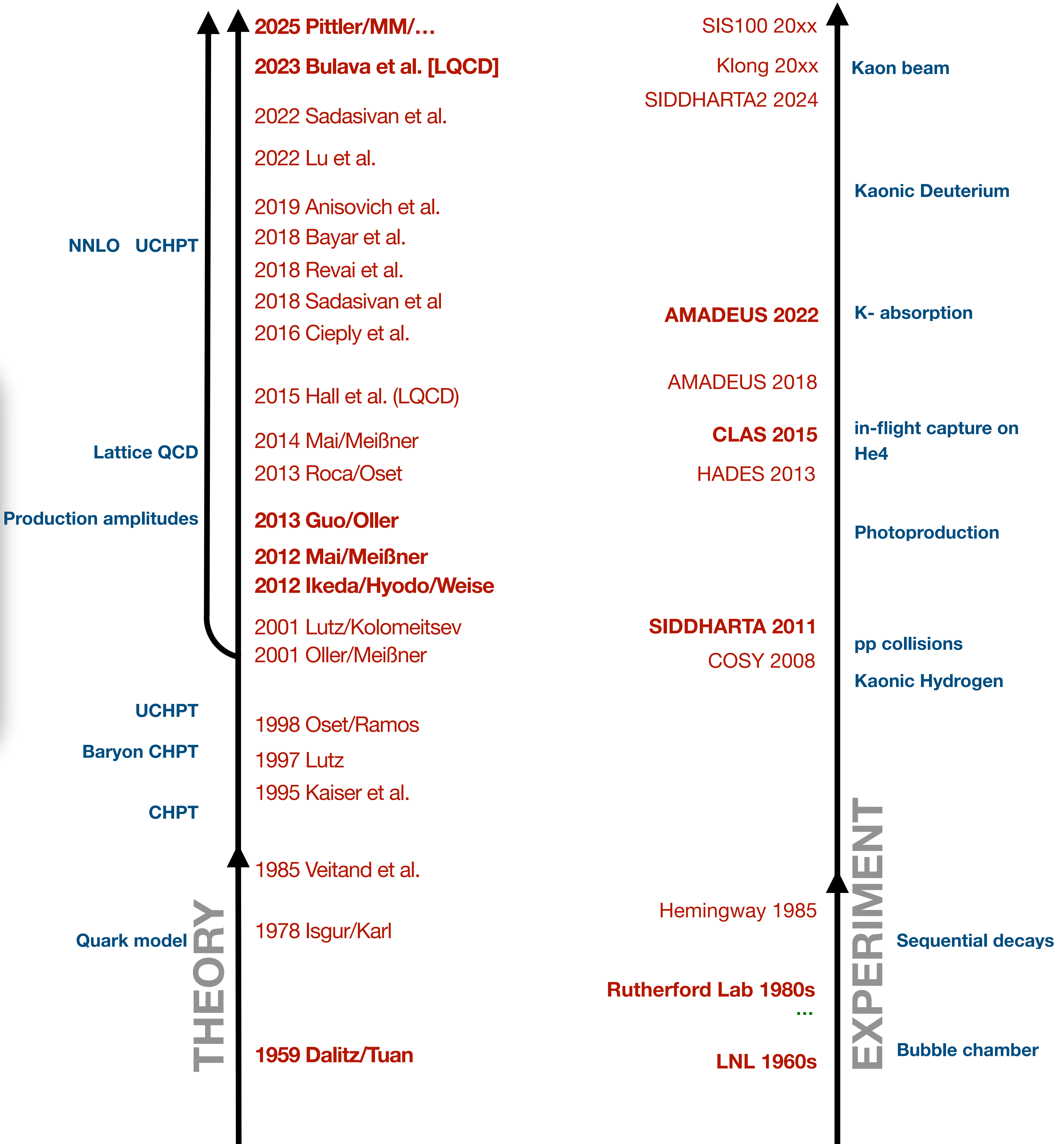


Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6

STATUS AND HISTORY OF $\Lambda(1405)$

Intertwined experimental/theoretical developments

- Sub-($\bar{K}N$)-threshold $\Lambda(1405)$ resonance
  Dalitz/Tuan 1959
- Second state $\Lambda(1380)$ predicted from UCHPT
  Oller/Meißner 2001
- No direct experimental verification
  [CLAS] Moriya et al. Phys.Rev.Lett. 112 (2014) 8
  MM/Meißner Eur.Phys.J.A 51 (2015) 3, 30
- Confirmed by many critical tests & LQCD
  Bulava+ (BaSc) PRL132, 051901 (2024)



STRANGENESS

Phenomenology

- New facilities CLAS12@JLab, Klong@JLab, SIS100@FAIR, AMBER@CERN
- $\bar{K}NN$ & $\bar{K}NNN$ bound states searches JPARC/...
- Femtoscopy/Correlations ALICE/ALICE2...

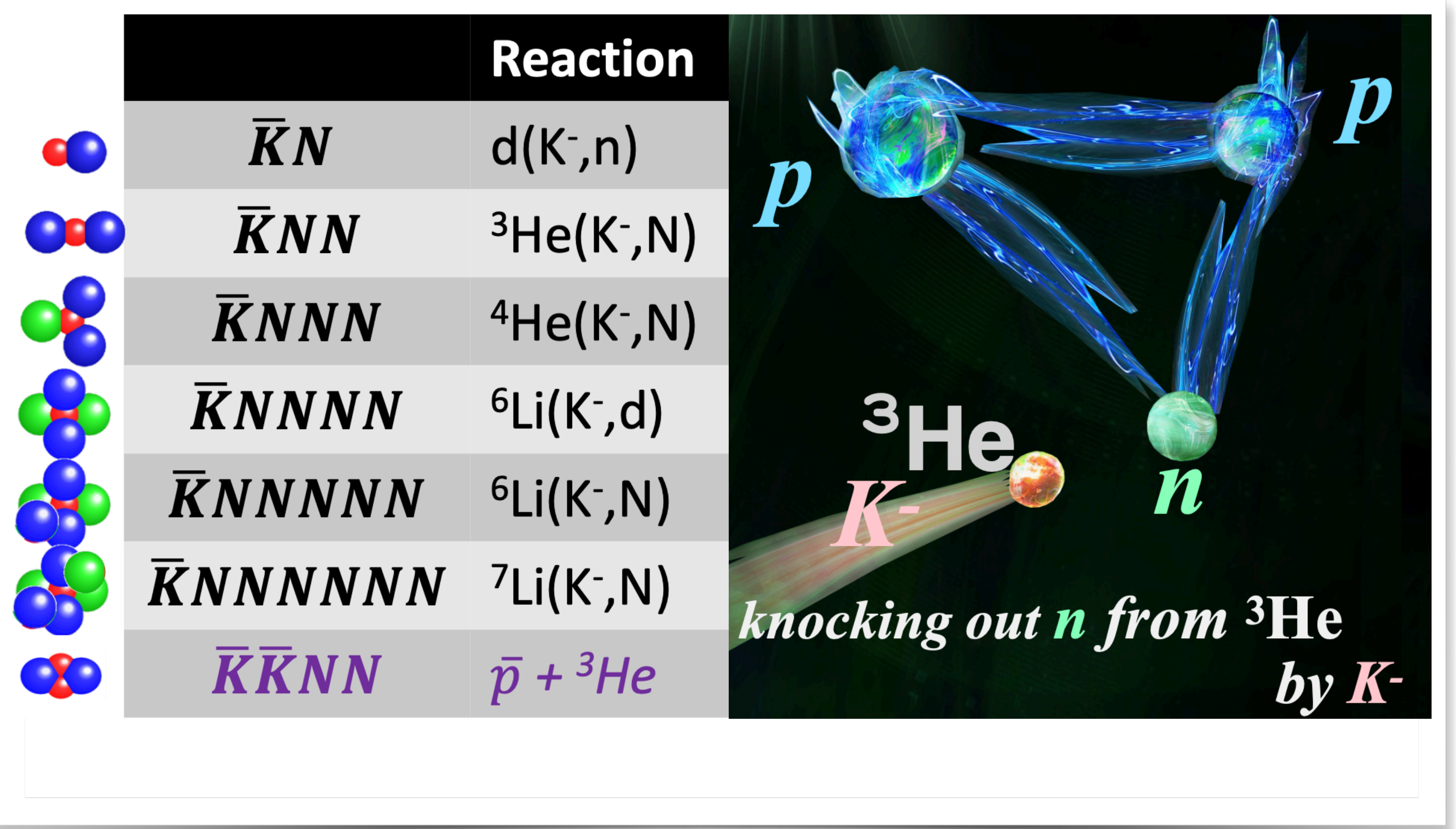
Theory

- K^- in medium and condensate
- Equation of State of neutron stars
- SU(3) dynamics

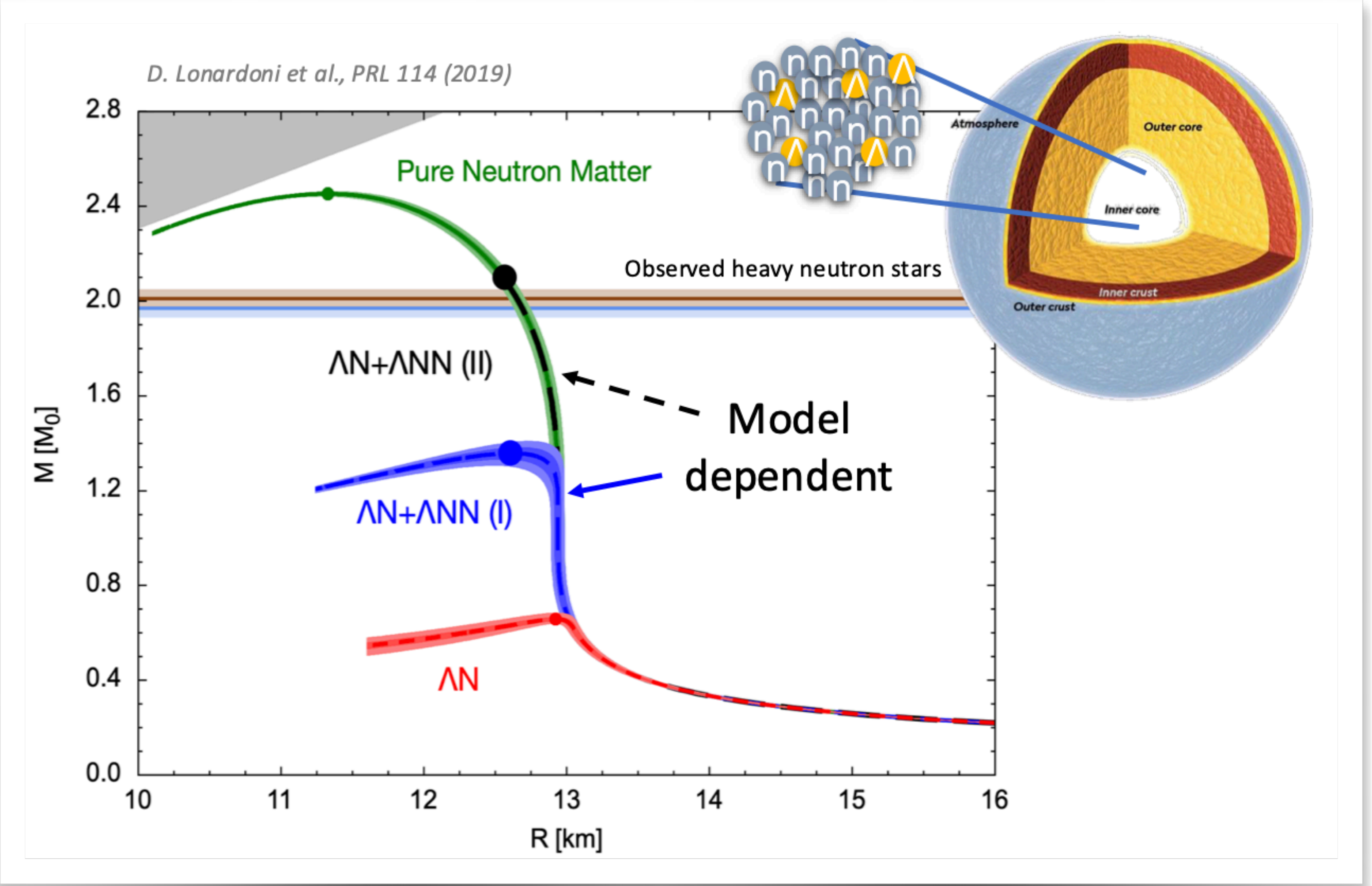
new Hyperons?

quark mass dependence

$$\{m_u, m_d\} < m_s \ll \{m_c, m_b, m_t\}$$

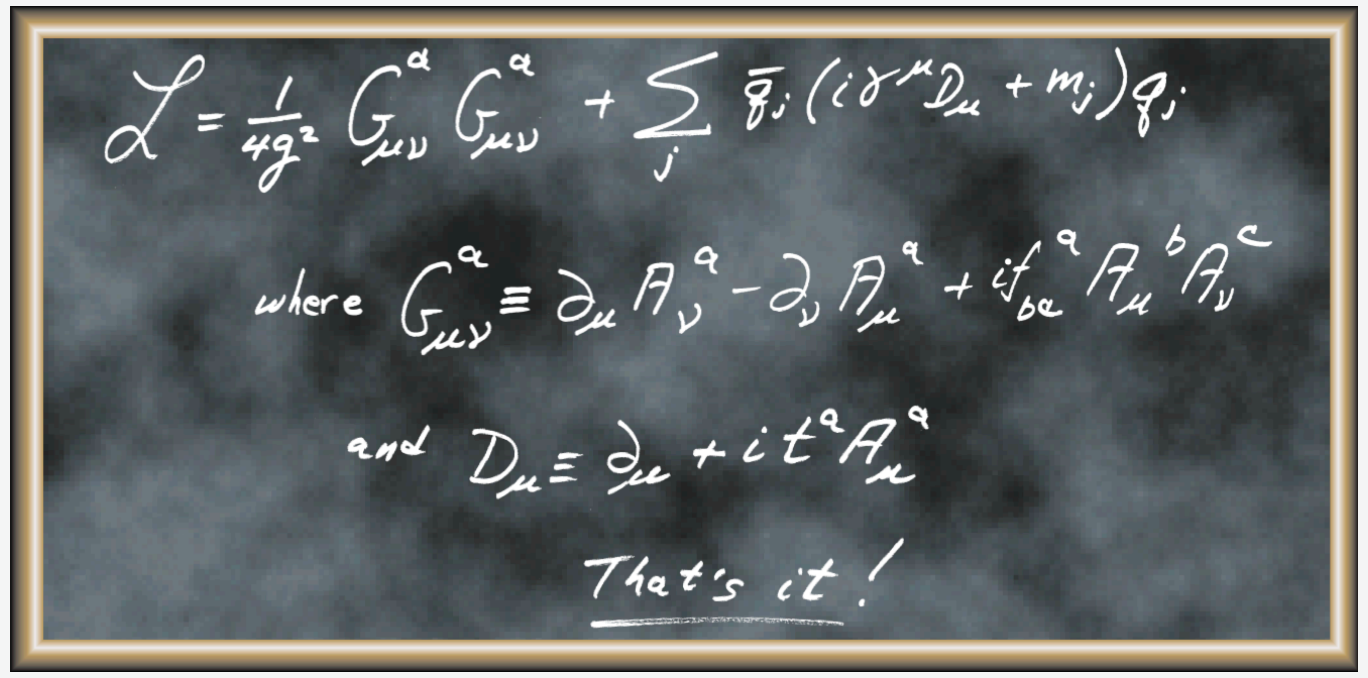


Sakuma/Iwasaki #J-PARC @EXAP/LEAP2024



Review: Gal/Hungerford/Millener (2016); Iwasaki et al. Phys.Rev.C 110 (2024) 1, 014002, …
Mareš et al. Acta Phys. Polon. B 51, 129 (2020), Hrtánkova et al. Phys.Lett. B 785, 90 (2018), …
Cassing/Tolos/Bratkovskaya/Ramos Nucl.Phys.A 727 (2003) 59-94
Michael Annan Lisa et al, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402, Fabbietti et al., ARNPS 71 (2021), 377-402

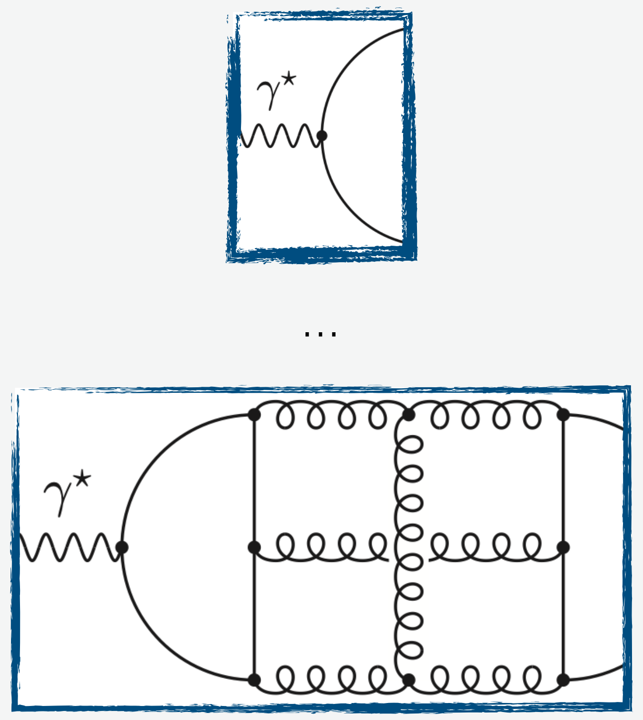
THEORY



www.frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Quantum Chromodynamics (QCD)

- Degrees of freedom: quarks and gluons
- pQCD: well-defined set of rules (Feynman diagrams) to calculate transition rates



LO

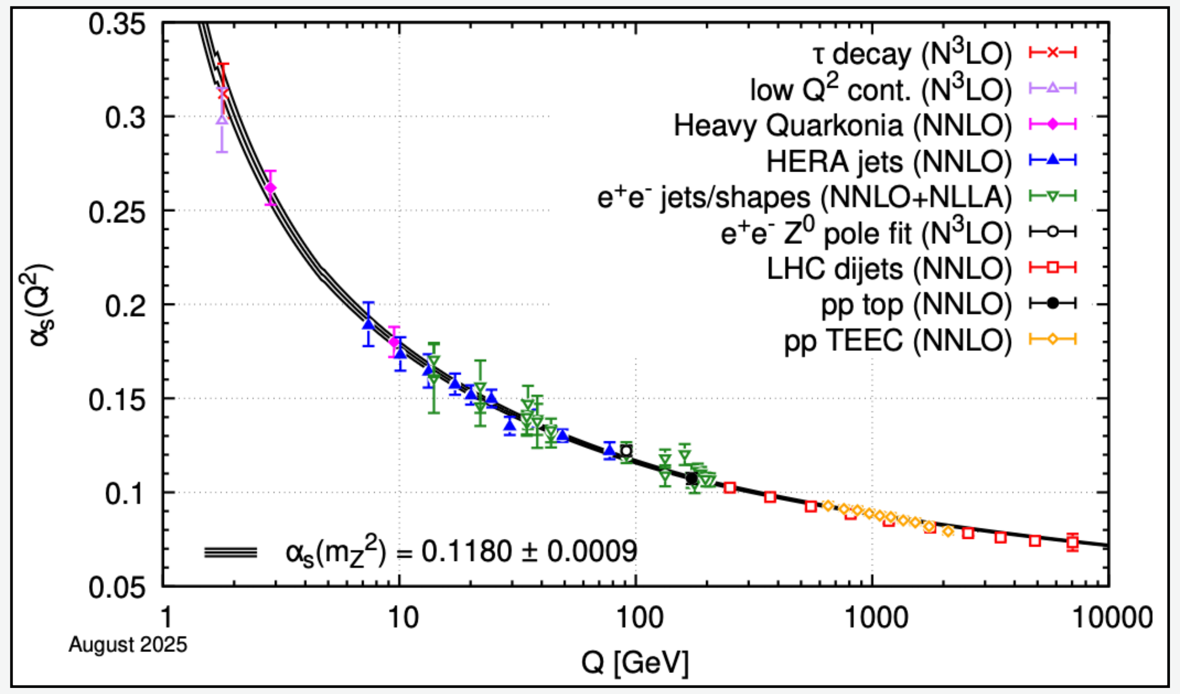
N4LO

JHEP 08 (2017) 113

Hadron spectroscopy

- Virtual effects: $\alpha_s \propto g_s^2$ depends on the momenta
  - $\alpha_s(Q) > 1$ *for small exchanged momenta*
  - *asymptotic states = stable hadrons* $p, n, \pi, \ldots$

←Spectroscopy pQCD→



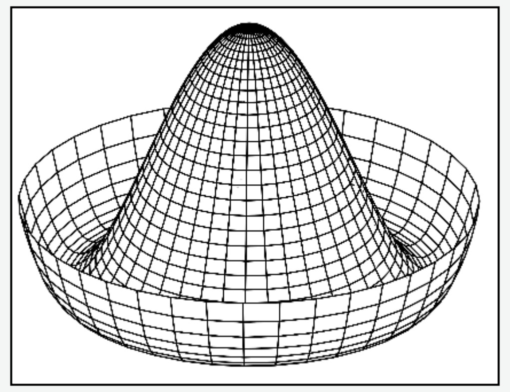
Particle Data Group 2025

NON-PERTURBATIVE APPROACHES

Effective Field Theory (EFT/CHPT)

Weinberg (1979) Gasser, Leutwyler (1981), …

Reviews: Ann. Rev. Nucl. Part. Sci. 57, 33 (2007), Adv. Nucl. Phys. 27, 277 (2003), …



$$Z[J] = \int [DU] e^{\int i d^4x \, \mathscr{L}_{\text{eff}}(U,v,a,s,p)}$$

[+] Effective/Hadronic degrees of freedom

[-] Infinitely many, unknown *low-energy constants*

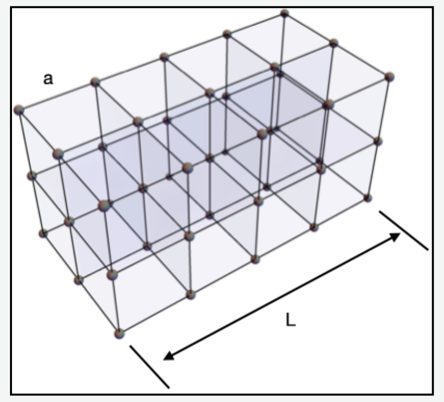
[+] Well-defined power counting

[+] quark mass dependence

Lattice Gauge Theory (LQCD)

Wilson, Phys. Rev. D10 (1974) 2445 , …

Reviews: hep-lat/9807028 [hep-lat] Rev.Mod.Phys. 90 (2018) Rept.Prog.Phys. 86 (2023)



$$Z[J] = \int [DU] e^{-S_E} \det[M[U]]$$

[+] QCD degrees of freedom

[-] discretized space-time / Euclidean metric / finite-volume

[+/-] unphysical quark masses

[+] non-perturbative access to QCD Green's functions

S-matrix theory

Further approaches: Functional methods, holography, K-matrix, dynamical models, …

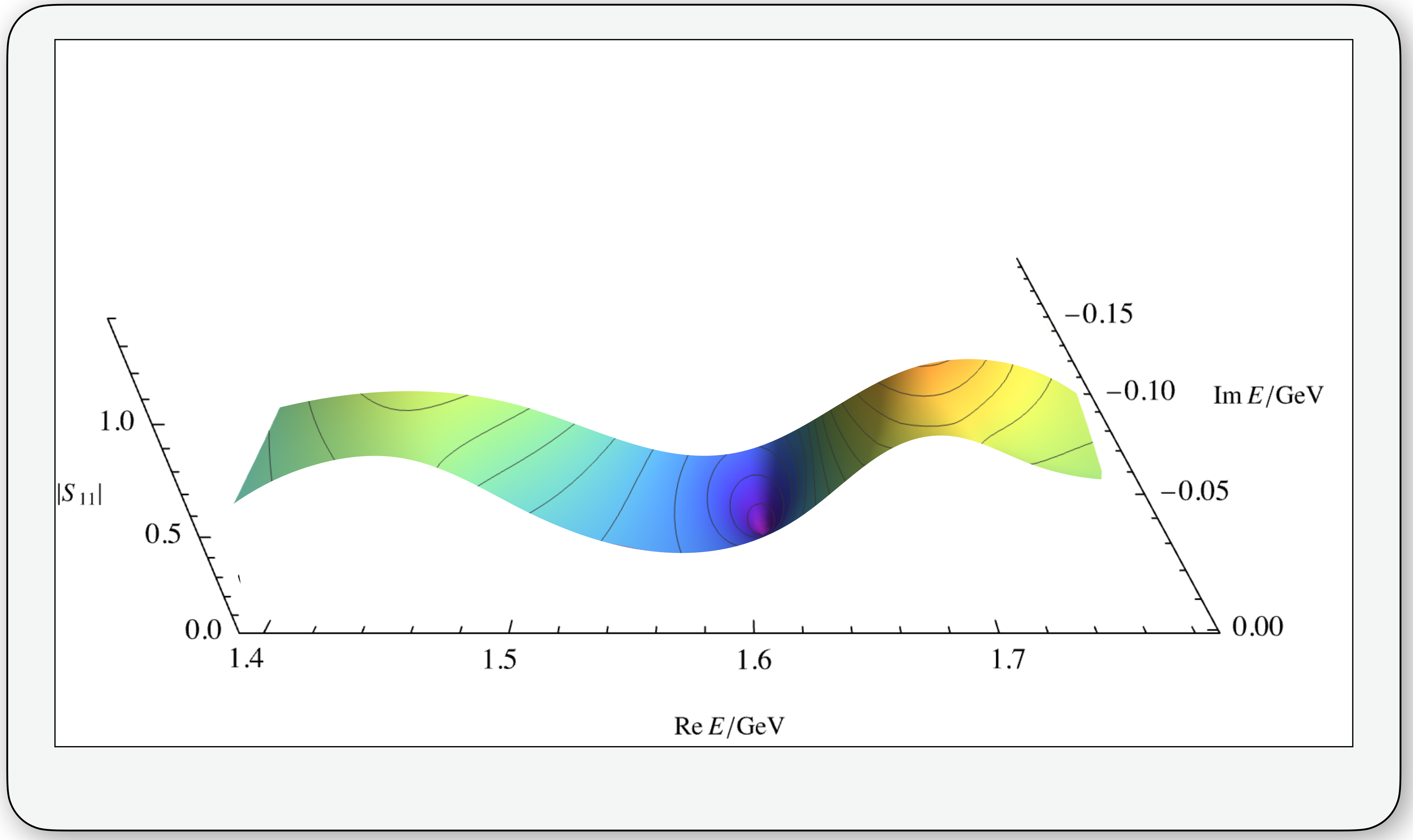
Review: Eichmann/Sanchis-Alepuz/Alkofer/Fischer Prog.Part.Nucl.Phys. 91 (2016) 1-100

Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6

Review: Döring/Haidenbauer/Sato/MM PPNP(2025)

S-MATRIX

- ✴ *Crossing symmetry (particle/antiparticle)*
- ✴ *Unitarity (probability conservation)*
- ✴ *Analyticity (causality)*



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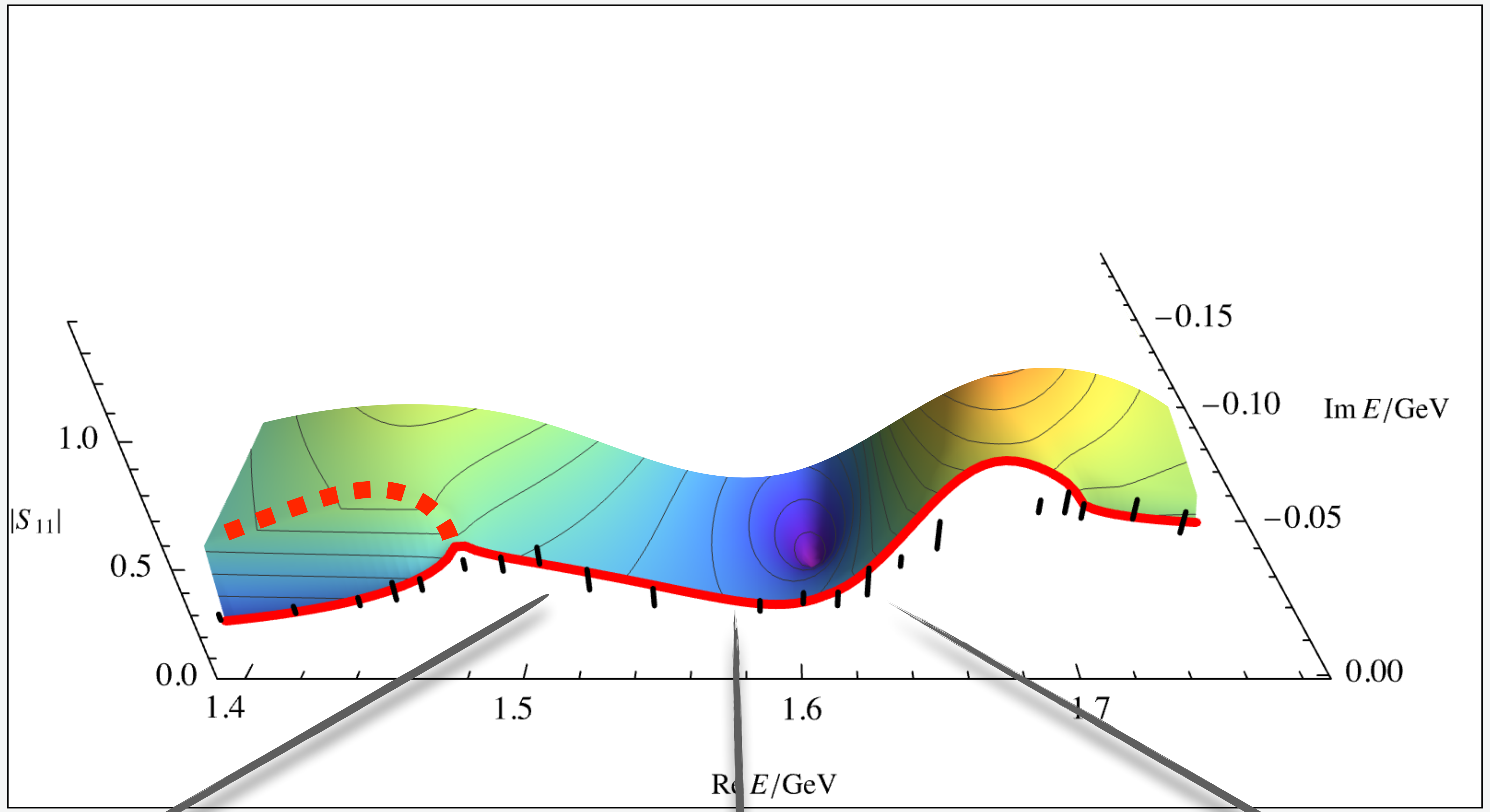
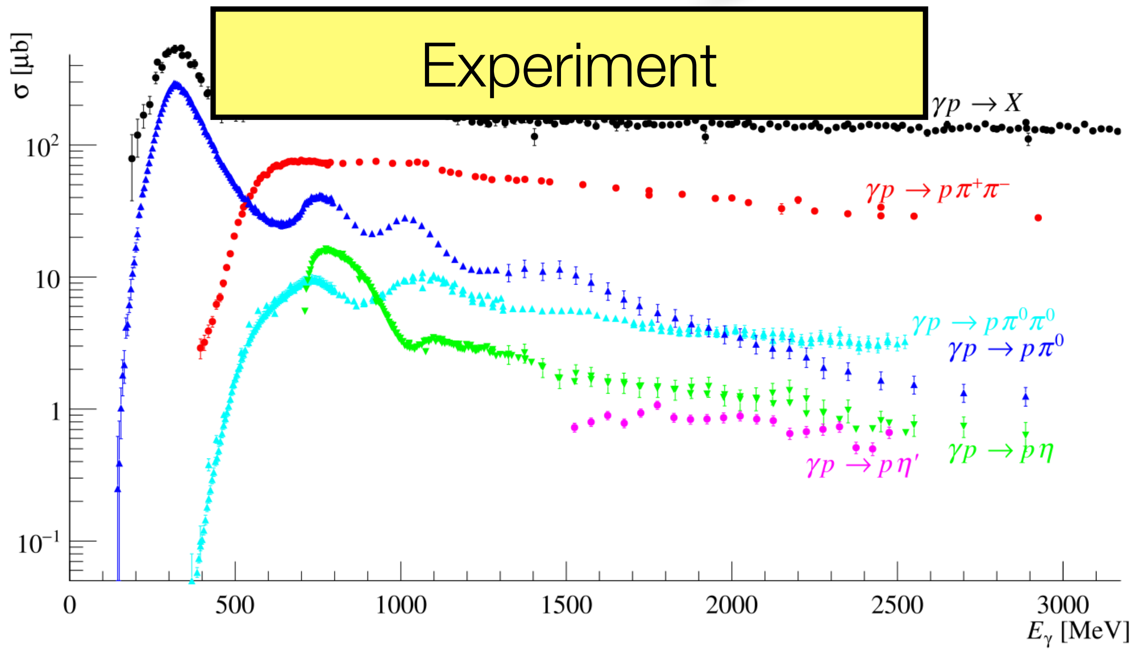
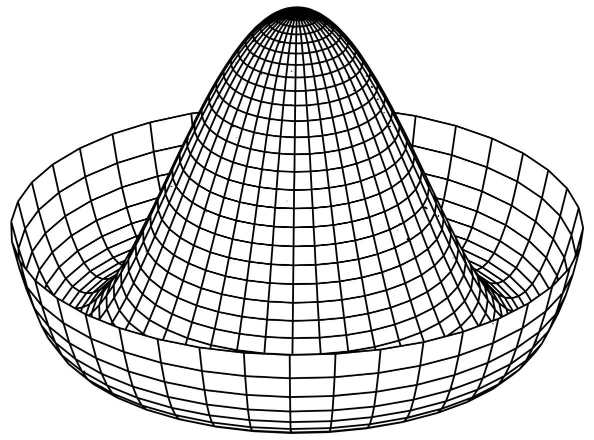


Experiment



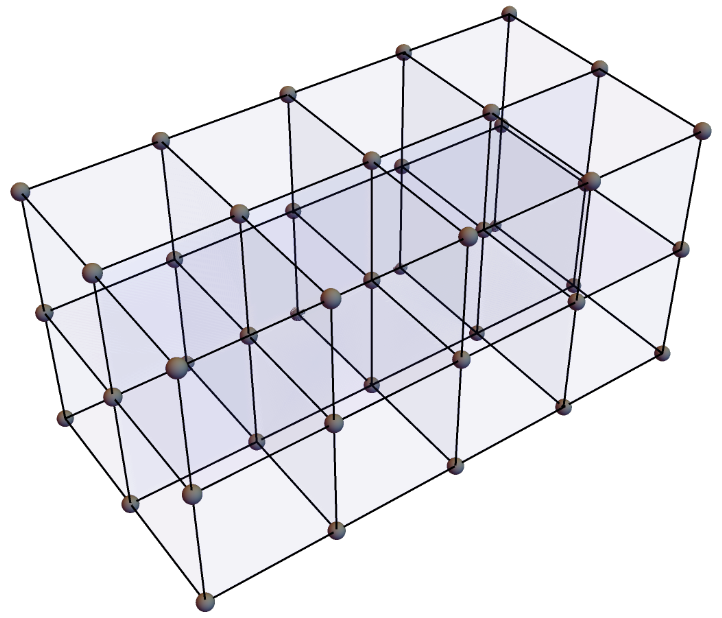
Data: JLAB, ELSA, MAMI CLAS12, GlueX, …
Plot: Thiel+ PPNP 125 (2022) 103949

Effective field theories



Data: SAID: PRC 74 (2006) 045205
Model: MM+ PRD 86 (2012) 094033

Lattice QCD



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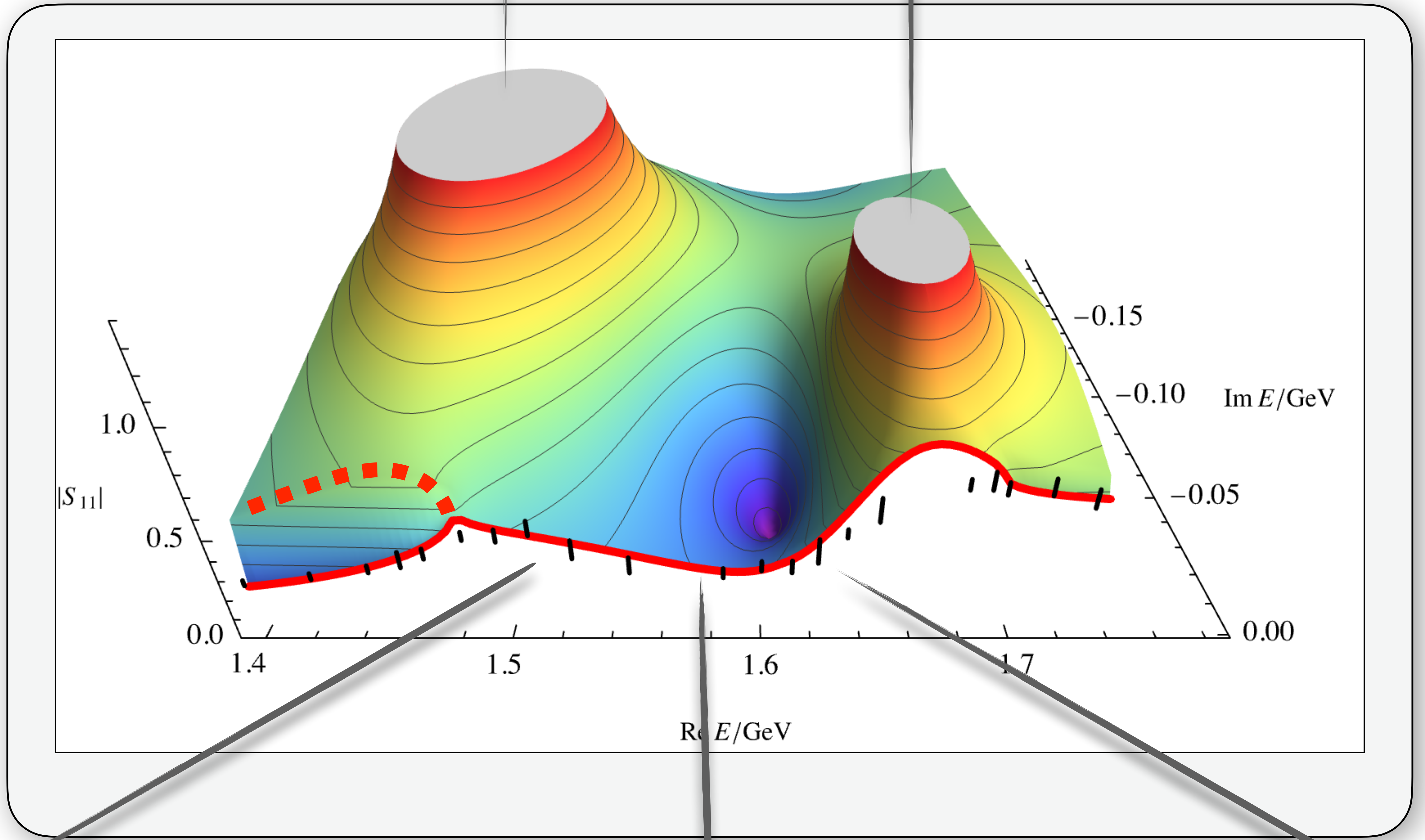
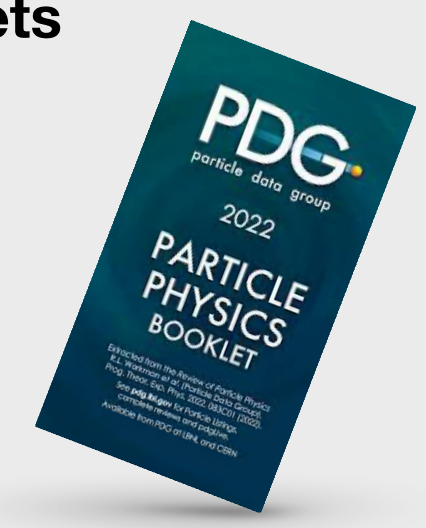
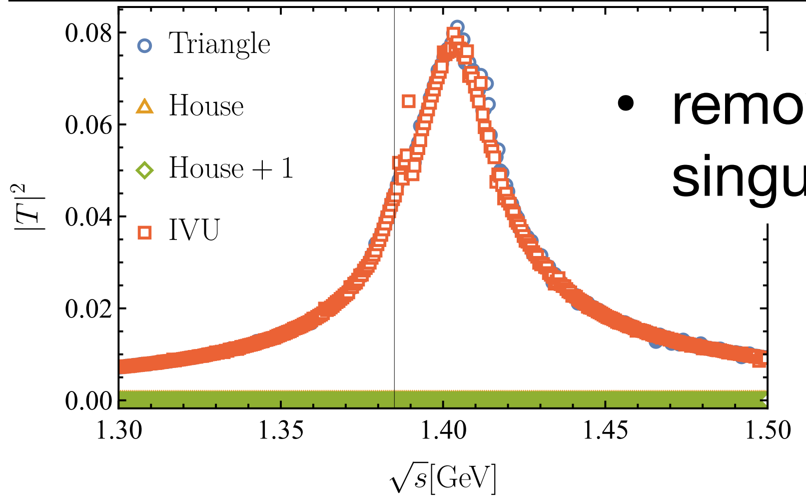


Poles on unphysical Riemann Sheets

- Universal resonance parameter
- Singularities (cuts, …) included
- **Hadron mass** $M = \mathrm{Re}\, E_{\mathrm{pole}}$
- **Decay width** $\Gamma = -2\, \mathrm{Im}\, E_{\mathrm{pole}}$

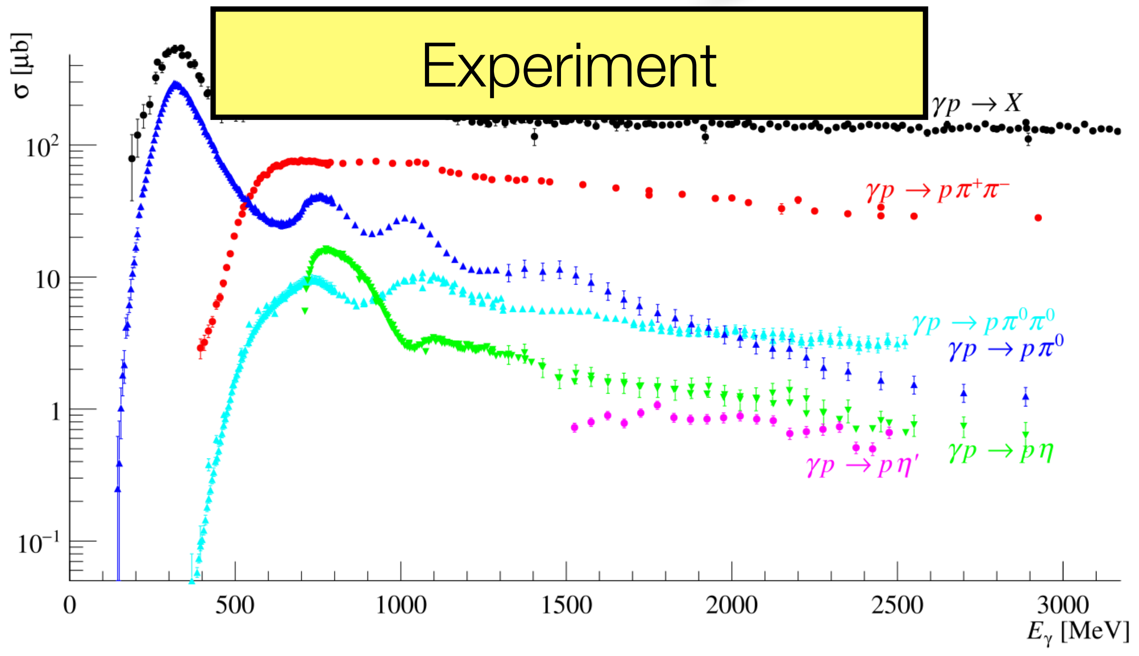




- removes triangle/Landau singularities

Sakthivasan et al. *JHEP* 10 (2024) 246

Experiment



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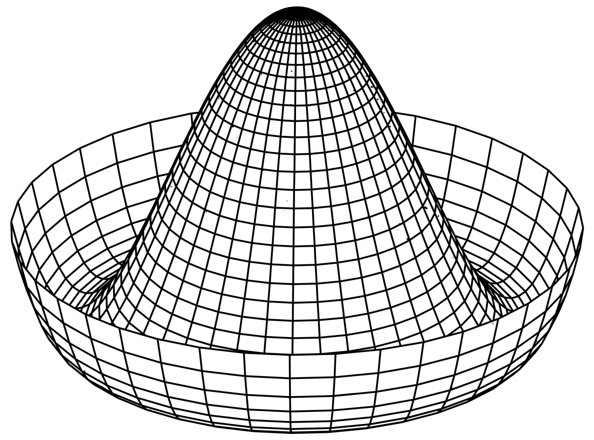

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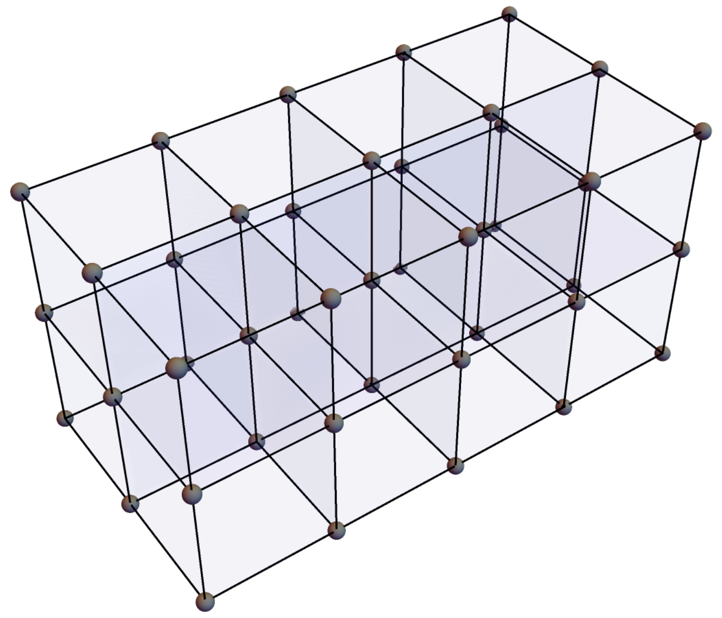

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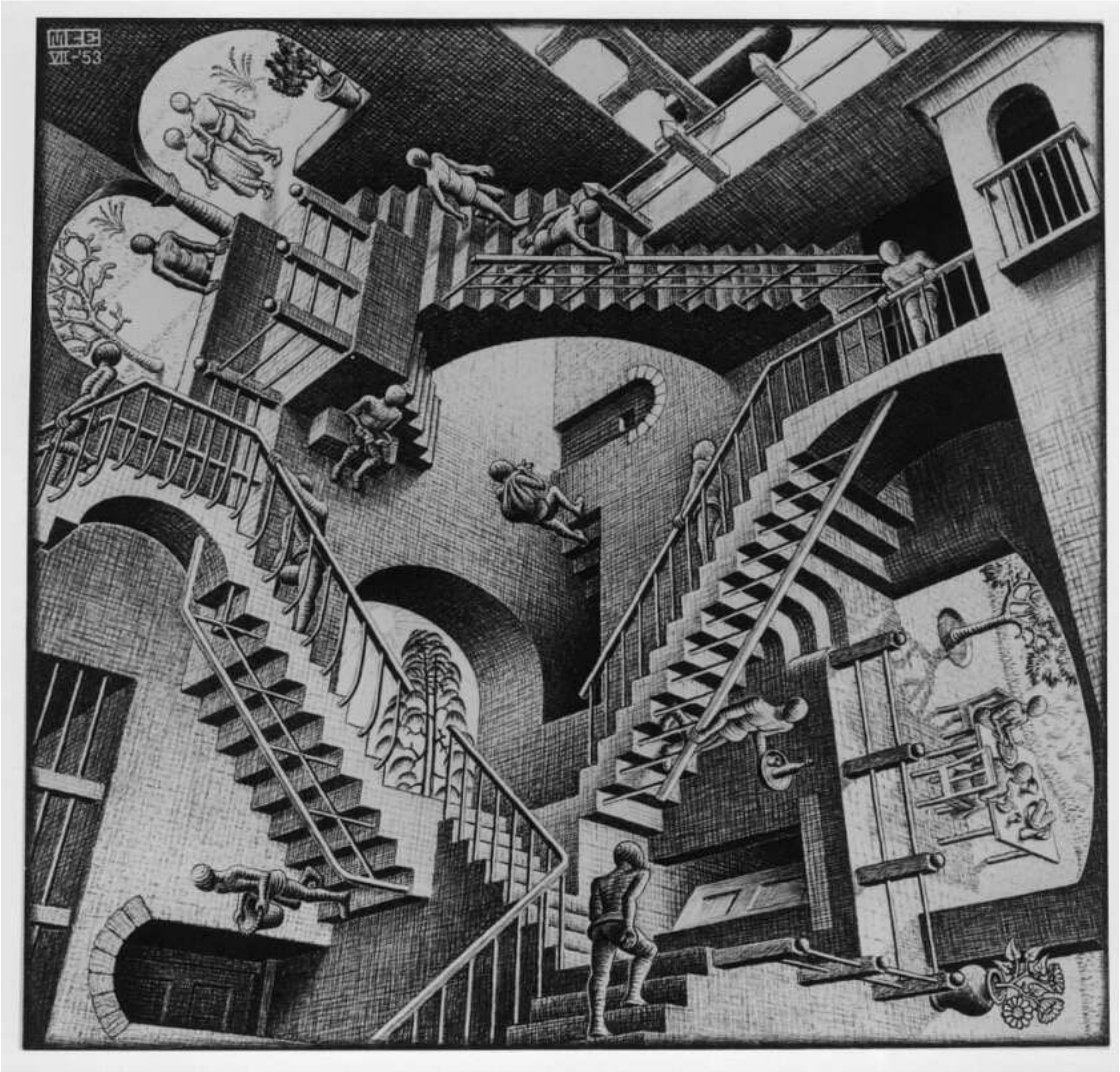
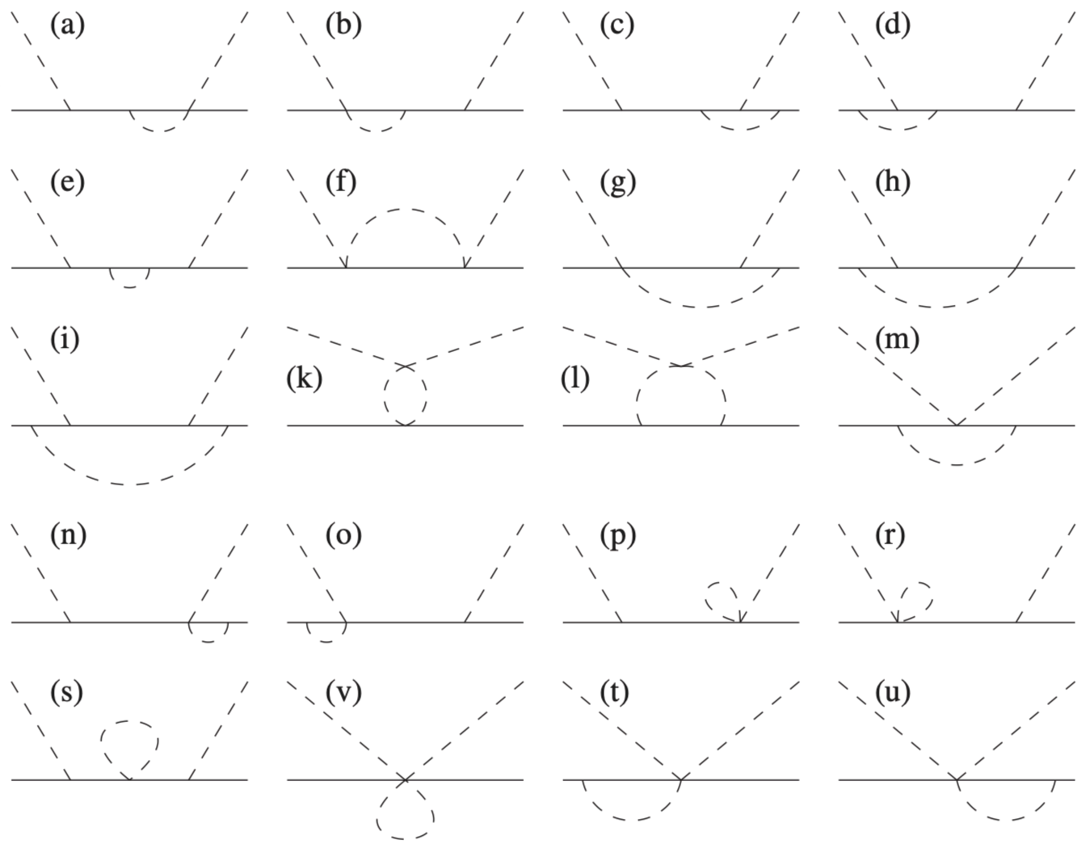


Relativity (1953) – MC Escher

BARYON CHIRAL PERTURBATION THEORY

$$\mathcal{L}^{(2)}_{\phi B} = b_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + b_{1/2}\langle \bar{B}[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle\bar{B}\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle\bar{B}B\rangle\langle u_\mu u^\mu\rangle$$
$$+ i\sigma^{\mu\nu}(b_{5/6}\langle\bar{B}[[u_\mu,u_\nu],B]_\mp\rangle + b_7\langle\bar{B}u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle\bar{B}\gamma^\mu[u_\mu,[u_\nu,[D^\nu,B]]_\mp]\rangle + \langle\bar{B}\gamma^\mu[D_\nu,[u^\nu,[u_\mu,B]]_\mp]\rangle)$$
$$+ \frac{ib_{10}}{2m_0}(\langle\bar{B}\gamma^\mu\{u_\mu,\{u_\nu,[D^\nu,B]\}\}\rangle + \langle\bar{B}\gamma^\mu[D_\nu,\{u^\nu,\{u_\mu,B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle\bar{B}\gamma^\mu[D_\nu,B]\rangle\langle u_\mu u^\nu\rangle$$
$$+ \langle\bar{B}\gamma^\mu B\rangle\langle[D_\nu,u_\mu]u^\nu + u_\mu[D_\nu,u^\nu]\rangle),$$

$$u_\mu = -i\frac{\partial_\mu\phi}{F} + \mathcal{O}(\phi^3)$$



Channel =	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)$	$+\mathcal{O}(q^3)_{\rm HB}$	$\Sigma_{\rm HB}$
$a^{(3/2)}_{\pi N}$ =	-0.12	$+0.05^{+0.02}_{-0.03}$	$-0.06^{+0.00}_{+0.00}$	$-0.13^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi N}$ =	$+0.21$	$+0.05^{+0.02}_{-0.03}$	$+0.00^{+0.00}_{+0.00}$	$+0.26^{+0.03}_{-0.03}$
$a^{(3/2)}_{\pi\Xi}$ =	-0.12	$+0.04^{+0.03}_{-0.03}$	$-0.09^{+0.00}_{+0.00}$	$-0.17^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi\Xi}$ =	$+0.23$	$+0.04^{+0.03}_{-0.03}$	$-0.03^{+0.00}_{+0.00}$	$+0.23^{+0.03}_{-0.03}$
$a^{(2)}_{\pi\Sigma}$ =	-0.24	$+0.07^{+0.01}_{-0.01}$	$-0.07^{+0.00}_{+0.00}$	$-0.24^{+0.01}_{-0.01}$

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

Meson-baryon scattering from CHPT

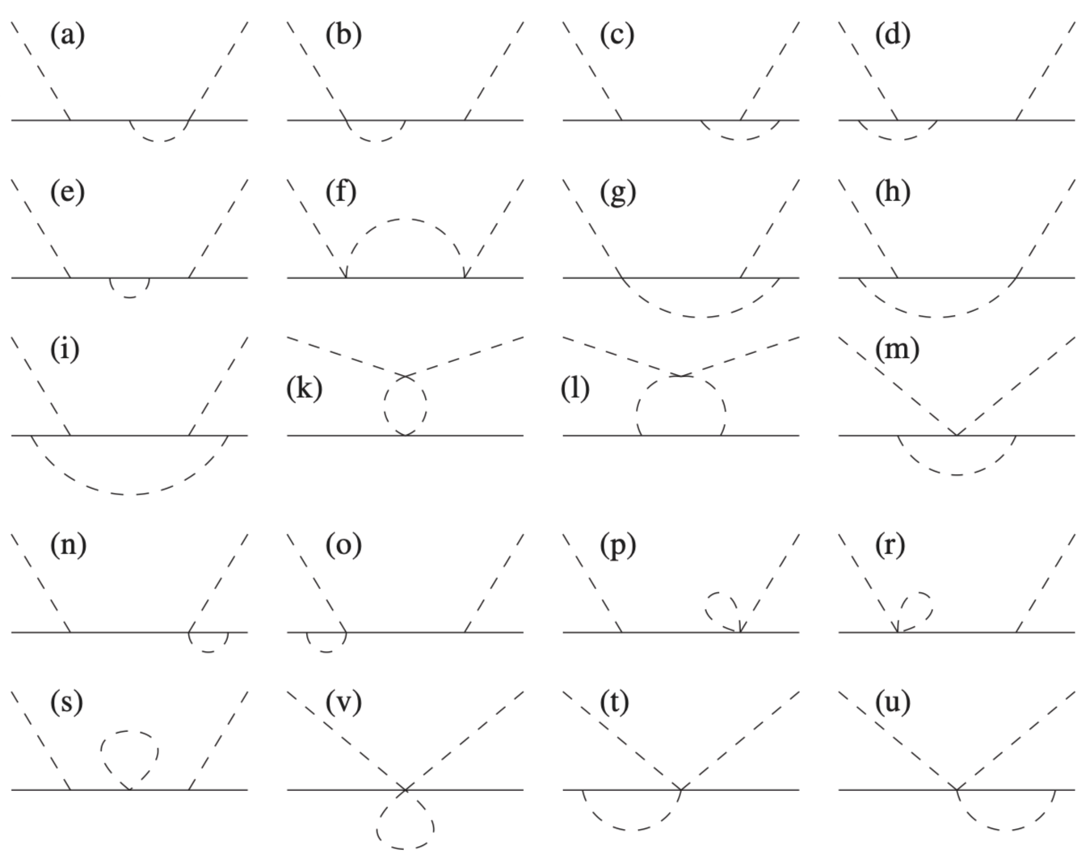
- full SU(3) dynamics near threshold
- agrees with experiment in many cases
- provides predictions for not measured channels

Torok/Beane/Detmold/Luu/… Phys.Rev.D 81 (2010) 074506

BARYON CHIRAL PERTURBATION THEORY

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle \bar B[\chi_+, B]_\pm\rangle + b_0\langle\bar B B\rangle\langle\chi_+\rangle + b_{1/2}\langle\bar B[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle\bar B\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle\bar B B\rangle\langle u_\mu u^\mu\rangle \\
& + i\sigma^{\mu\nu}(b_{5/6}\langle\bar B[[u_\mu,u_\nu],B]_\mp\rangle + b_7\langle\bar B u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle\bar B\gamma^\mu[u_\mu,[u_\nu,[D^\nu,B]]_\mp]\rangle + \langle\bar B\gamma^\mu[D_\nu,[u^\nu,[u_\mu,B]]_\mp]\rangle) \\
& + \frac{ib_{10}}{2m_0}(\langle\bar B\gamma^\mu\{u_\mu,\{u_\nu,[D^\nu,B]\}\}\rangle + \langle\bar B\gamma^\mu[D_\nu,\{u^\nu,\{u_\mu,B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle\bar B\gamma^\mu[D_\nu,B]\rangle\langle u_\mu u^\nu\rangle \\
& + \langle\bar B\gamma^\mu B\rangle\langle[D_\nu,u_\mu]u^\nu + u_\mu[D_\nu,u^\nu]\rangle),
\end{aligned}
$$

$$u_\mu = -i\frac{\partial_\mu\phi}{F} + \mathcal{O}(\phi^3)$$



$$a^{I=0}_{\bar KN} = \Big((+0.53)_{\rm LO} + (+0.97)_{\rm NLO} + (-0.40 + 0.22i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,,$$
$$a^{I=1}_{\bar KN} = \Big((+0.20)_{\rm LO} + (+0.22)_{\rm NLO} + (-0.26 + 0.18i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,.$$

… Fails for resonant (strangeness) channel

- Kaon mass is large → convergence
- Relevant thresholds are widely separated → convergence
- Resonance just below $\bar KN$ threshold → non-perturbative effect

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

CHIRAL UNITARY APPROACH

Good

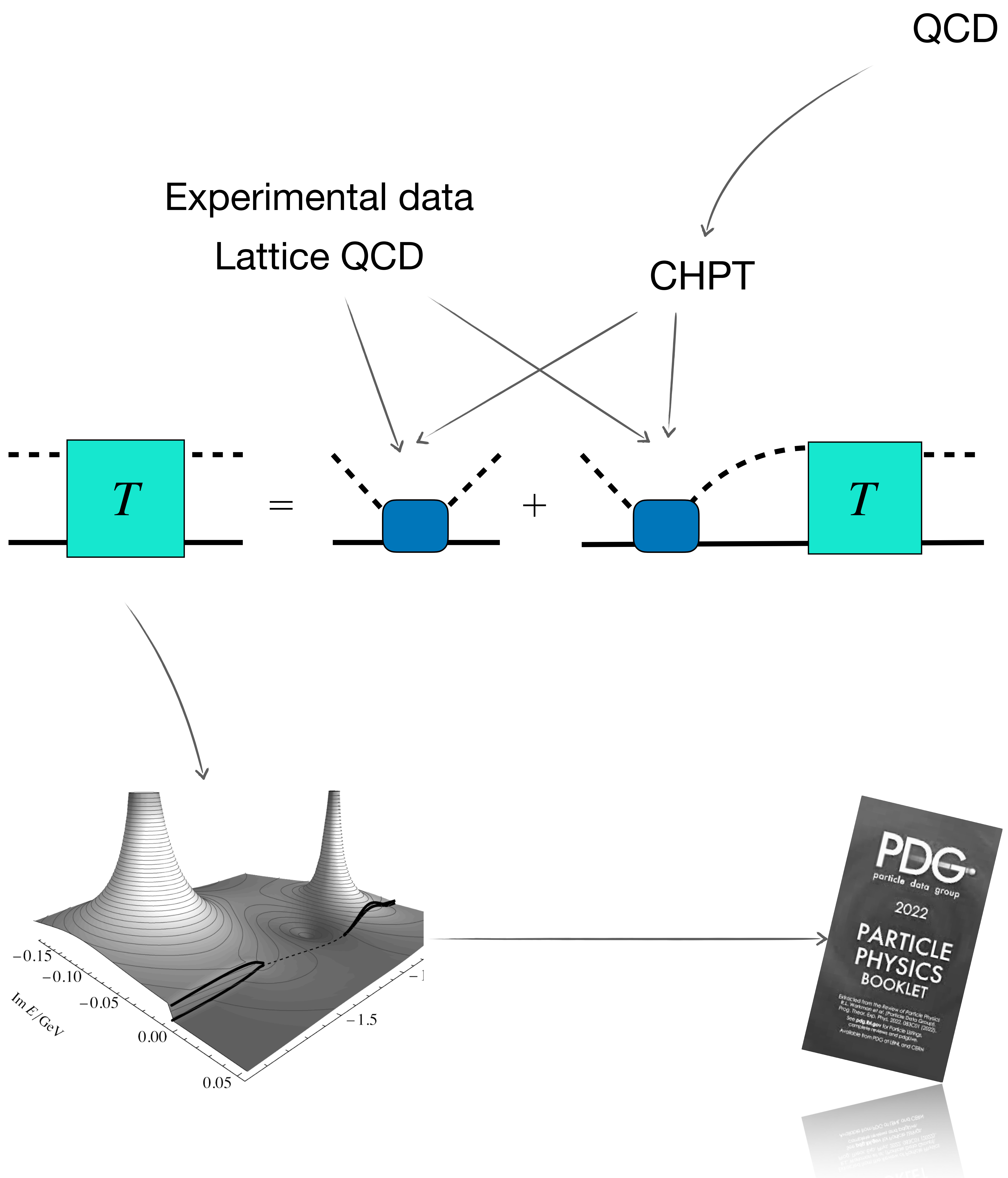
- Non-perturbative (resonant channels/higher energies)
- Record complex pole-positions (*II Riemann Sheet*)
- Coupled-channel

$$\boxed{\mathcal{S}=\{K^-p,\bar{K}^0n,\pi^0\Lambda,\pi^0\Sigma^0,\pi^+\Sigma^-,\pi^-\Sigma^+,\eta\Lambda,\eta\Sigma^0,K^+\Xi^-,K^0\Xi^0\}}$$

- Examples: $N(1535), N(1650), \Lambda(1405), \Lambda(1380), \ldots$

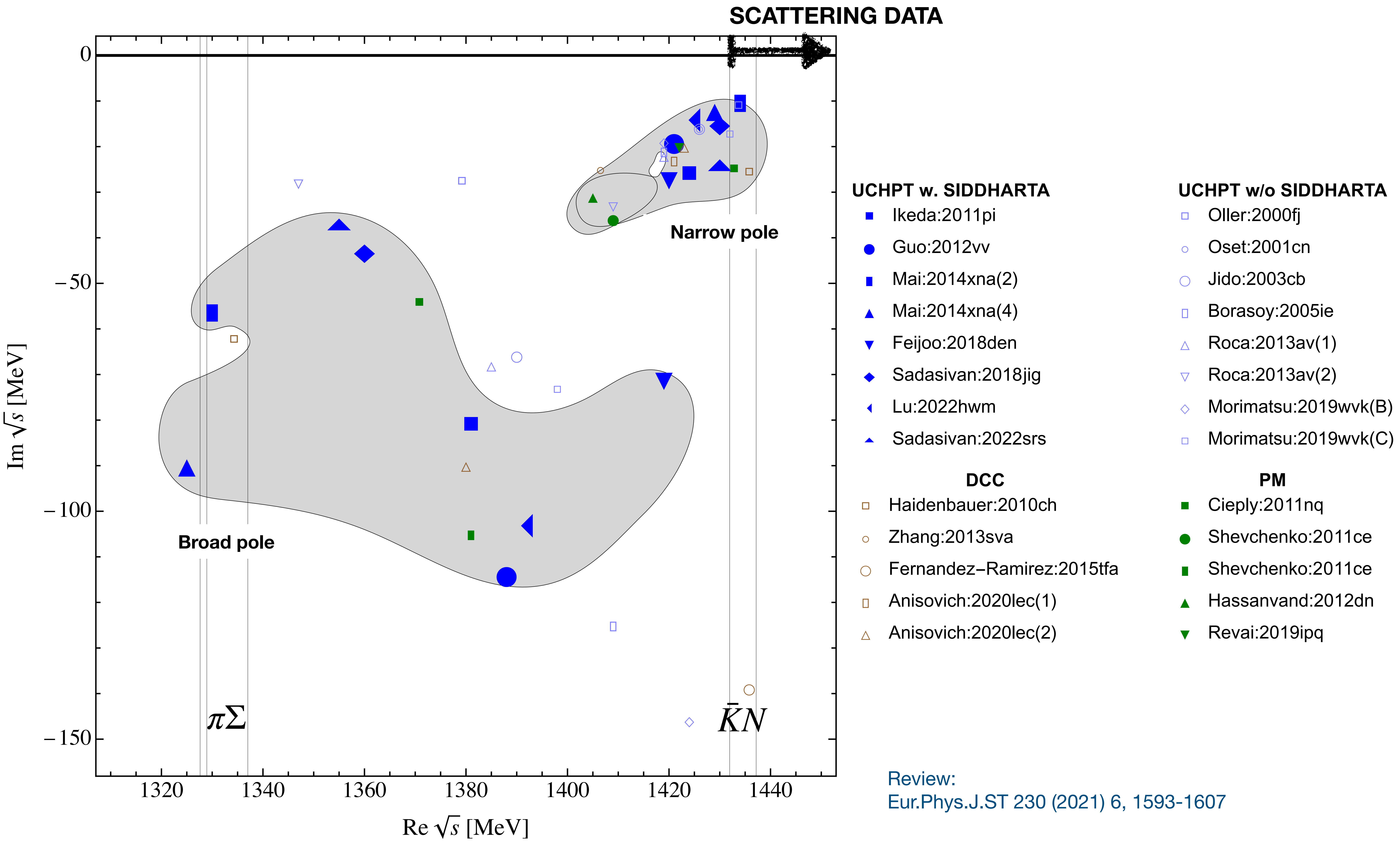
Attention — model dependence

- Renormalisation (*perturbatively*)
- Crossing symmetry (*perturbatively*)
- Power counting (*perturbatively/kernel*)



Kaiser/Siegel/Weise Phys.Lett.B 362 (1995); Lutz/Soyeur Nucl.Phys.A 773 (2006); MM et al. Phys.Lett.B 697 (2011); …
Review: MM, Eur.Phys.J.ST 230 (2021) 6, 1593-1607

STATUS (~2022)



Review:
Eur.Phys.J.ST 230 (2021) 6, 1593-1607

STUDIES BEYOND S=-1

UCHPT kernel respects

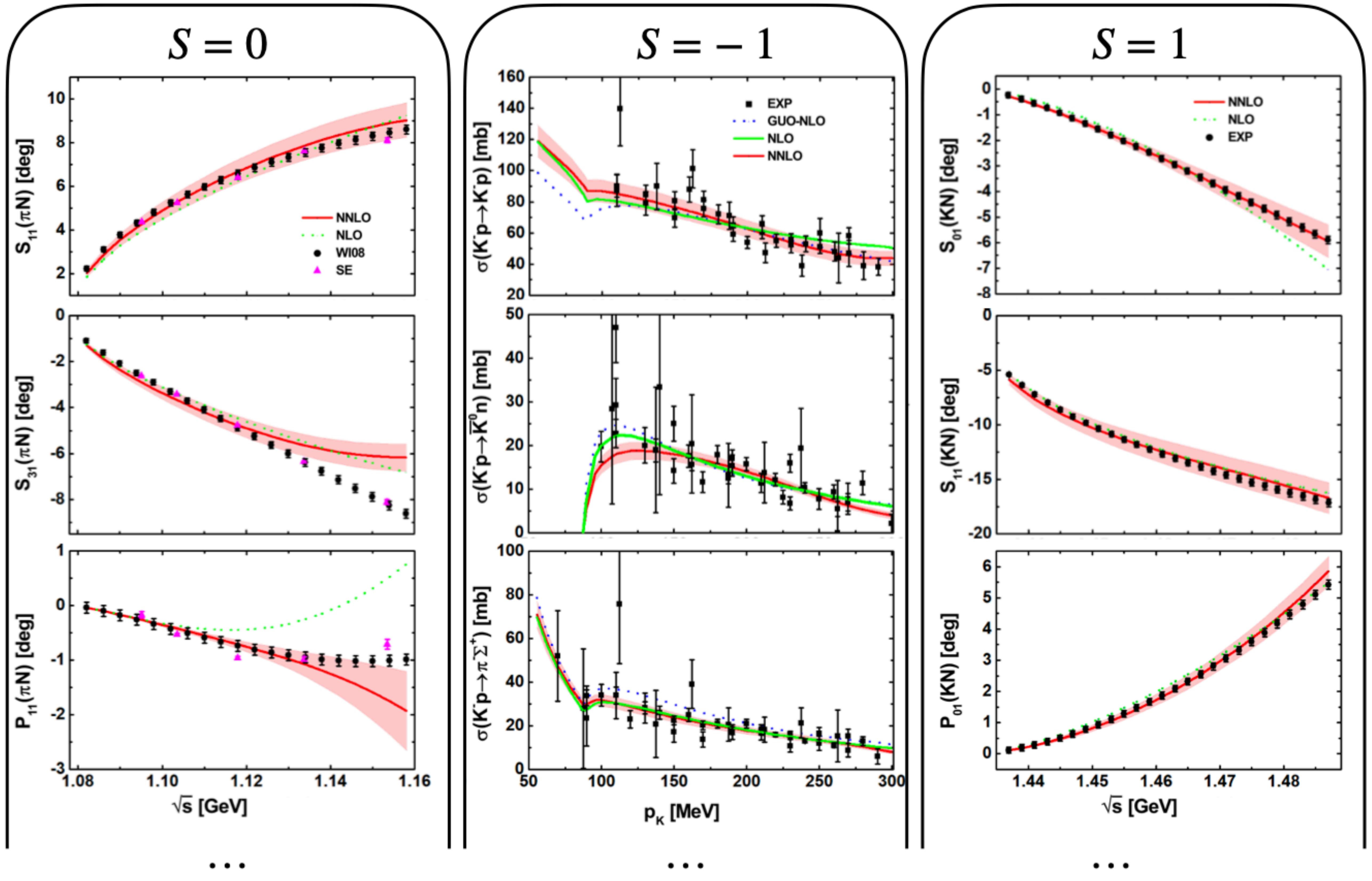
- chiral symmetry
- power counting (systematic expansion)

  … LO→NLO→NNLO

  Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023) 7
- SU(3) symmetry

  … connecting different strangeness sectors

  Lutz/Kolomeitsev *Nucl.Phys.A* 700 (2002) 193-308
  Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023) 7



Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023) 7

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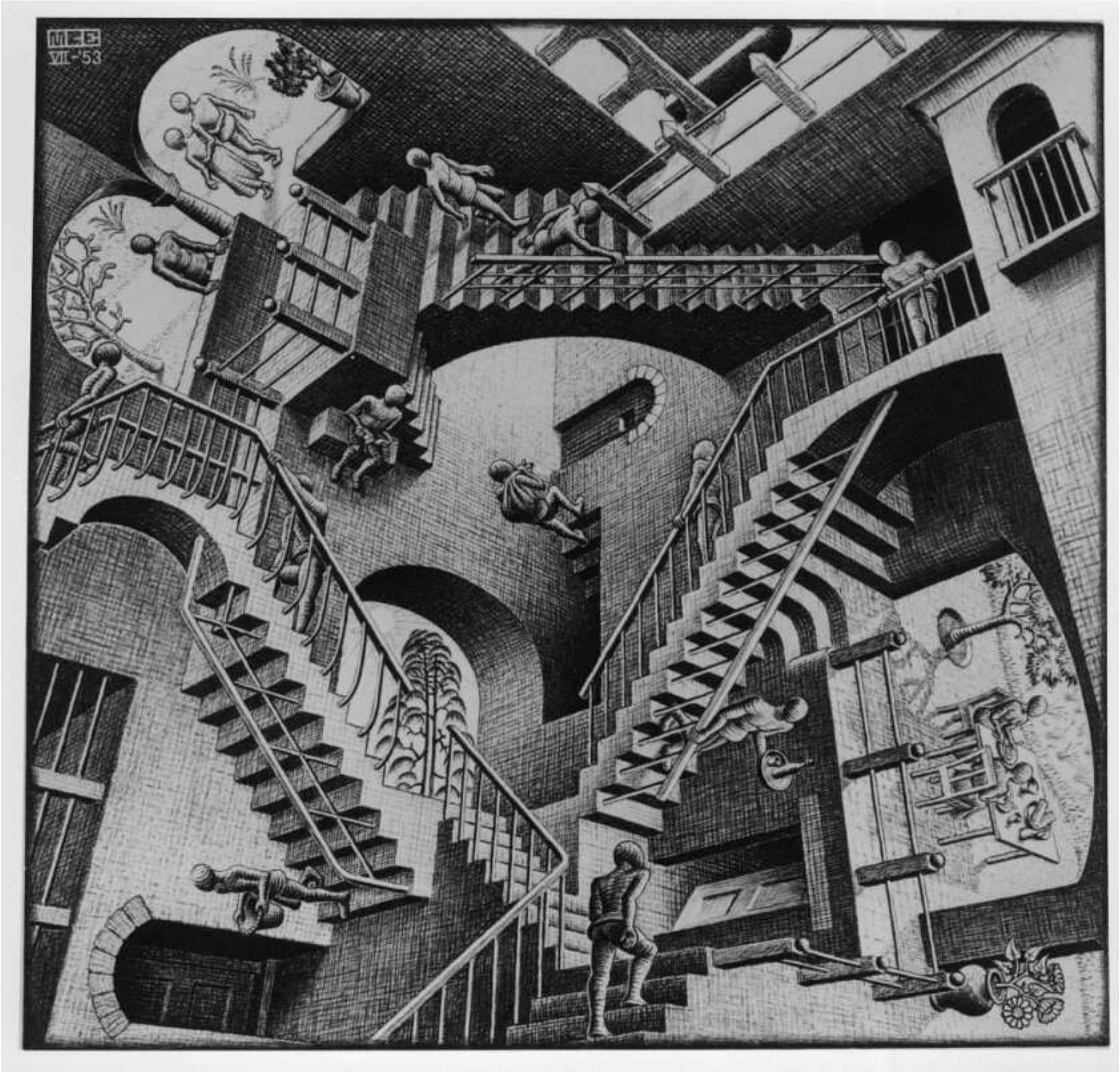


Relativity (1953) – MC Escher

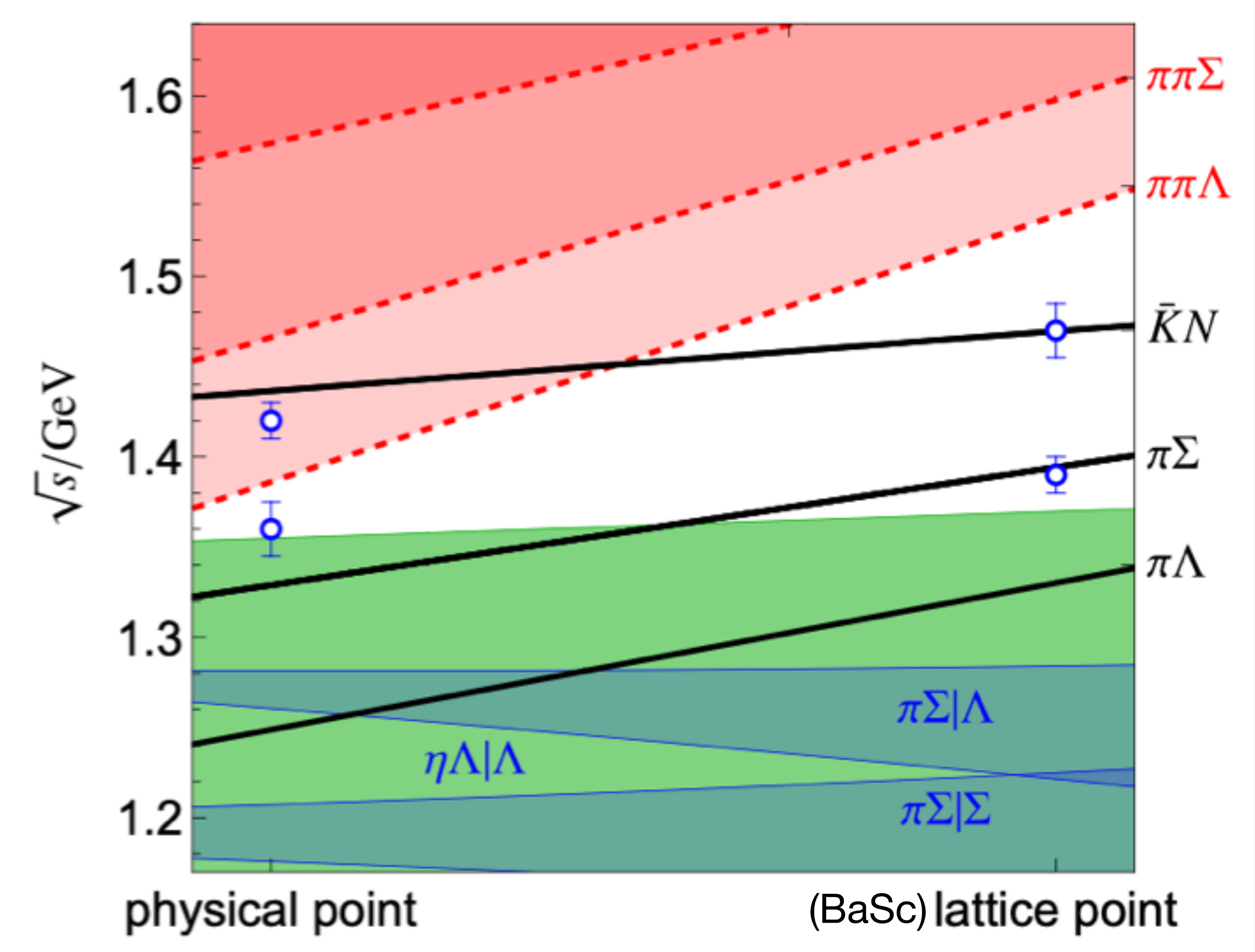
HEAVY UNIVERSE

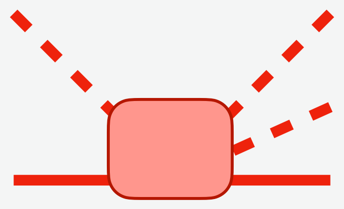
Lattice QCD calculations at unphysical quark masses

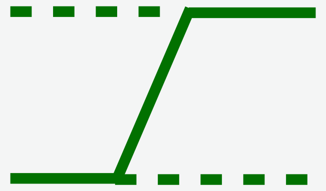
- **Extrapolation to physical point necessary**
  - ➡ Chiral Perturbation Theory dictates quark mass dependence
  - ➡ New insights into hadron spectrum
- **Finite-volume methods are simpler**
  - ➡ inelastic (rhc/lhc) thresholds* are more widely separated

see also anomalous thresholds Korpa/Lutz/… *Phys.Rev.D* 107 (2023) 3, L031505

Phys.Rev.D 112 (2025) 7, 074037

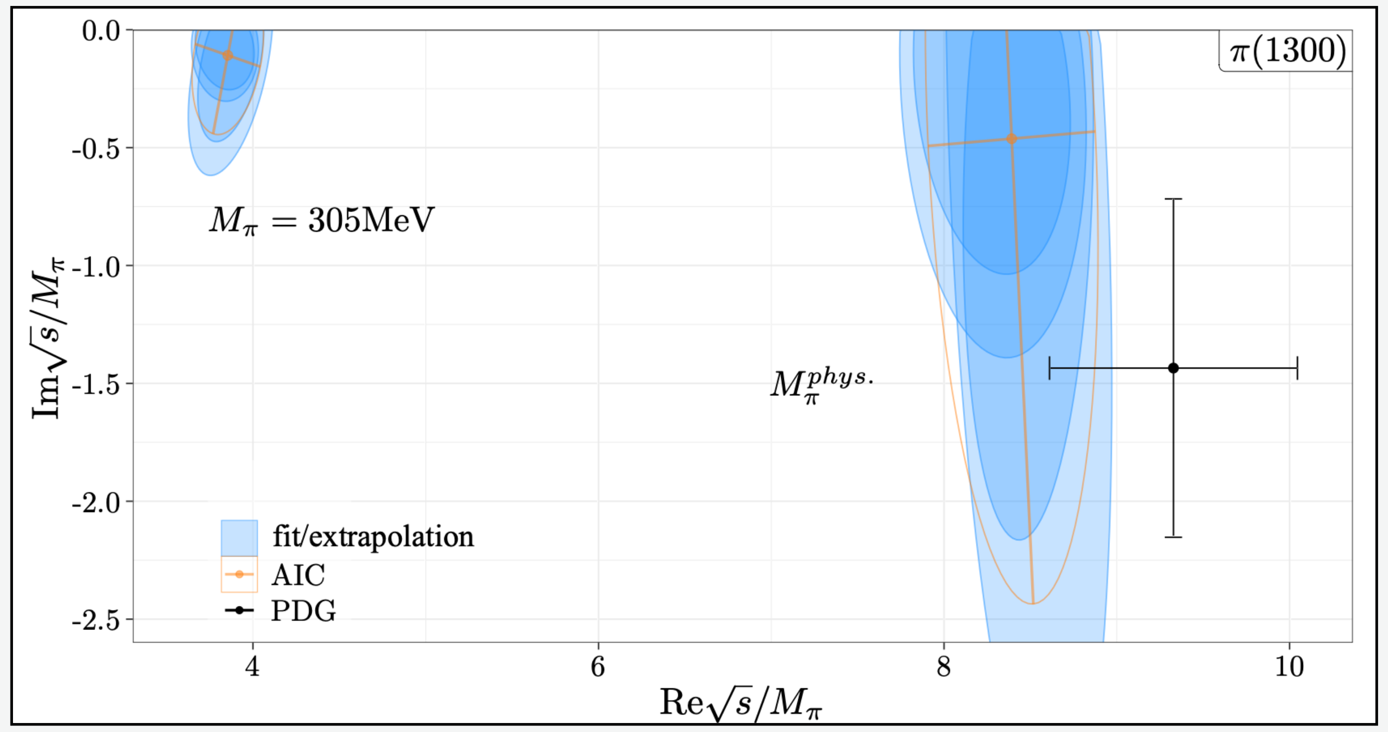




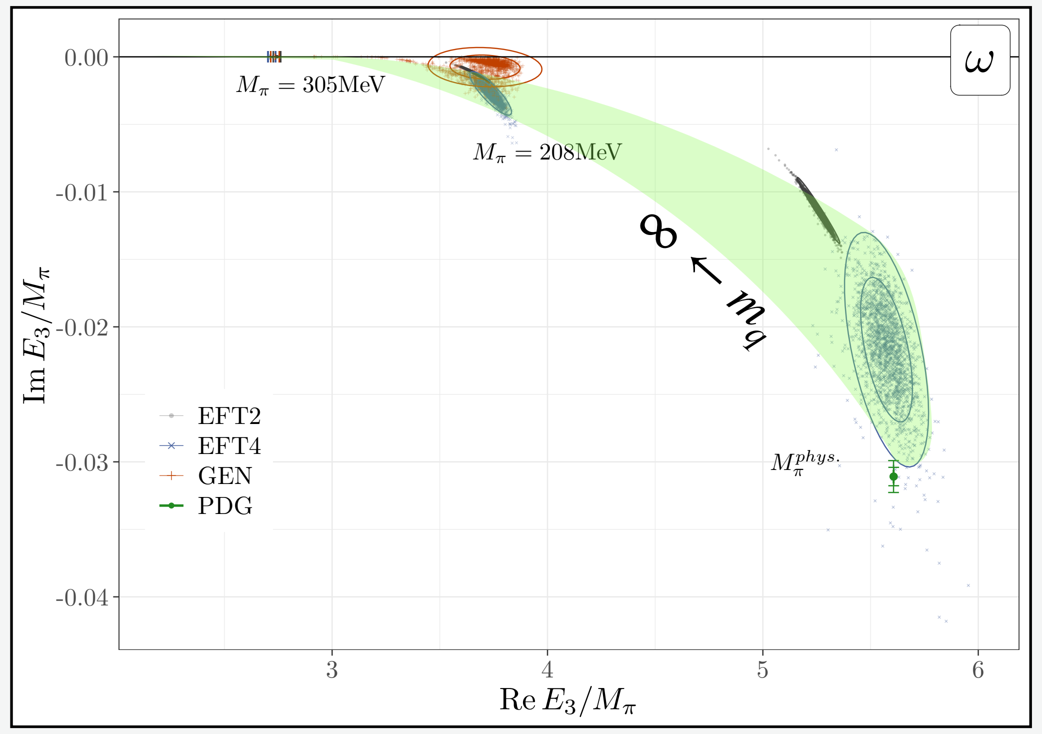


*) 3-particles on-shell:

- lots of progress on resonant 3b systems
- equations are complex
- more degrees of freedom



2510.09476 [hep-lat]



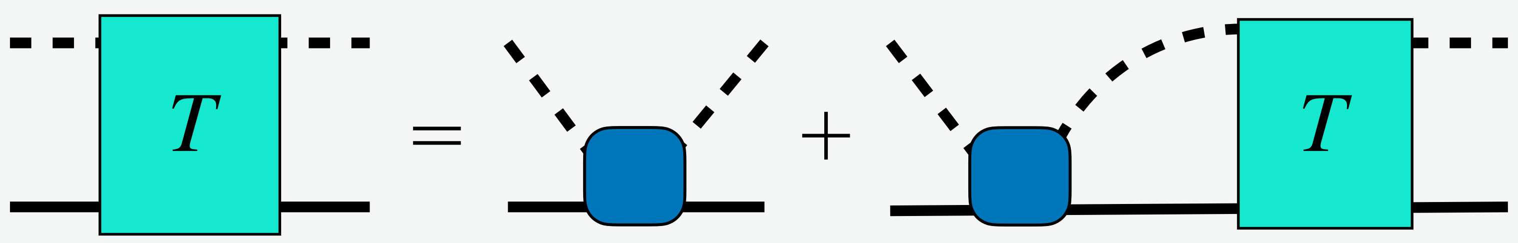
Yan/MM/Garofalo/Meißner/Liu/Liu/Urbach PRL 133 (2024) 21, 211906

CHIRAL TRAJECTORIES

Guo/Kamyia/MM/Meißner *Phys.Lett.B* 846 (2023) 138264

Input

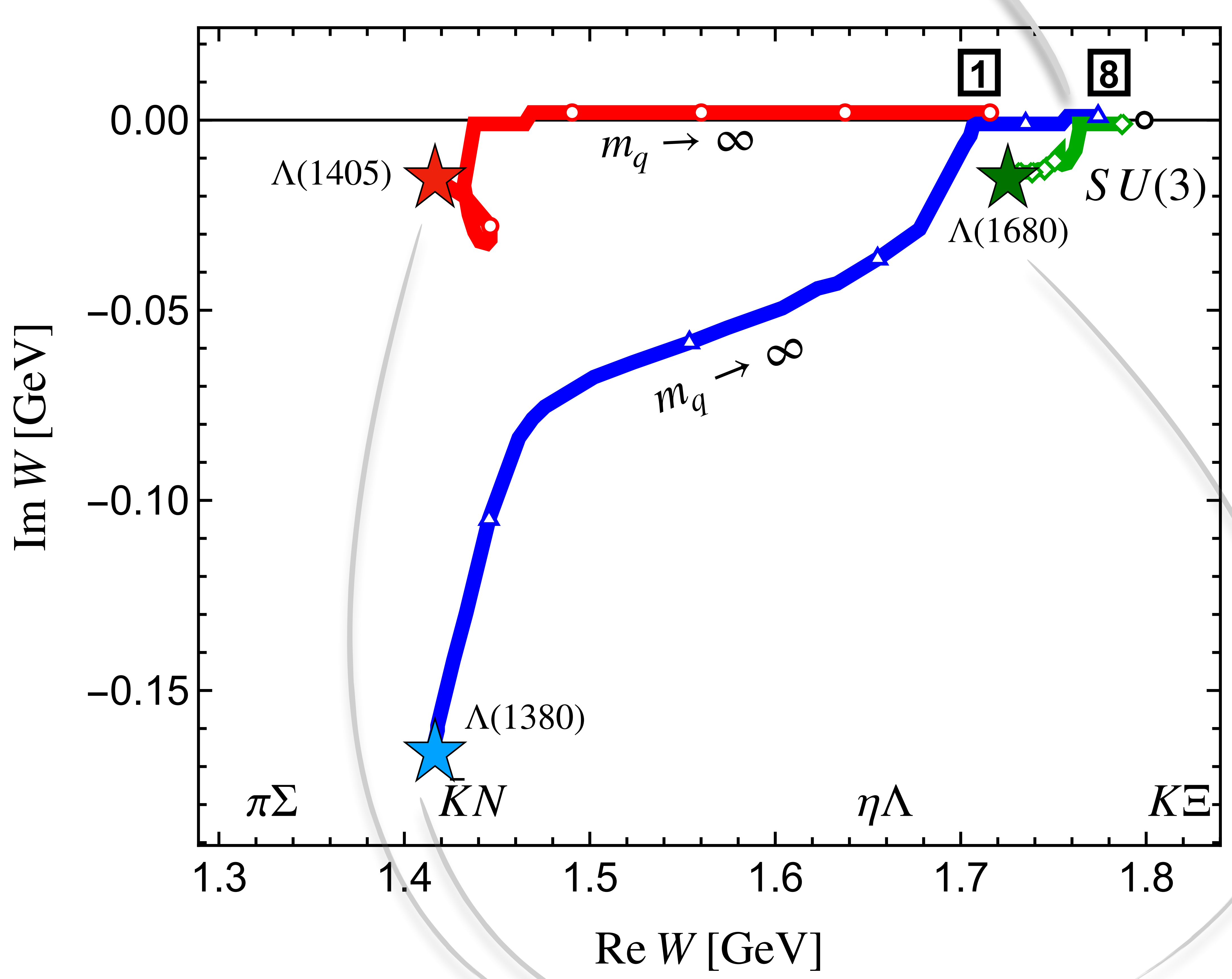
- S-matrix/CHPT/ExpData



Output/Prediction

- resonance parameter
- in our and in the "heavy Universe" $(m_q > m_q^{\text{phys}})$

Bound states in a "heavy universe"



Resonance @ physical point

Experimental data: Nowak/Sakitt/Tovee/Ciborowski/… (LBL/CERN/... 1970s)
Piscicchia+ (AMADEUS) PRC 108, 055201 (2023)
Bazzi+ (SIDDHARTA) PLB 704, 113–117 (2011)
https://github.com/maxim-mai/Experimental-Data/tree/master/Lambda1405

UNPHYSICAL QUARK MASSES

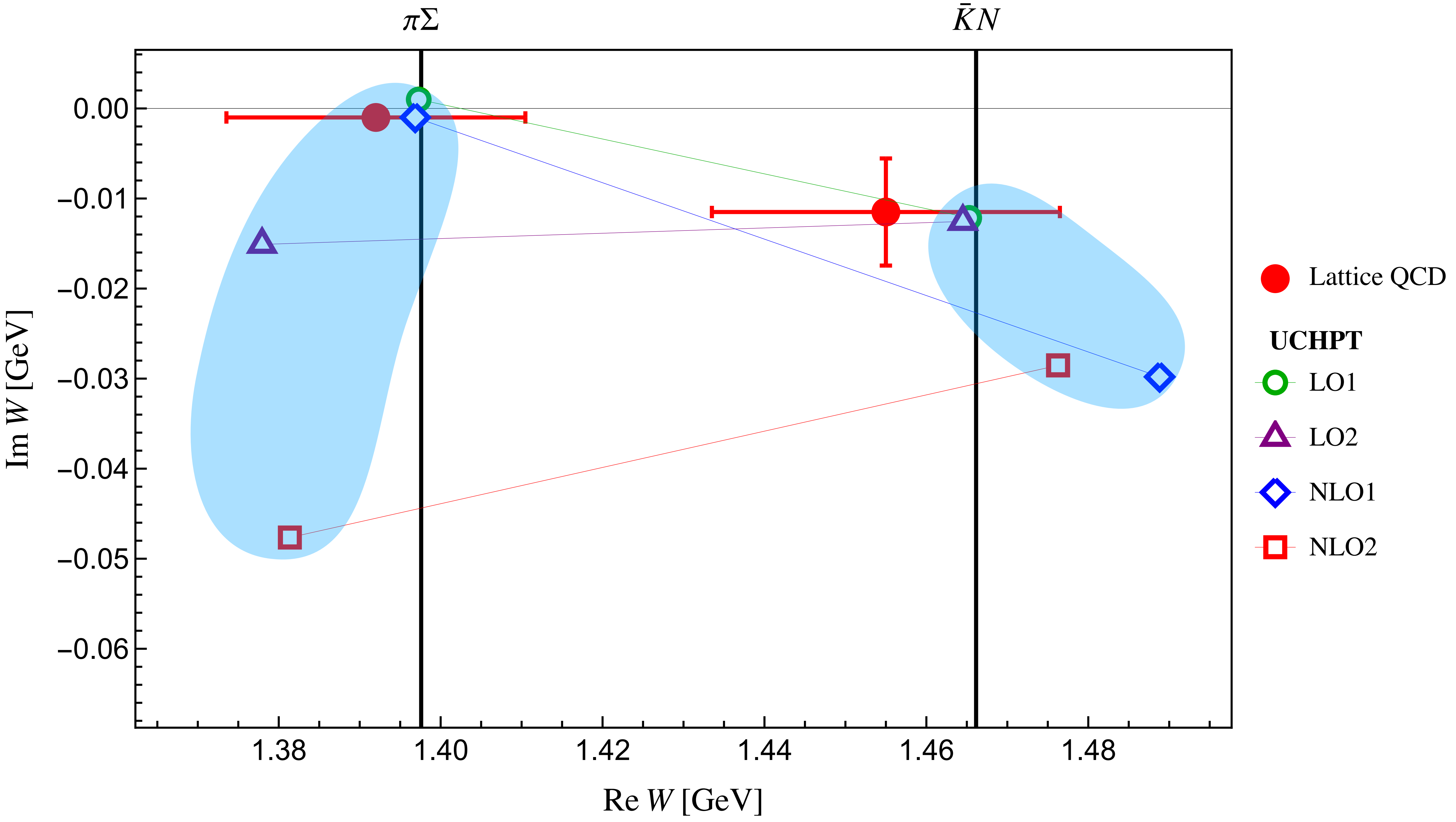
CHPT encodes quark mass dependence

- Available Lattice spectrum — BaSc setup[1]

$$M_\pi \approx 200\,\mathrm{MeV} \;\; M_K = \approx 487\,\mathrm{MeV}$$

$$M_\pi L = 4.181(16) \quad a = 0.0633(4)(6)\,\mathrm{fm}$$

- Compare to prediction of UHPT[2]



[1] [BaSc] Bulava et al. Phys.Rev.Lett. 132 (2024) 5; 2307.13471
[2] Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)

GLOBAL STUDY

Phys.Rev.D 112 (2025) 7, 074037

Include:

S-matrix/CHPT/ExpData + **[BaSC]LatticeSpectrum**

$$\det(1-\tilde{K}B^{\vec{P}})=0$$

Morningstar+ Nucl. Phys. B 924, 477–507 (2017)

- **Many model variations assumed**

  (M1/M2/M3) chiral order truncation (kernel)

  (S1/S2/S3) regularization schemes

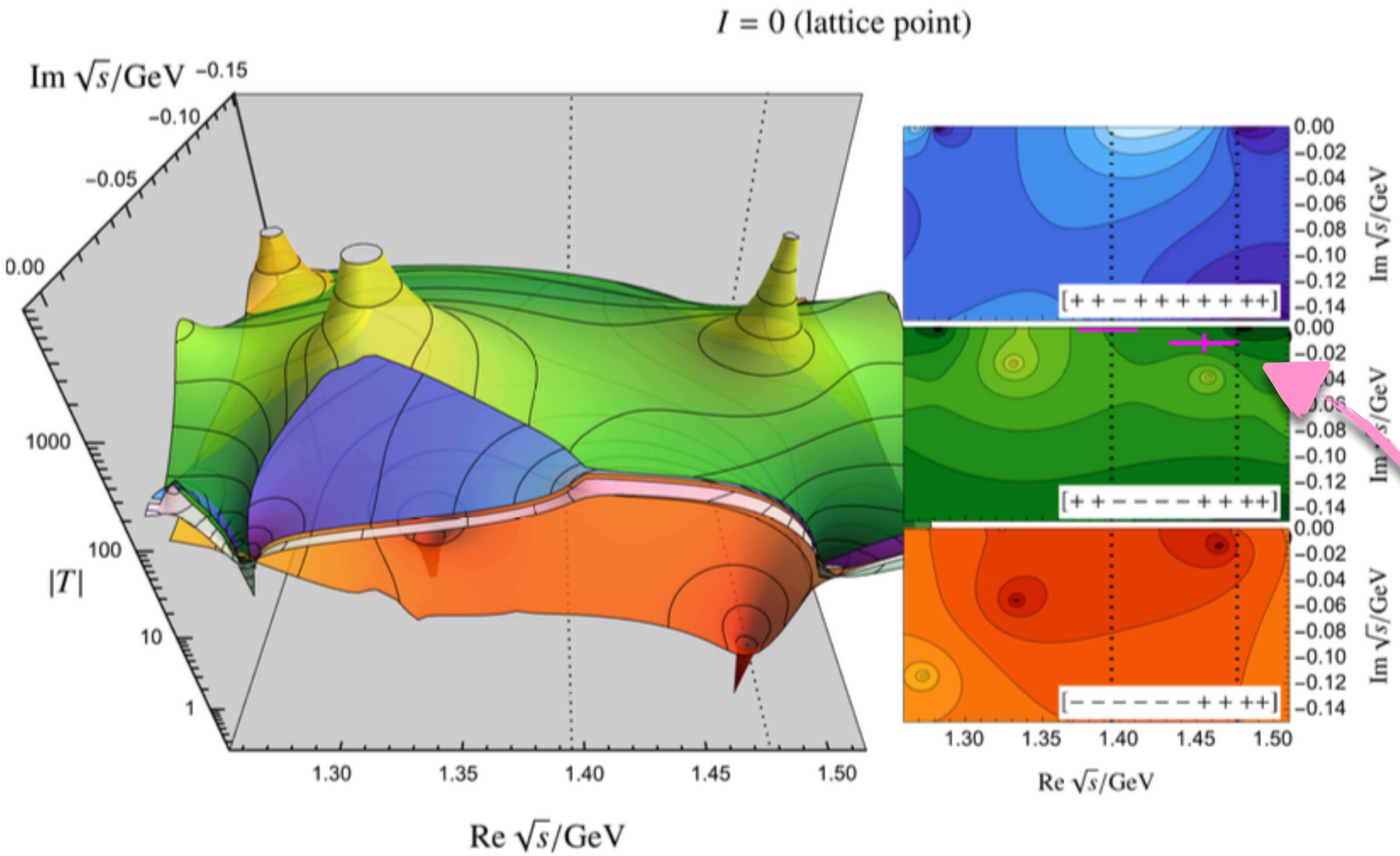
- **Data imperfections**

  (P) Experimental data has unknown systematics

  (L) Lattice input has higher quality (+correlations)

$$\chi^2_{\rm dof}=\frac{\sum_a N_a}{A((\sum_a N_a)-N_{\rm par})}\chi^2_{\rm wt}\quad\text{with } \chi^2_{\rm wt}=\sum_{a=1}^{A}\frac{\chi^2_a}{N_a}$$

Fit	UCHPT type	$N_{\rm data\vert exp.+lat.+m}$	$N_{\rm par.}$	$\chi^2_{\rm dof}$	AIC	BIC
F_{19}	M1S1L	$0+14+0$	3	1.36	7.4	9.3
F_{31}	M1S2L	$0+14+0$	0	2.89	2.9	2.9
F_{18}	M1S3L	$0+14+0$	1	4.42	6.4	7.1
F_{20}	M2S1L	$0+14+0$	3	1.42	7.4	9.3
F_{32}	M2S2L	$0+14+0$	0	2.68	2.7	2.7
F_{25}	M2S3L	$0+14+0$	1	3.54	5.5	6.2
F_{01}	M3S1L	$0+14+4$	10	0.96	21.0	29.9
F_{15}	M3S2L	$0+14+4$	7	0.90	14.9	21.1
F_{10}	M3S3L	$0+14+4$	8	0.92	16.9	24.0
F_{21}	M1S1P	$258+0+0$	6	4.23	16.2	37.5
F_{28}	M1S2P	$258+0+0$	0	25.58	25.6	25.6
F_{27}	M1S3P	$258+0+0$	1	30.28	32.3	35.8
F_{22}	M2S1P	$258+0+0$	6	8.87	20.9	42.2
F_{29}	M2S2P	$258+0+0$	0	48.16	48.2	48.2
F_{26}	M2S3P	$258+0+0$	1	18.69	20.7	24.2
F_{30}	M3S1P	$258+0+0$	16	1.51	33.5	90.4
F_{13}	M3S2P	$258+0+0$	7	1.85	15.8	40.6
F_{11}	M3S3P	$258+0+0$	8	1.50	17.5	45.9
F_{24}	M1S3PL	$258+14+0$	1	27.56	29.6	33.2
F_{23}	M2S3PL	$258+14+0$	1	17.82	19.8	23.4
F_{17}	M3S1PL	$258+14+4$	16	1.44	33.4	91.4
F_{16}	M3S2PL	$258+14+4$	7	2.11	16.1	41.5
F_{12}	M3S3PL	$258+14+4$	8	2.23	18.2	47.2

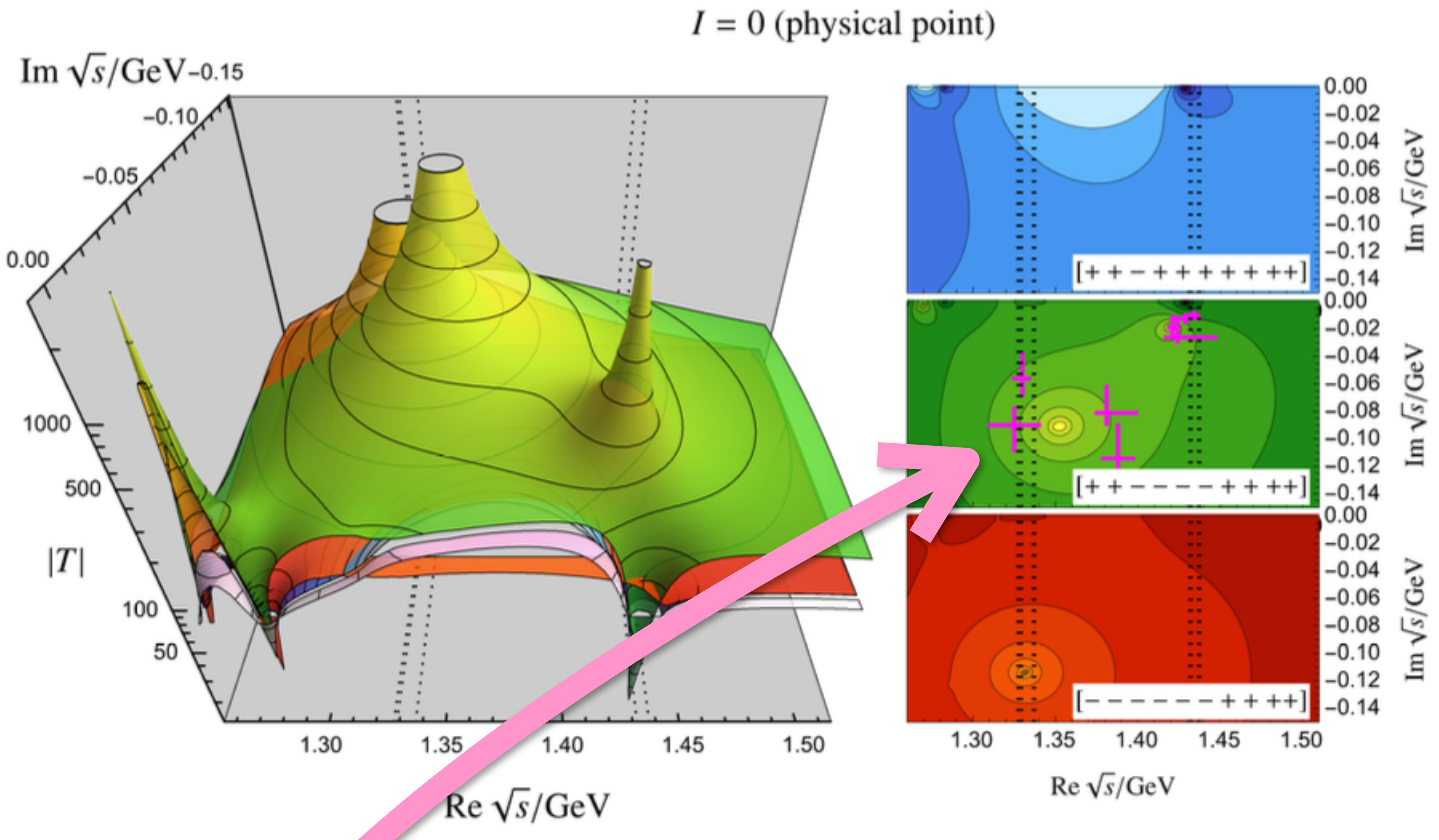


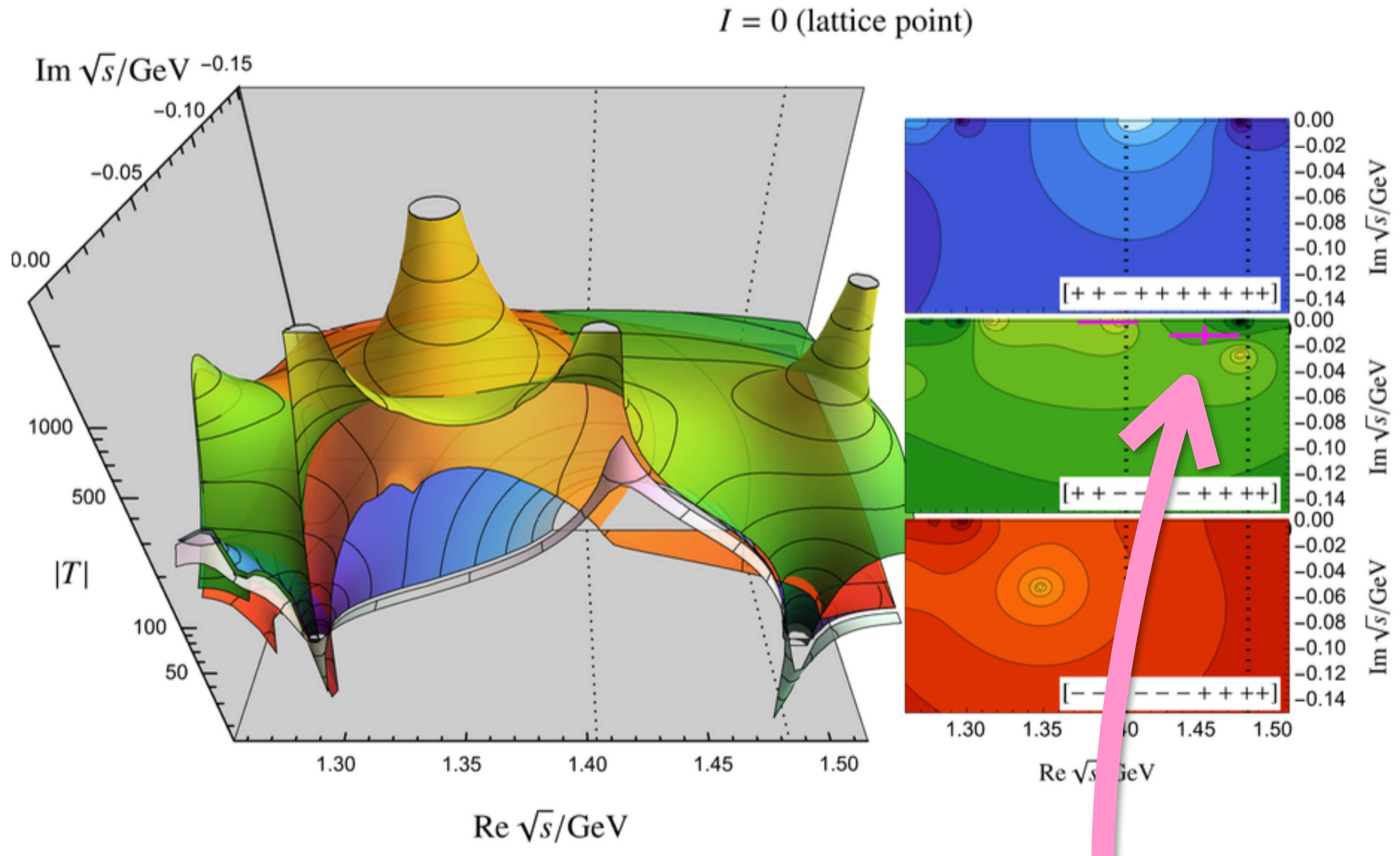
Bulava+ (BaSc) PRL132, 051901 (2024)

M3SXL

- UCHPT do not agree always with generic parametrization (PRL132(2024))
- not enough data? too much freedom?

M3S3PL (F_{12})



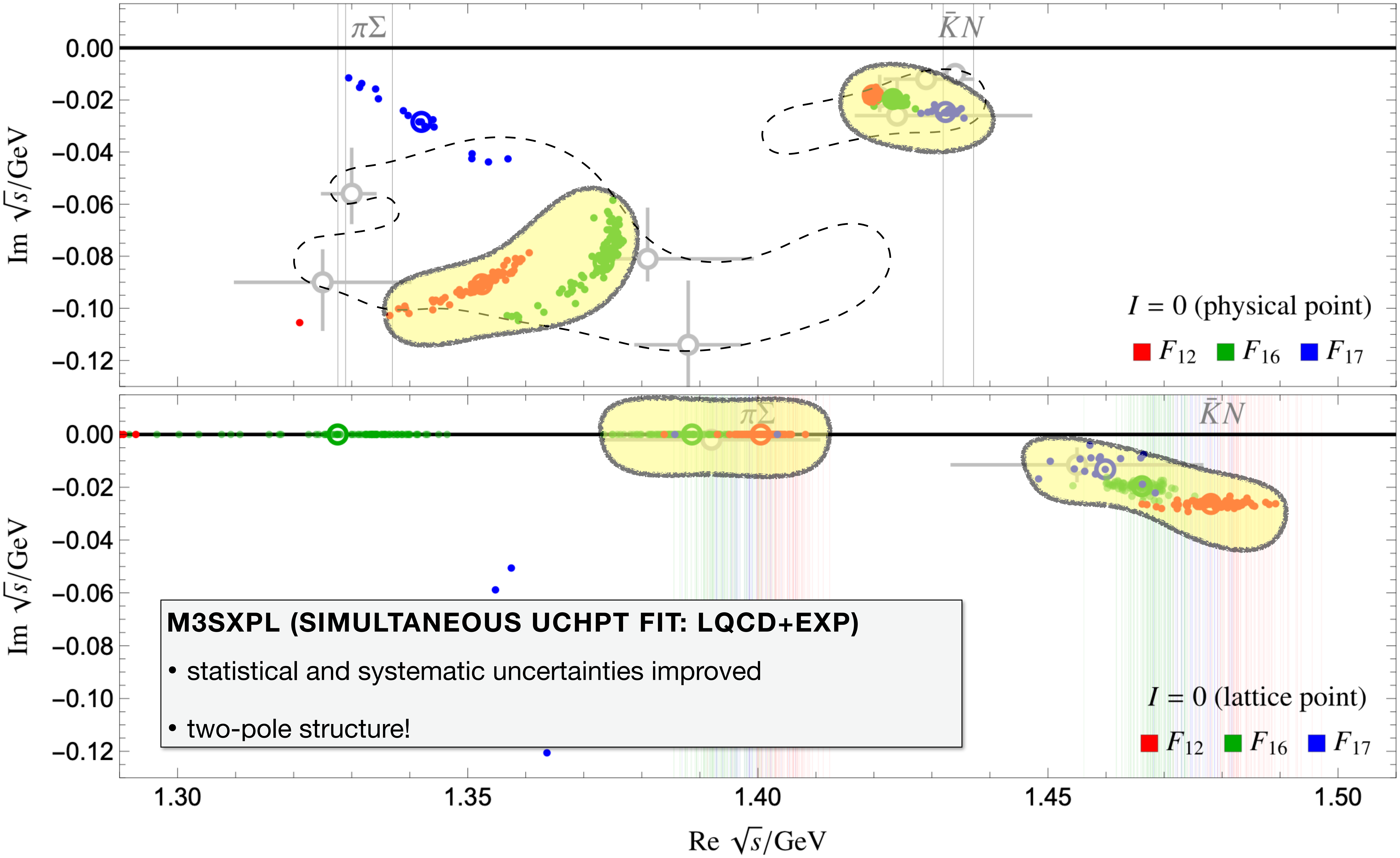


PDG+UCHPT

Bulava+ (BaSc) PRL132, 051901 (2024)

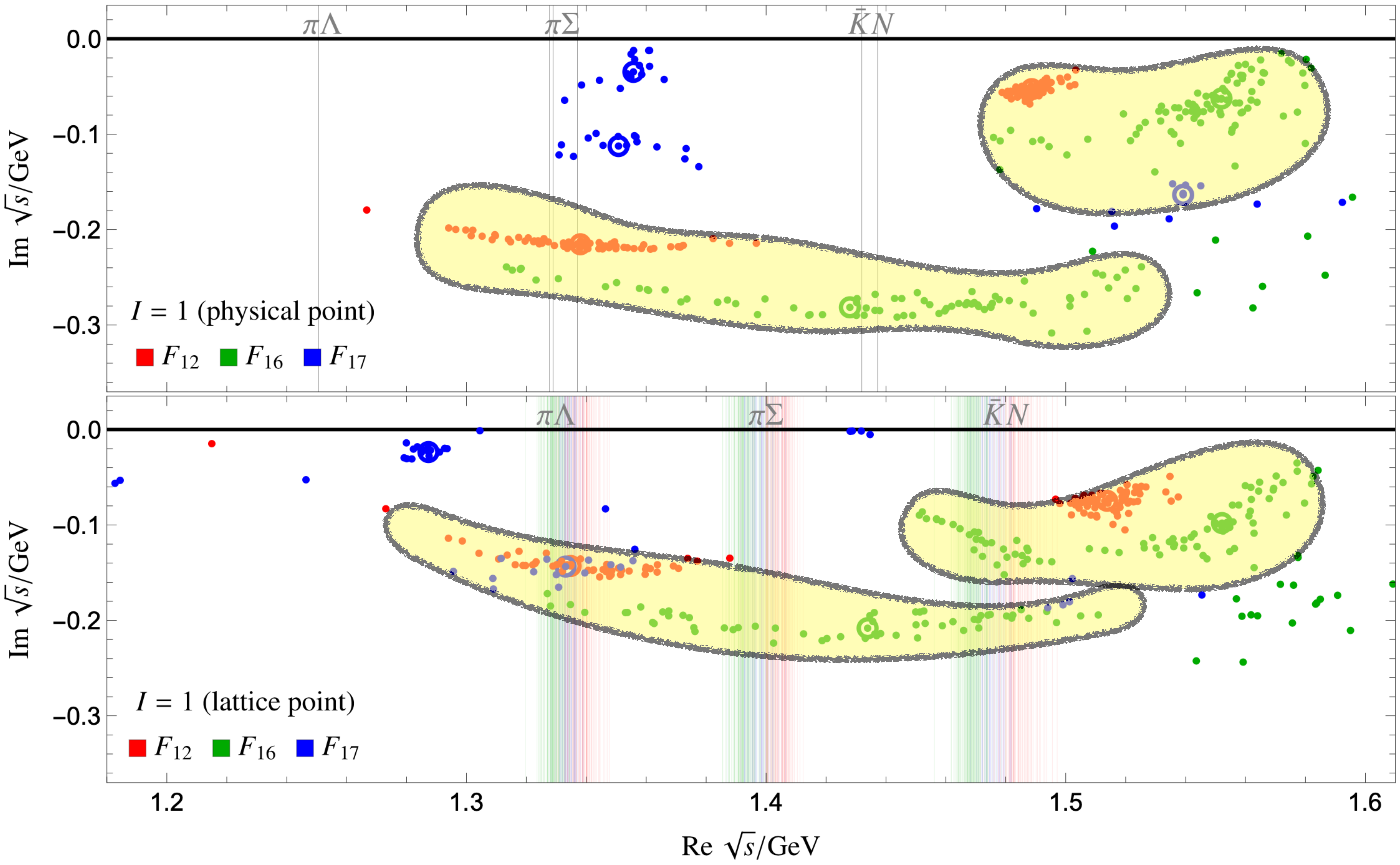
M3SXP

- UCHPT with experimental input only
- extrapolates to similar results (PRL132(2024)) at the lattice point



M3SXPL (SIMULTANEOUS UCHPT FIT: LQCD+EXP)

- statistical and systematic uncertainties improved
- two-pole structure!



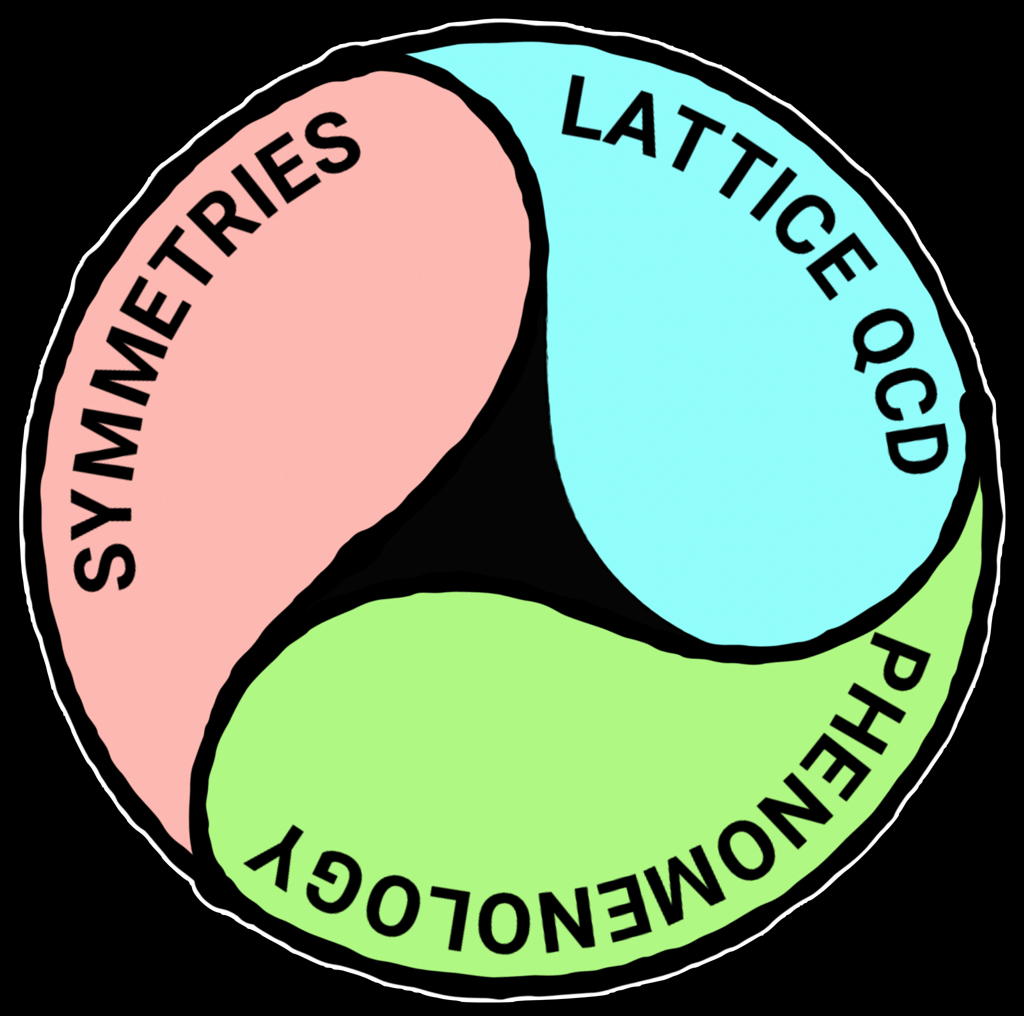
M3SXPL (SIMULTANEOUS LQCD+EXP UCHPT FITS)

- **2-pole structure of I=1 state**
- **Poles are partially hidden (Thresholds)**

Effective Field Theories
- quark-mass dependence
- analytical tools
- dynamically generated resonances

S-matrix
- Mathematical constraints on transitions
- Universal resonance parameter

Lattice QCD:
- ab-initio calculations
- universal tool for physical und unphysical scenarios
- many new advances and results



SUMMARY

Combined UCHPT+EXP+LQCD studies

- Two-pole structure: $\Lambda(1405)$, $\Lambda(1380)$
- Chiral trajectories of resonances

OUTLOOK

1) Predicted I=1 states

- Can one see those on the lattice? [lattice homework]

2) Are there robust predictions for Hyperons [UCHPT homework]

3) Some fits to lattice spectra disagree with BaSC

- BaSC functional basis flexible enough? [somebody's omework]
- What does this mean for the Nevanlinna approach (*PRD* 112 (2025) 11)? [somebody's homework]

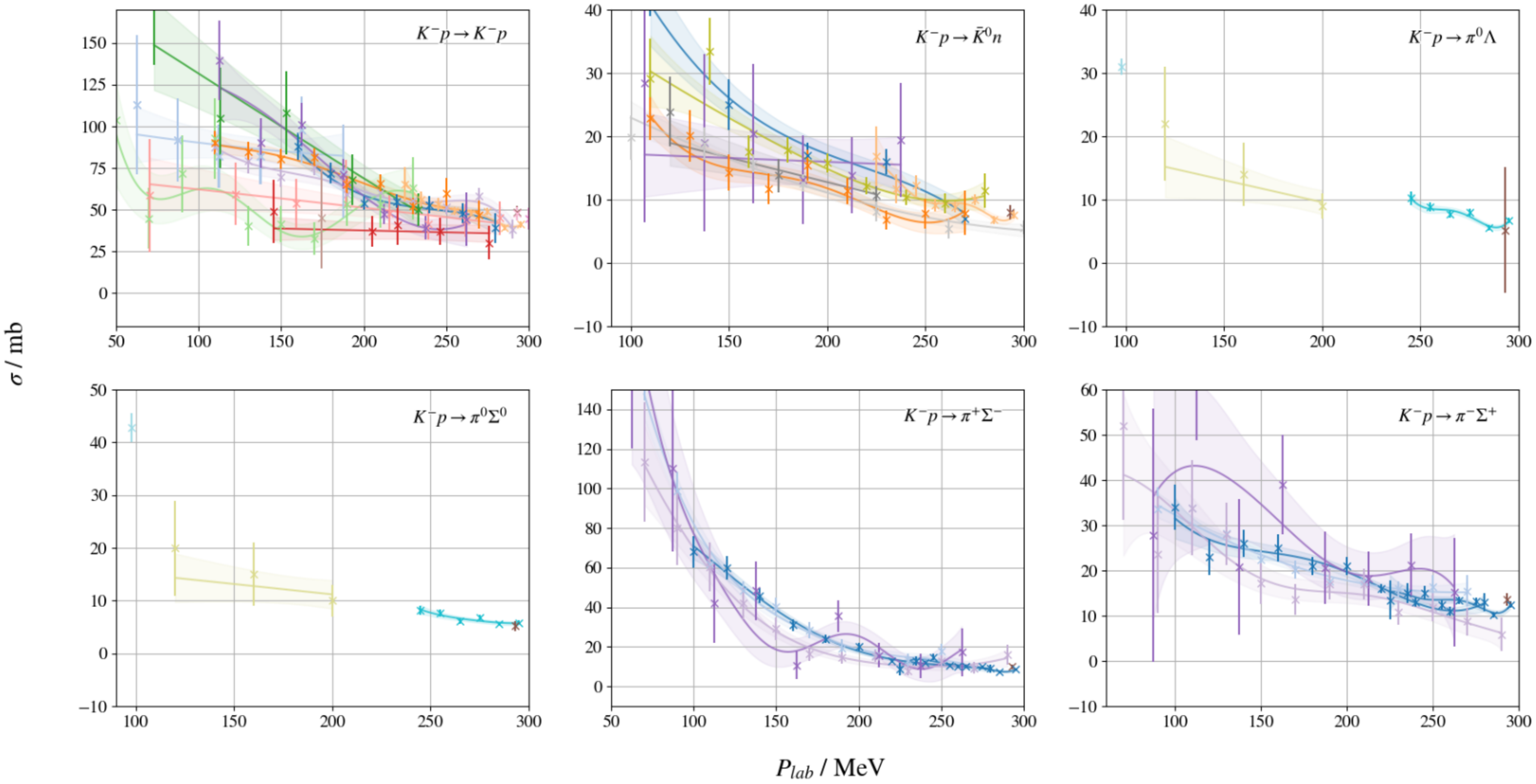
- D200 ensemble of the CLS collaboration
- 2+1 flavor of non-perturbatively improved Wilson fermions
- Lüscher-Weisz gauge action
- $(\pi(\vec{p}_1)\Sigma(\vec{p}_2), \bar{K}(\vec{p}_1)N(\vec{p}_2))$ operators

L[fm]	a[fm]	M_π[GeV]	M_K[GeV]	M_η[GeV]	m_N[GeV]	m_Σ[GeV]	m_Λ[GeV]	m_Ξ[GeV]
4.05(4)	0.0633(7)	0.2036(8)	0.4864(5)	0.5511	0.979(11)	1.193(6)	1.132(4)	1.322(3)

$$QC = 1 - \tilde{K}B;$$

$$\mathcal{B} = diag(u_a R^a_{00});$$

$$R_{\ell m} = \frac{1}{\gamma \pi^{3/2} u_a^{\ell+1}} \mathcal{Z}_{\ell m}(s_a, \gamma, u_a^2)$$



Nowak/Sakitt/Tovee/Ciborowski/... (LBL/CERN/... 1970s)
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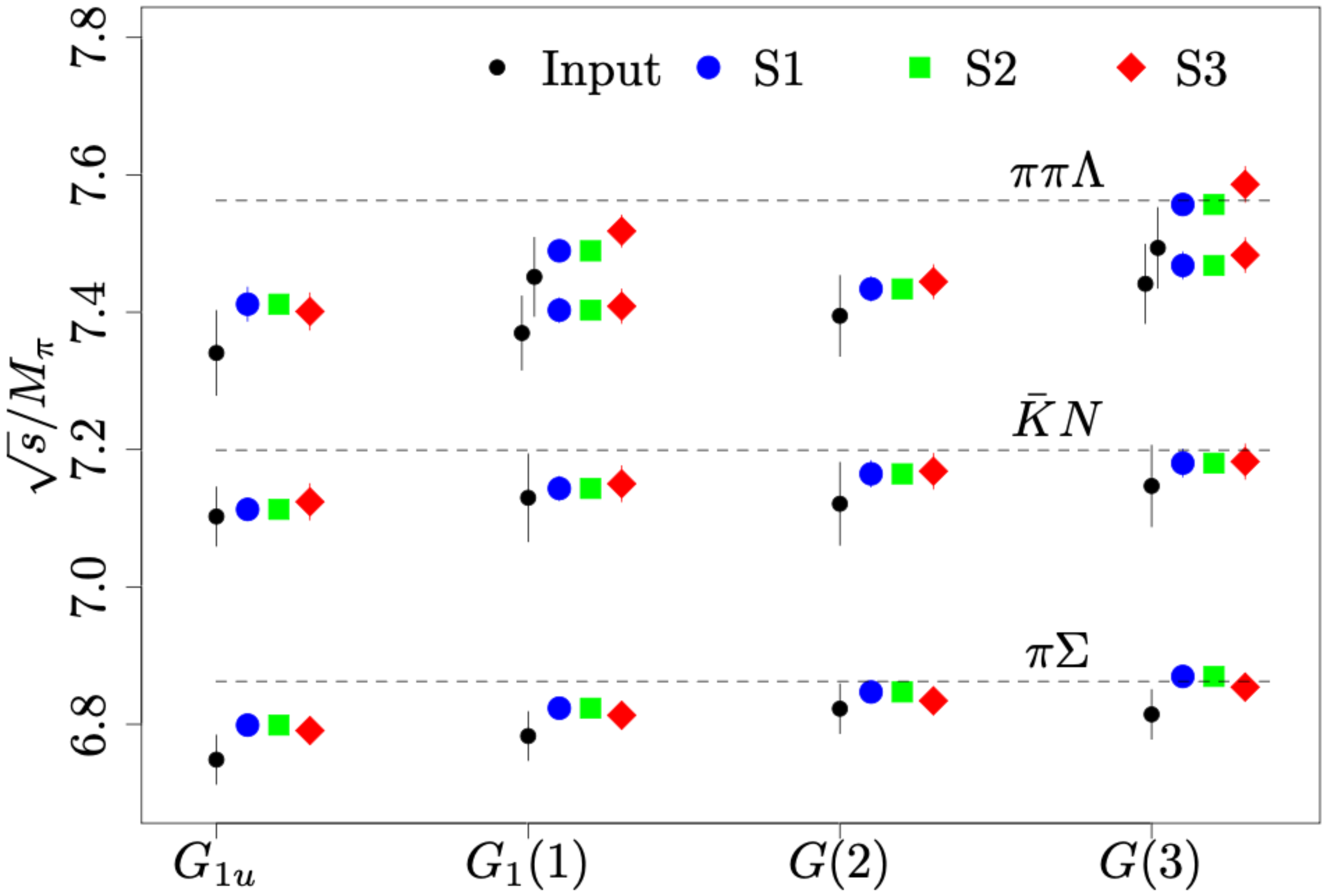

Input
S1
S2
S3
$\pi\pi\Lambda$
$\bar{K}N$
$\pi\Sigma$
7.8
7.6
7.4
7.2
7.0
6.8
$\sqrt{s}/M_\pi$
G_{1u}
$G_1(1)$
$G(2)$
$G(3)$

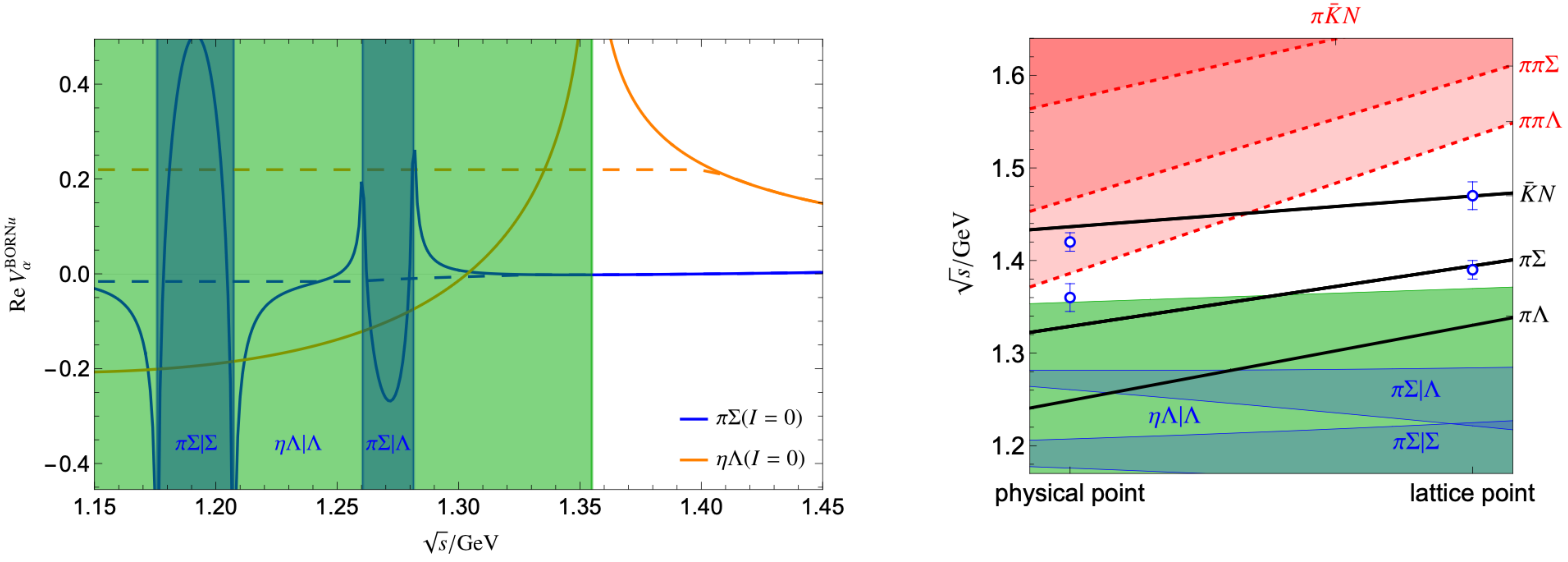

Figure 5. Three-body related singularities for the physical and unphysical quark-mass setups. Left: Singularities of the u-channel Born diagram Eq. (4.10) projected to $I = 0$ for the initial/final states as specified in the legend for physical quark masses. Long-dashed lines represent the potential implemented in the UCHPT amplitude mitigating the appearance of u-channel left-hand cuts. Right: Relevant short left-hand cut associated with the u-channel exchange (green, blue areas), c.f. left figure. Black solid and red dashed lines denote the position of the right-hand cut branching points with respect to two- and three-body states, respectively. Energy region with no allowed three-body onshell states is the remaining white area. Blue dots with error bars denote the averaged result from the global analysis discussed in Sect. VI.