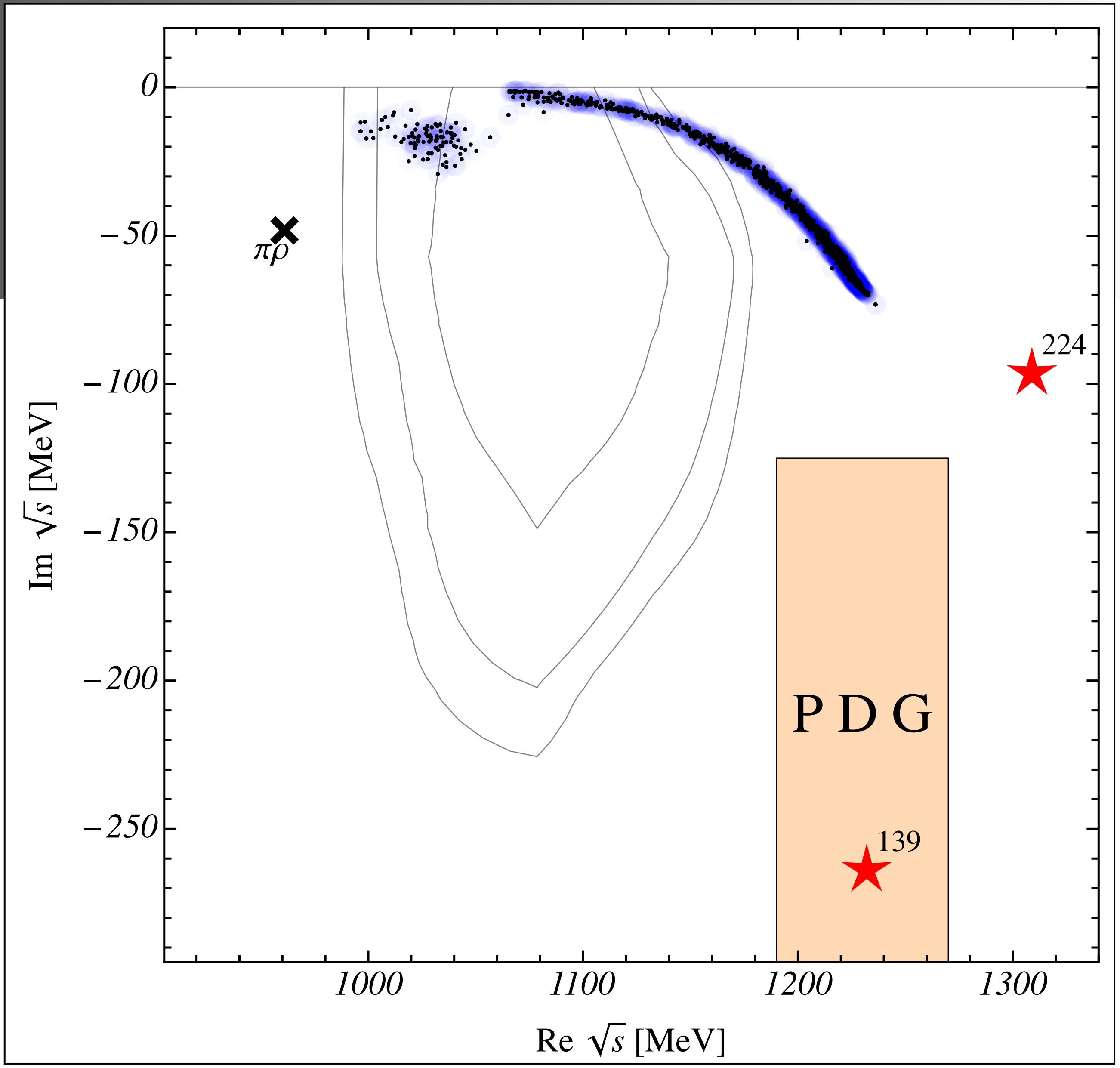
Three-body dynamics of the $a_1(1260)$ resonance from lattice QCD

2107.03973 [hep-lat]



Maxim Mai, *A. Alexandru, R. Brett, C. Culver,*

M. Döring, F. Lee, D. Sadasivan

[GWQCD]

DE-SC0016582
DE-FG02-95ER40907
PHY-2012289
Deutsche Forschungsgemeinschaft
DFG
MA-7156

EXCITED HADRONS

https://maxim-mai.github.io/talks/PWA21-MM.pdf

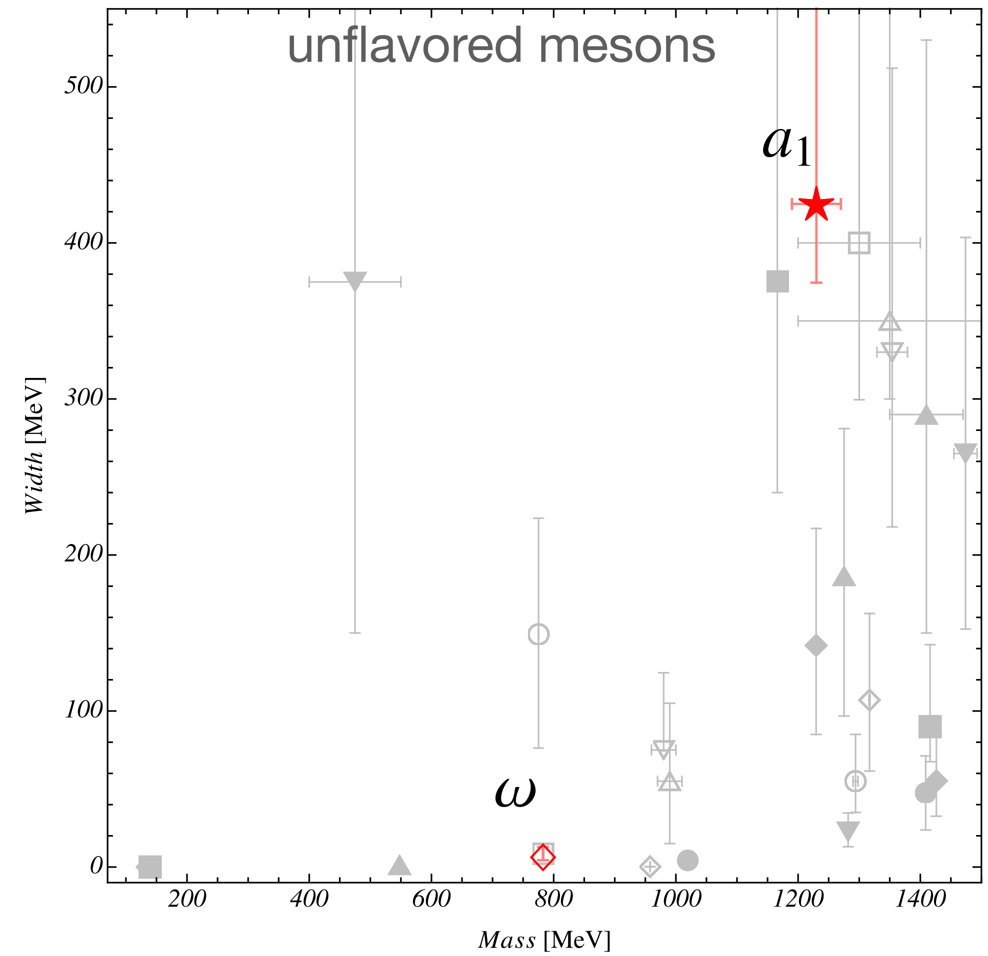
Pattern and **production mechanisms** of many excited hadrons are unresolved issues

Many questions are related to the hadronic 3-body problem

- Roper resonance *N*(1440)*
- exotic mesons: π_1(1600), ...

experimental searches @COMPASS, GlueX

- ω(782), a_1(1260), ...



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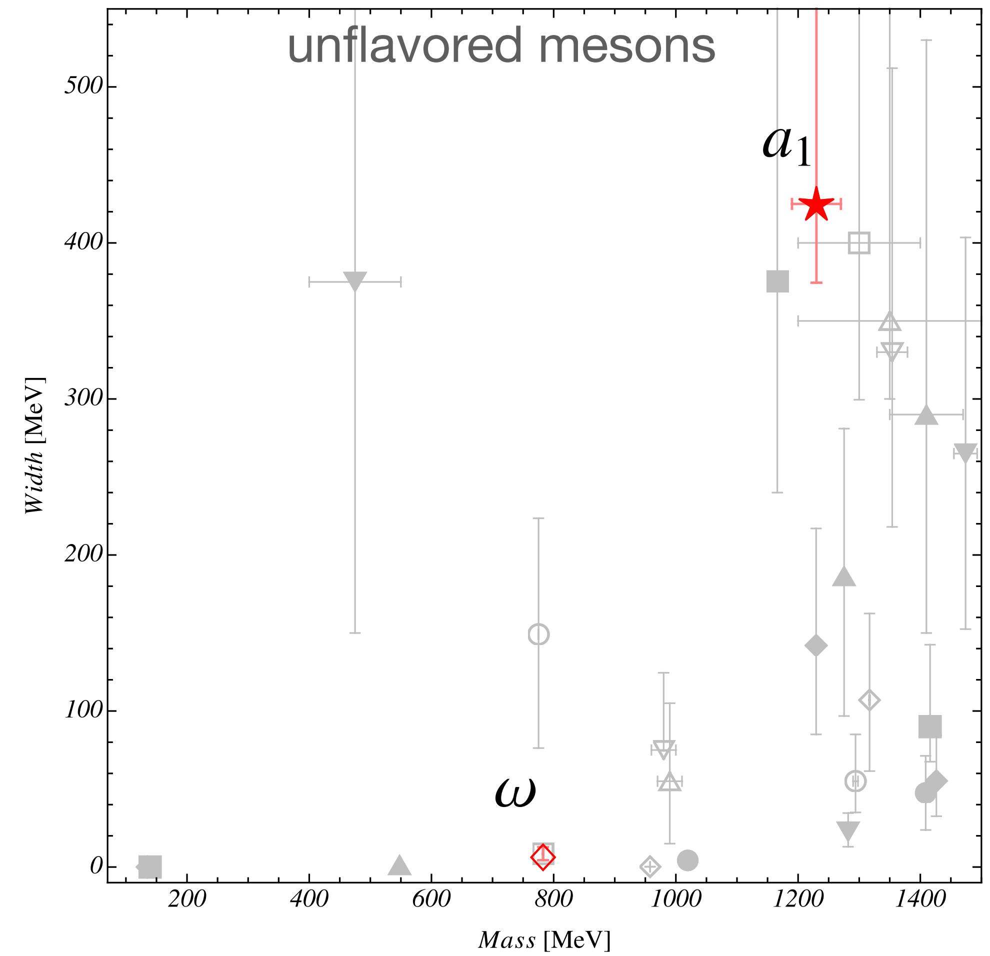


This work: a_1(1260) from lattice QCD

- Goal #1: Resonant three-body system in lattice QCD.
- Goal #2: First-ever determination of parameters of three-body resonance from QCD.

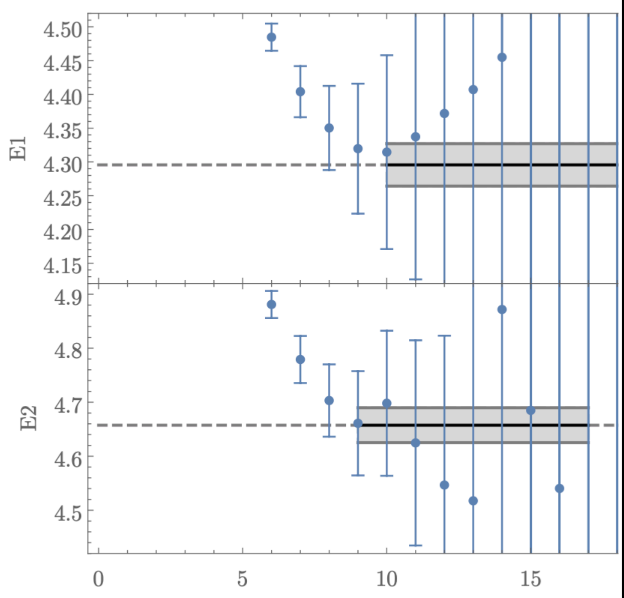
Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:

Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:

Finite-volume lattice QCD spectrum



EXCITED HADRONS

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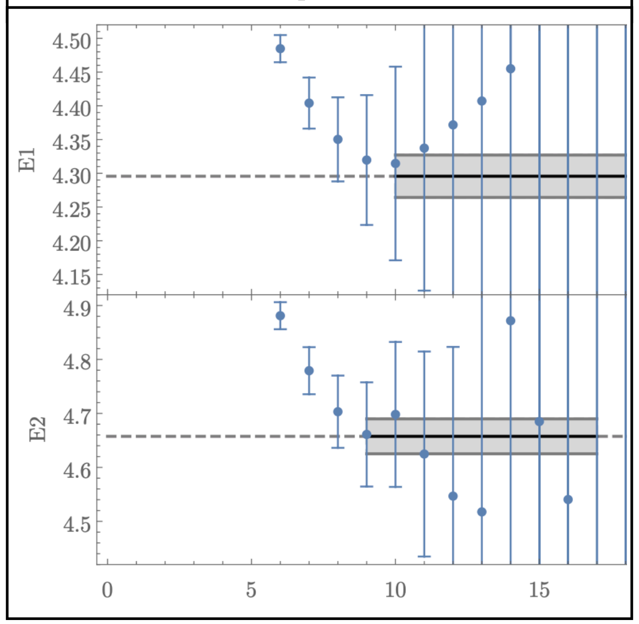
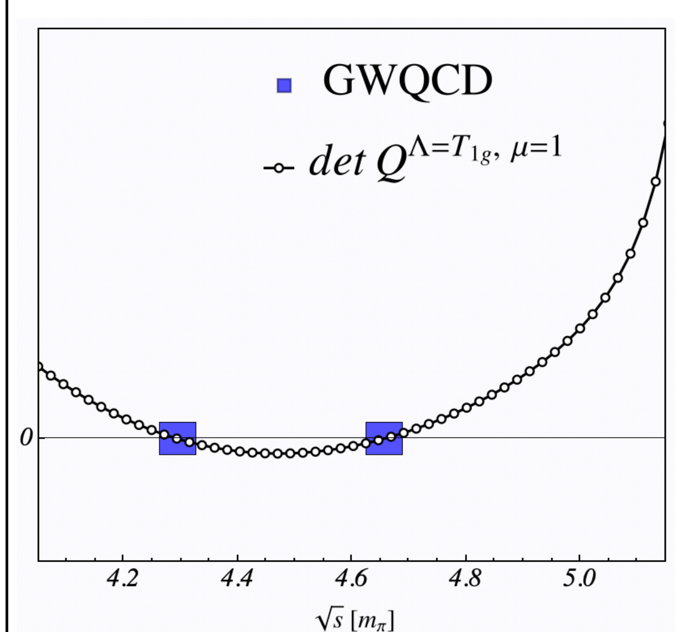


energy eigenvalues

3-body quantization conditions



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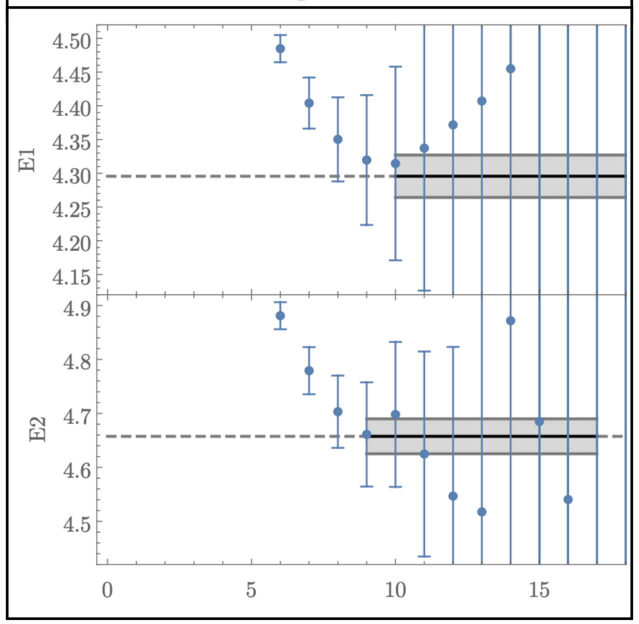

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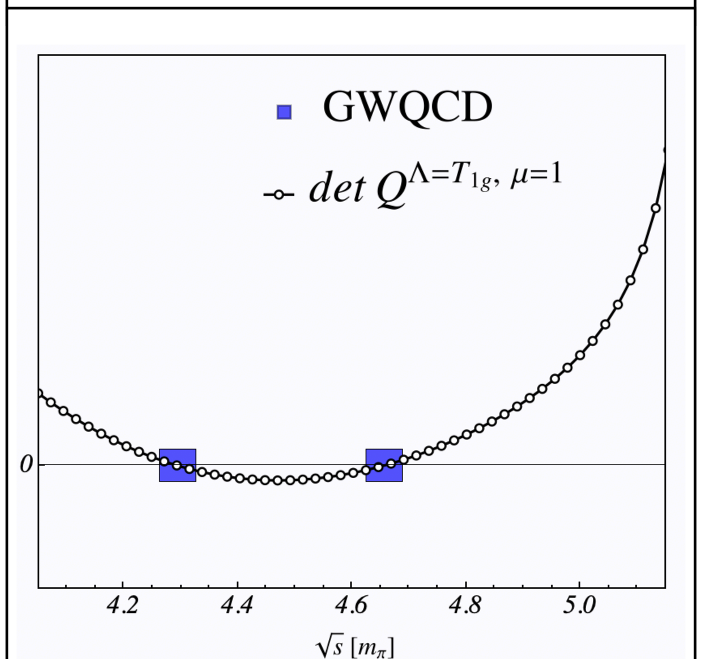
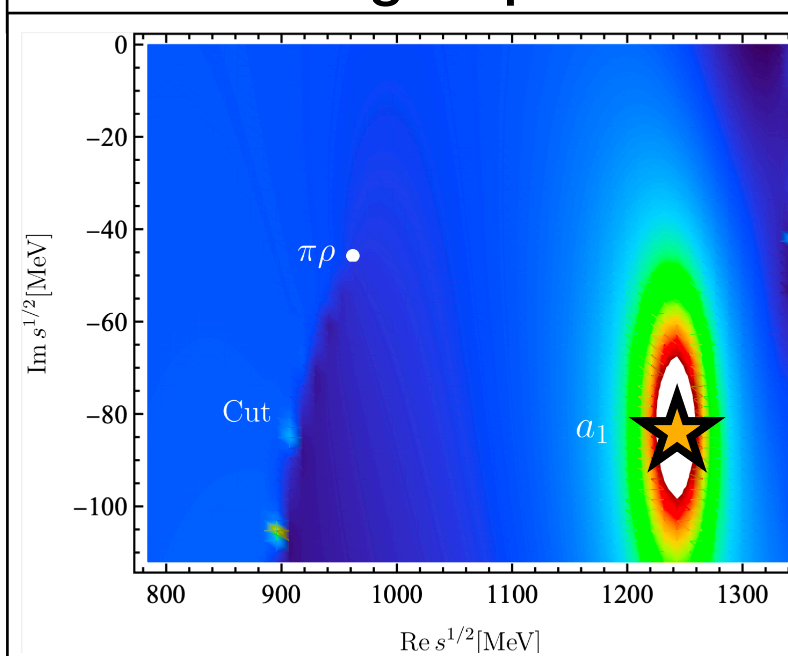


volume-independent quantities

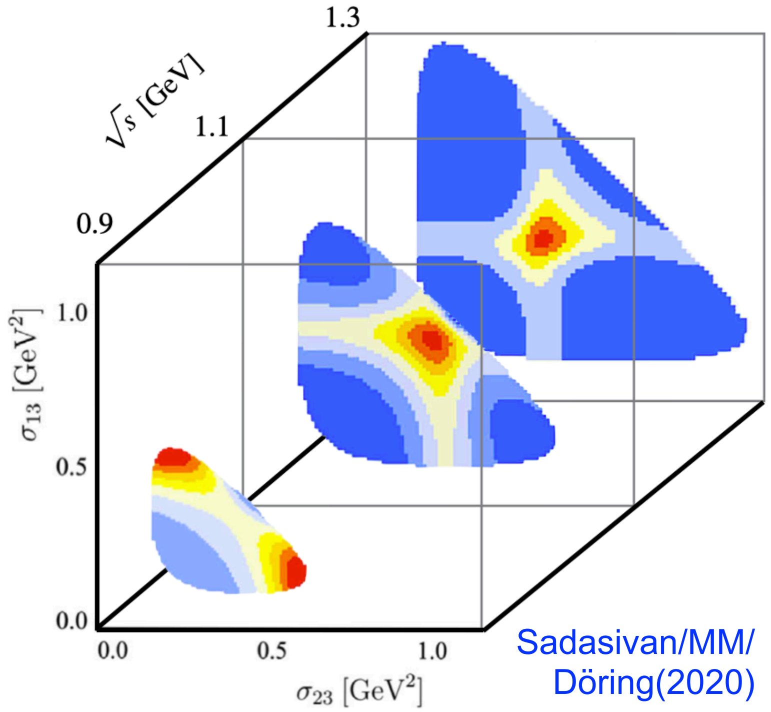
Riemann Surface: scattering amplitude



Phenomenological studies

- A clean way: a_1(1260) from τ-decays (ALEPH@CERN)

Mikhashenko et al. [JPAC] (2018) Sadasivan/MM/Döring(2020)



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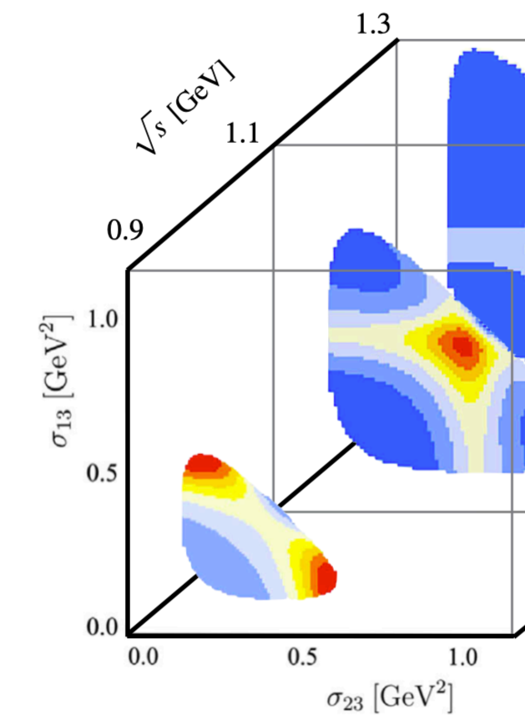
LATTICE QCD

https://maxim-mai.github.io/talks/PWA21-MM.pdf



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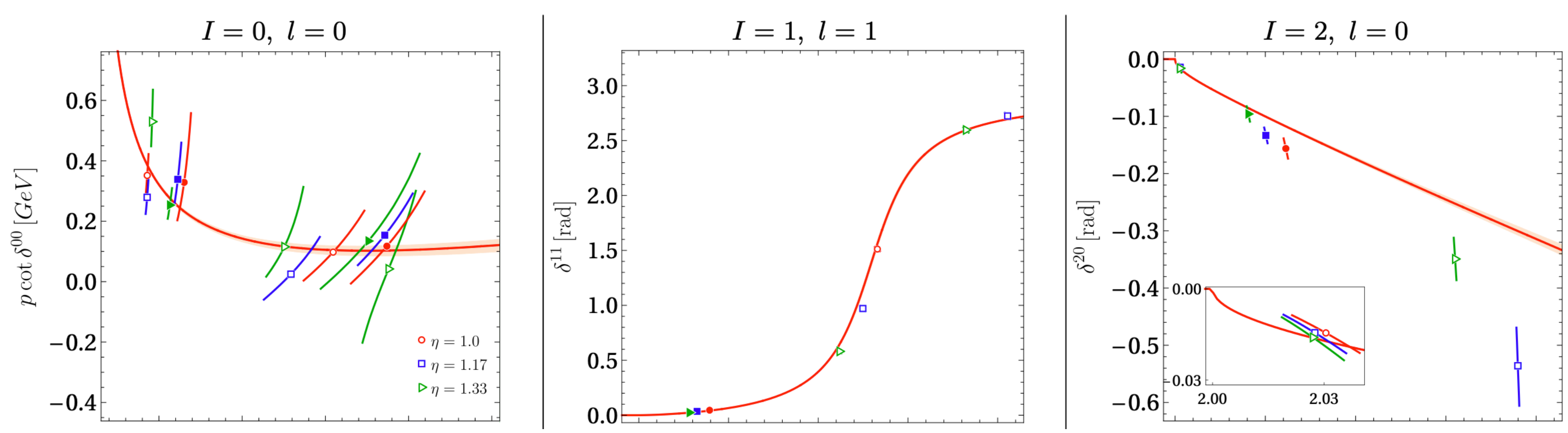
Theoretical ab-initio access: lattice QCD

- full quark-gluon dynamics with some technical issues

1.	Discretized Euclidean space-time	>> continuum limit
2.	Unphysical pion mass	>> chiral extrapolations
3.	Finite Volume	>> quantization condition

- many studies of 2-body systems

[NPLQCD], [RQCD], [ETMC], [HadSpec], ... **Review: Rev.Mod.Phys. 90 (2018) 2**



e.g. MM et al. [GWQCD] (2019)

- new calculations of non-resonant 3-body systems with maximal isospin: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...

[NPLQCD]; Hörz/Hanlon; [GWQCD]; [HadSpec]; Blanton et al.

LATTICE QCD

Key details of GWQCD lattice QCD calculation

Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2021)
PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

- $N_f = 2$ dynamical fermions, LapH smearing
- ***P***=(0,0,0), m_π=224 MeV, $m_\pi L$=3.3
- *GEVP with one-/two-/three-meson operators*
- *Relevant irrep(O_h) for* ***a1(1260)*** $I^G (J^{PC}) = 1^- (1^{++})$:

$\mathbf{P}$	Λ	$J^P\ (I^G = 1^-)$
$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
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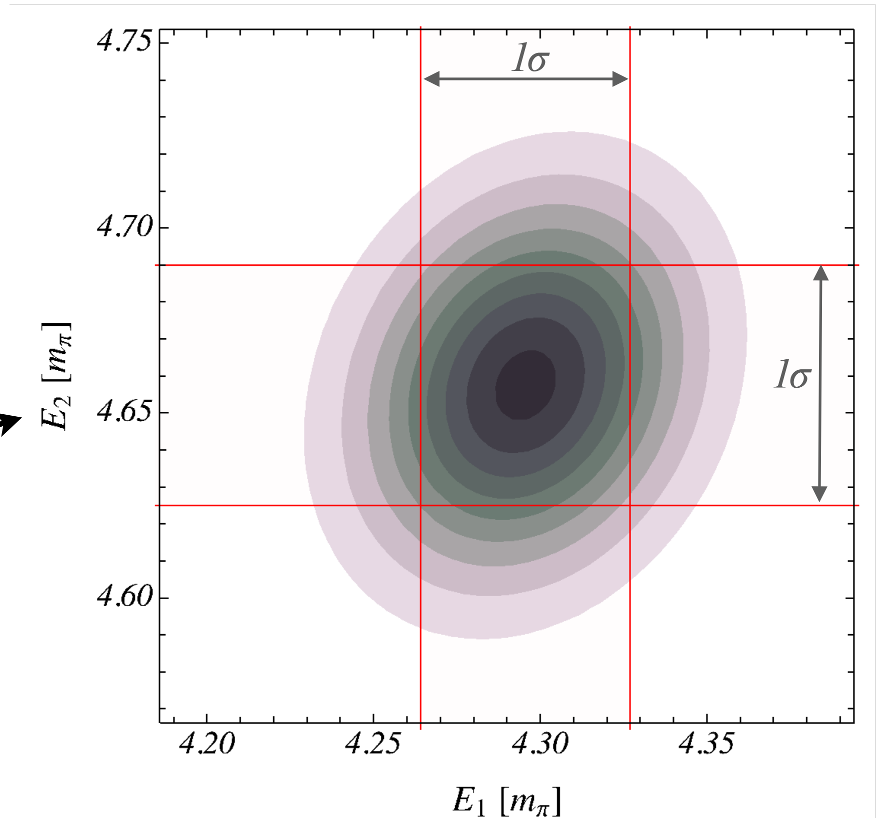
Key results:

- 3π operators essential for the excited-state extraction
  c.f. $\rho\pi$ operators in previous 2-meson a_1 calculation
  Lang et al. JHEP 04, 162 (2014)
- high-momentum states are required:
  π(0,0,0)π(+1,+1,0)π(-1,-1,0) etc..
- two levels exist below 5π threshold



3-BODY QUANTIZATION CONDITION

Discrete, real-valued finite-volume (lattice) spectrum ⟵ **Quantization condition** ⟶ **continuous complex-valued amplitudes**

- established in 2-body: Lüscher's method, extensions...

Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...

- 3-body methods matured

Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu

Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

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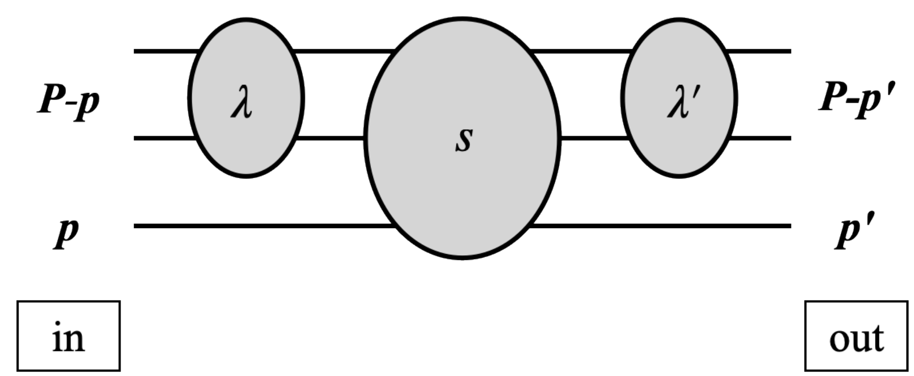
Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

Finite Volume Unitarity (FVU) MM, Döring EPJA (2017) PRL (2019)

- *extended to higher spin and coupled-channels: new degree of freedom (helicity, λ)*

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$



- *∞-dim. determinant equation in* $\mathbf{p} \in \frac{2\pi}{\mathbf{L}}\mathbf{Z}^3$ → *practical applications require truncation*
→ *common to all quantization conditions*

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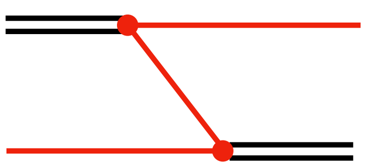
one-particle exchange

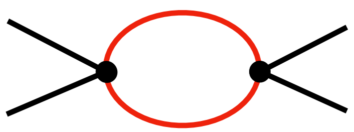
- fixed by 3b-unitarity



- no free parameters

two-body self-energy

- fixed by 2b-unitarity



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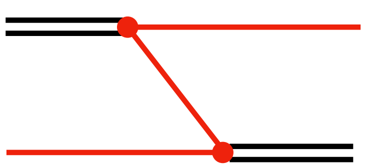

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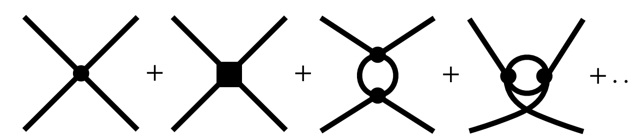
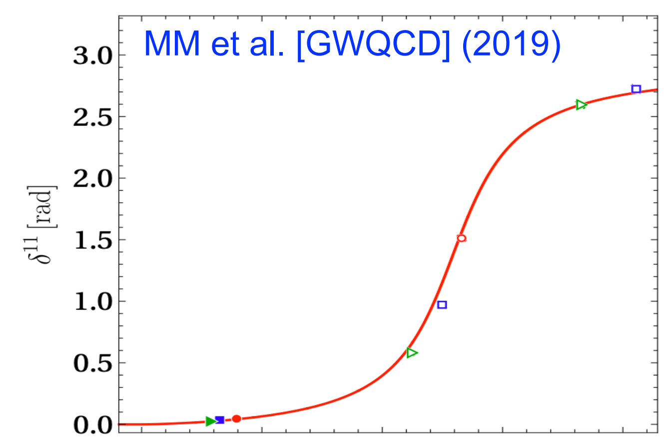
two-body kernel

- dynamics of *I=1* $\pi\pi$ system



$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- regular function ⇒ polynomial



- parameters *(a_0,a_1)* from cross-channel fit to $\pi\pi$ GWQCD levels

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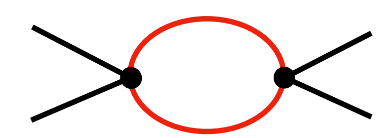

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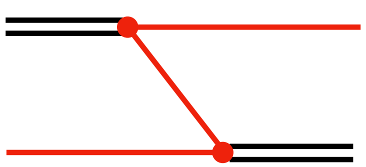

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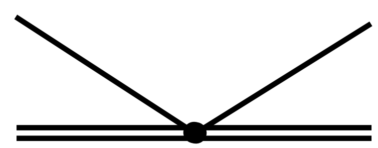
three-body force

- dynamics of $\rho\pi$ system



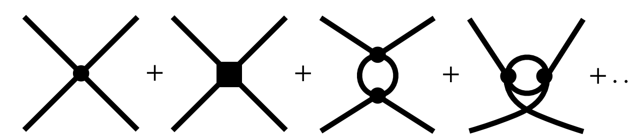
- regular function ⇒ Laurent series

$$C_{\ell'\ell}(s,\mathbf{p}',\mathbf{p}) = \sum_{i=-1}^{\infty} c^{(i)}_{\ell'\ell}(\mathbf{p}',\mathbf{p})(\mathbf{s}-\mathbf{m}^2_{\mathbf{a_1}})^{\mathbf{i}}$$

- fit to 3-body levels → *next slide*

two-body kernel

- dynamics of *I=1* $\pi\pi$ system



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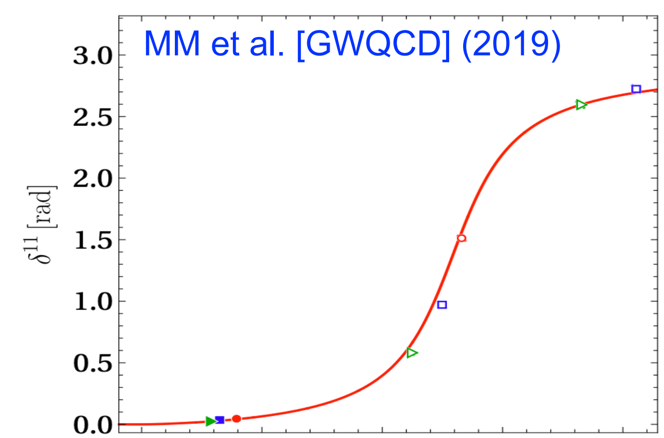

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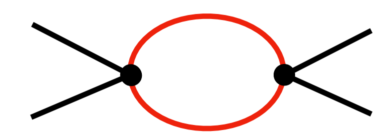

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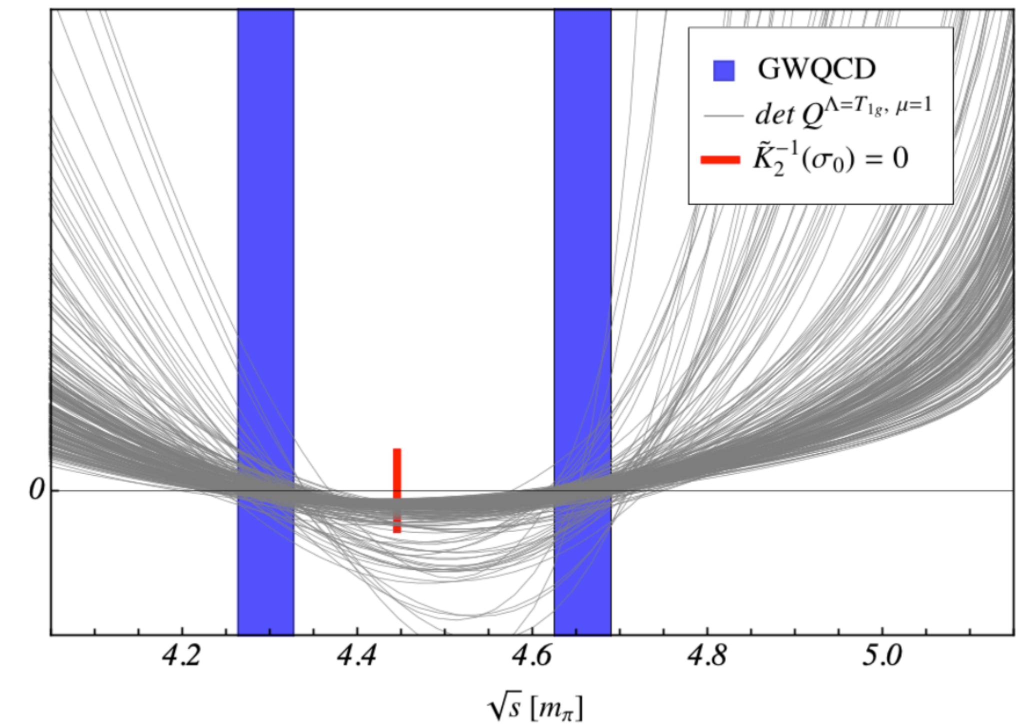
MAPPING & FITS

Last unknown (volume-independent) quantity: 3-body term *C*

- *a_1* pole is generated *<u>with/without explicit pole-term</u>*
- best description (with large correlations)

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{\mathbf{1}}{\mathbf{s}-\mathbf{m}_{\mathbf{a_1}}^{\mathbf{2}}}\mathbf{g}_{\ell}|\mathbf{p}|^{\ell}+\mathbf{c}\,\delta_{\ell'\mathbf{0}}\delta_{\ell\mathbf{0}}$$

- full resampling... accessing statistical errors



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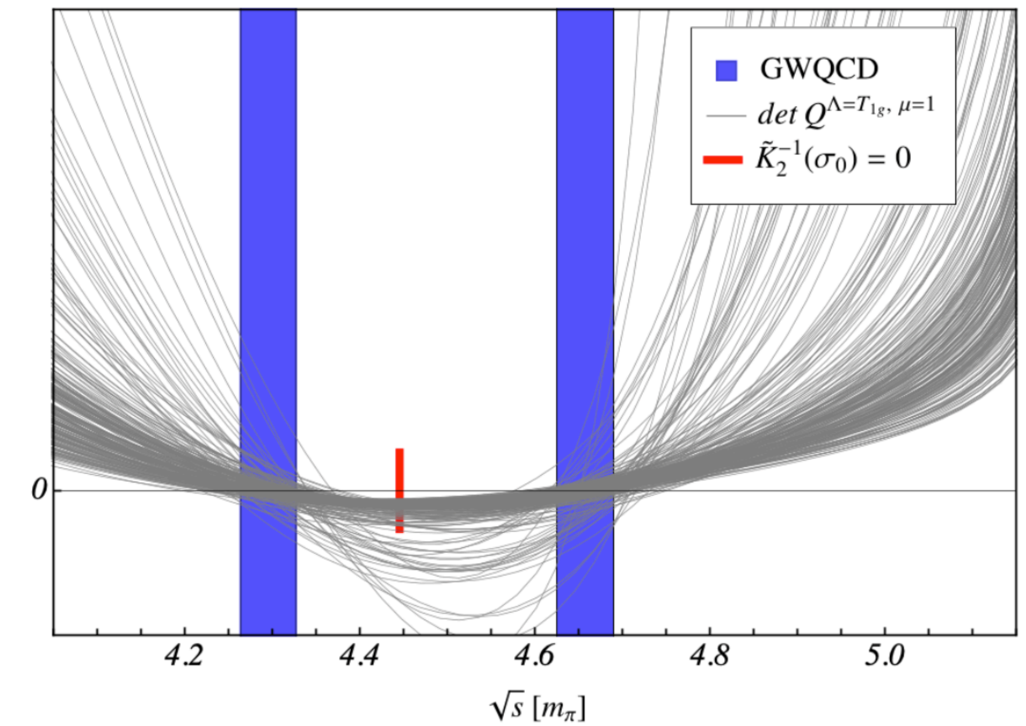
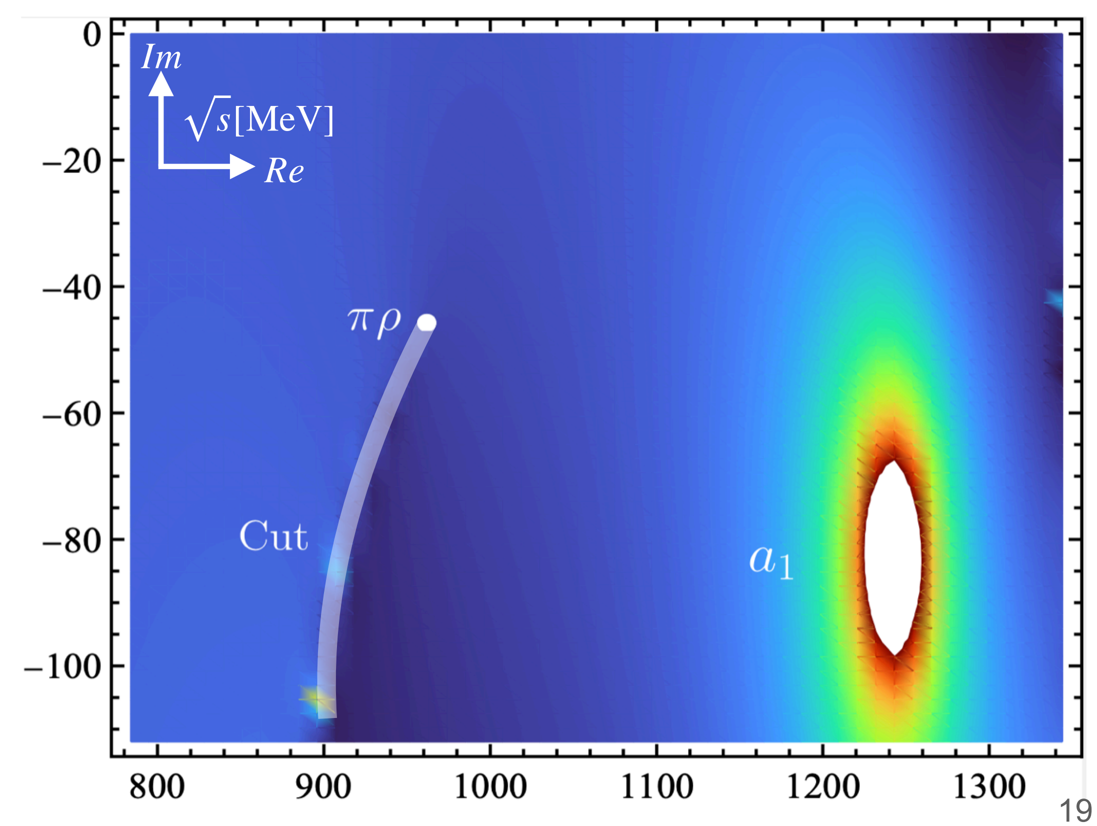


Infinite-volume scattering amplitude

- integral equation with all inputs fixed from lattice QCD results

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$

- contour deformation of spectator momenta
- analytic continuation to 2.Riemann sheet



PARAMETERS OF a₁(1260)

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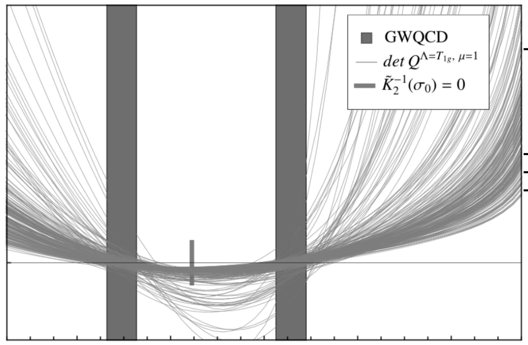
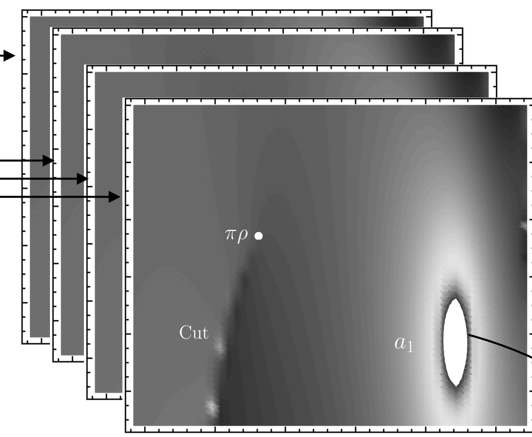
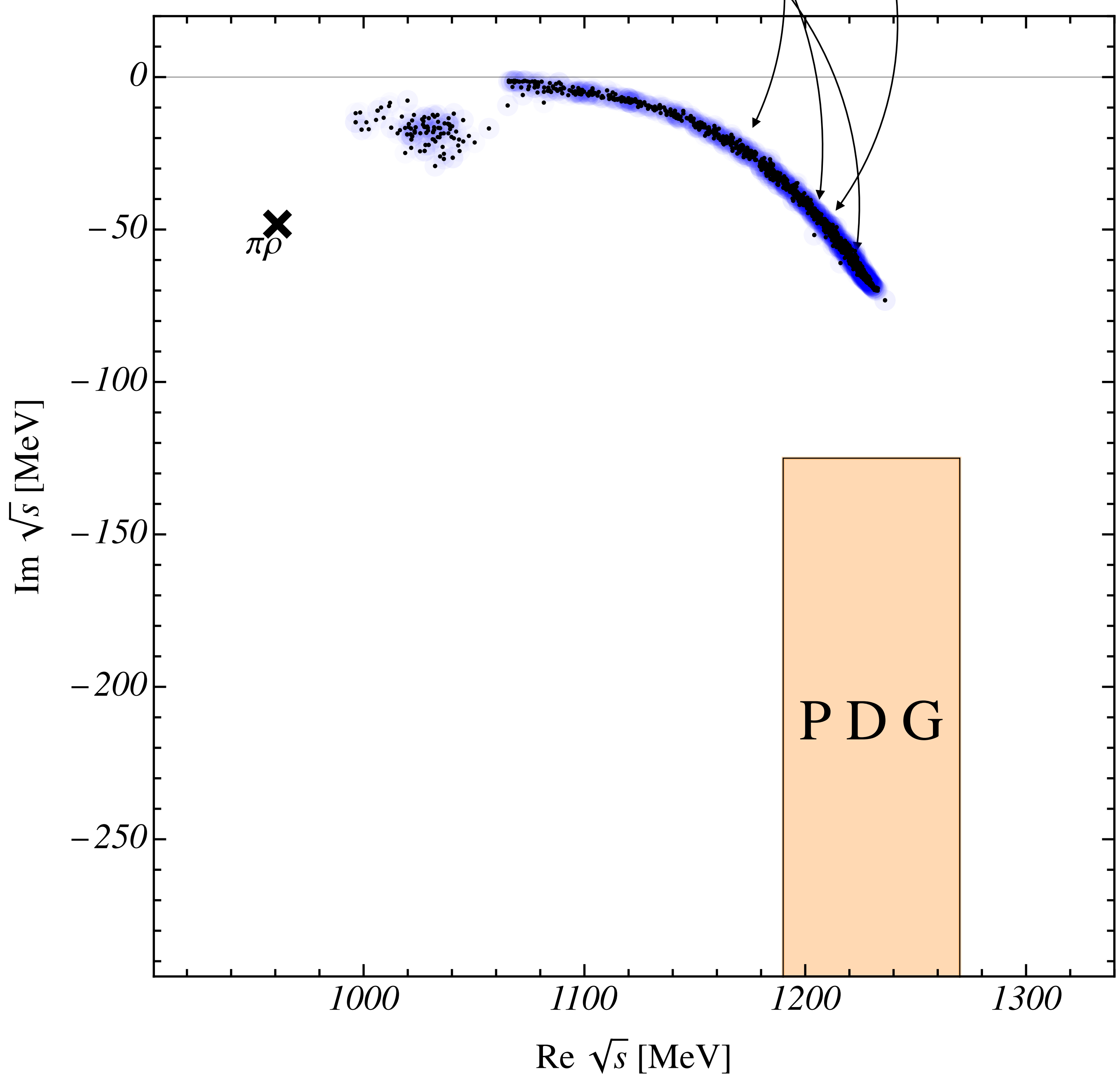




- m_π=224 MeV
- 2000 resampling sets
- distribution of poles is finite
- 90% @ ~((1110..1210) - (20..70)i) MeV



PARAMETERS OF $a_1(1260)$ - CHECKS

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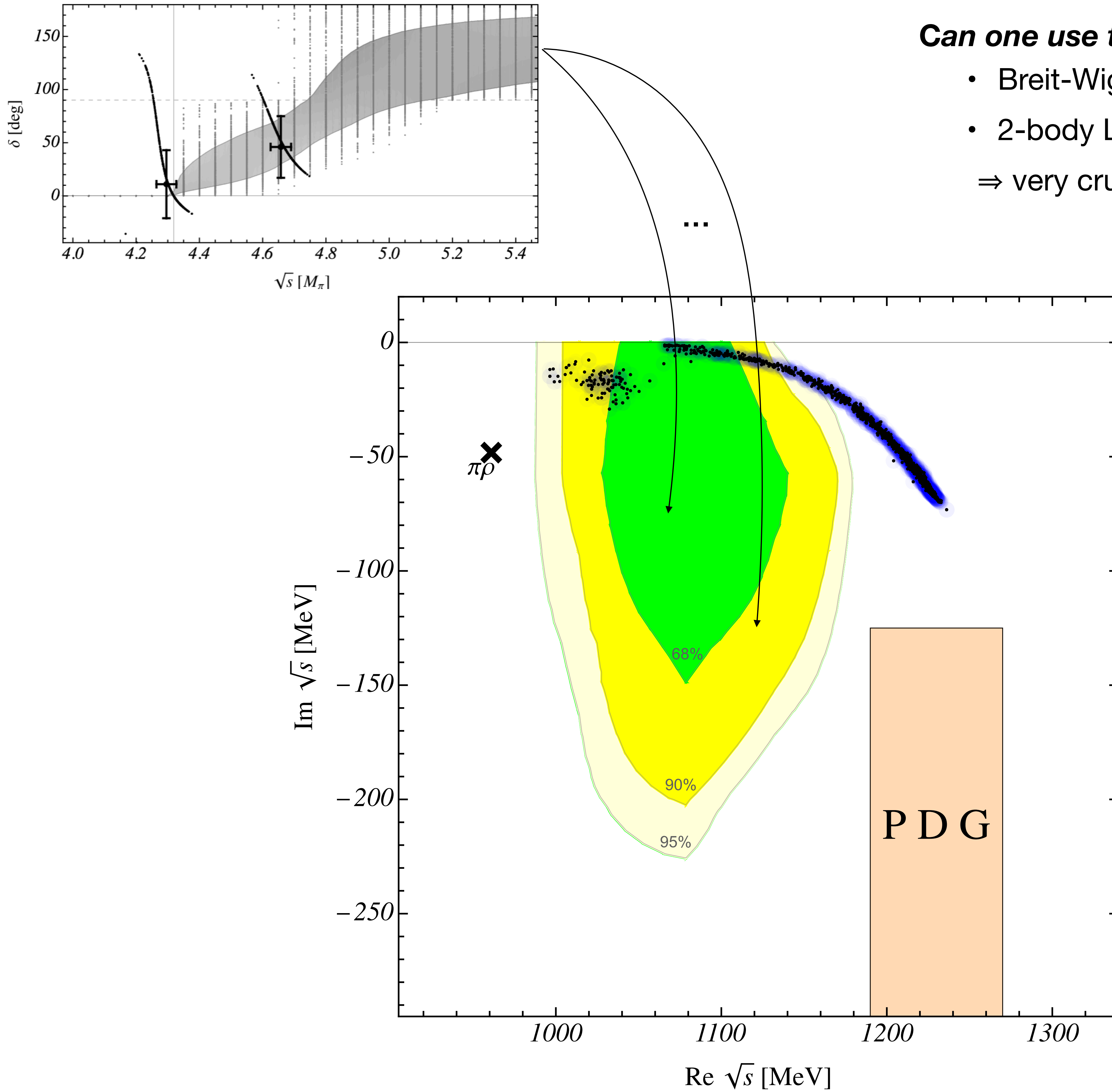
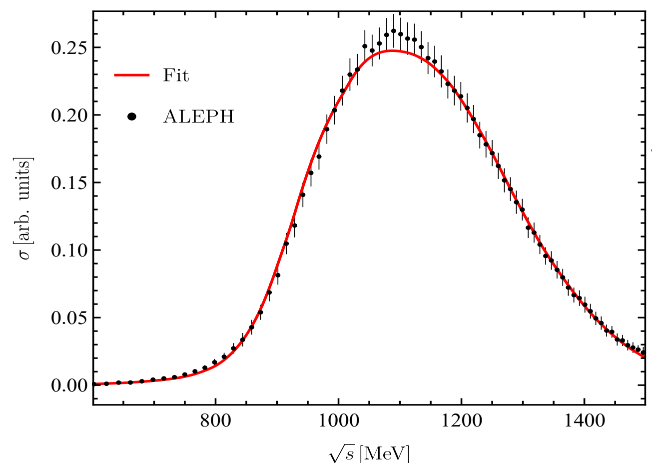


Can one use two-body approximation?

- Breit-Wigner parametrization
- 2-body Lüscher quantization condition

⇒ very crude approximation!

PARAMETERS OF $a_1(1260)$ - CHECKS



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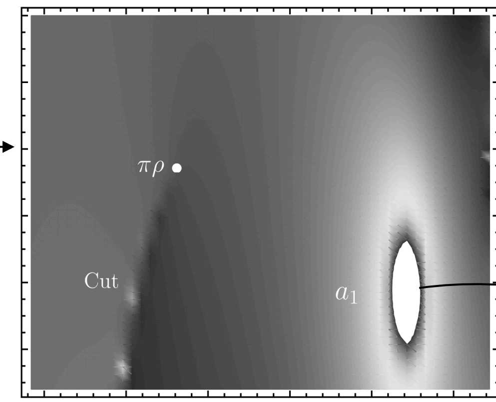
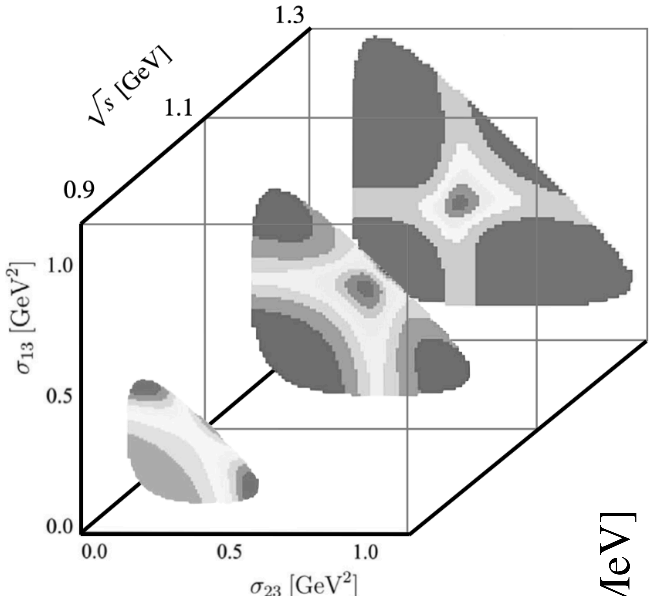
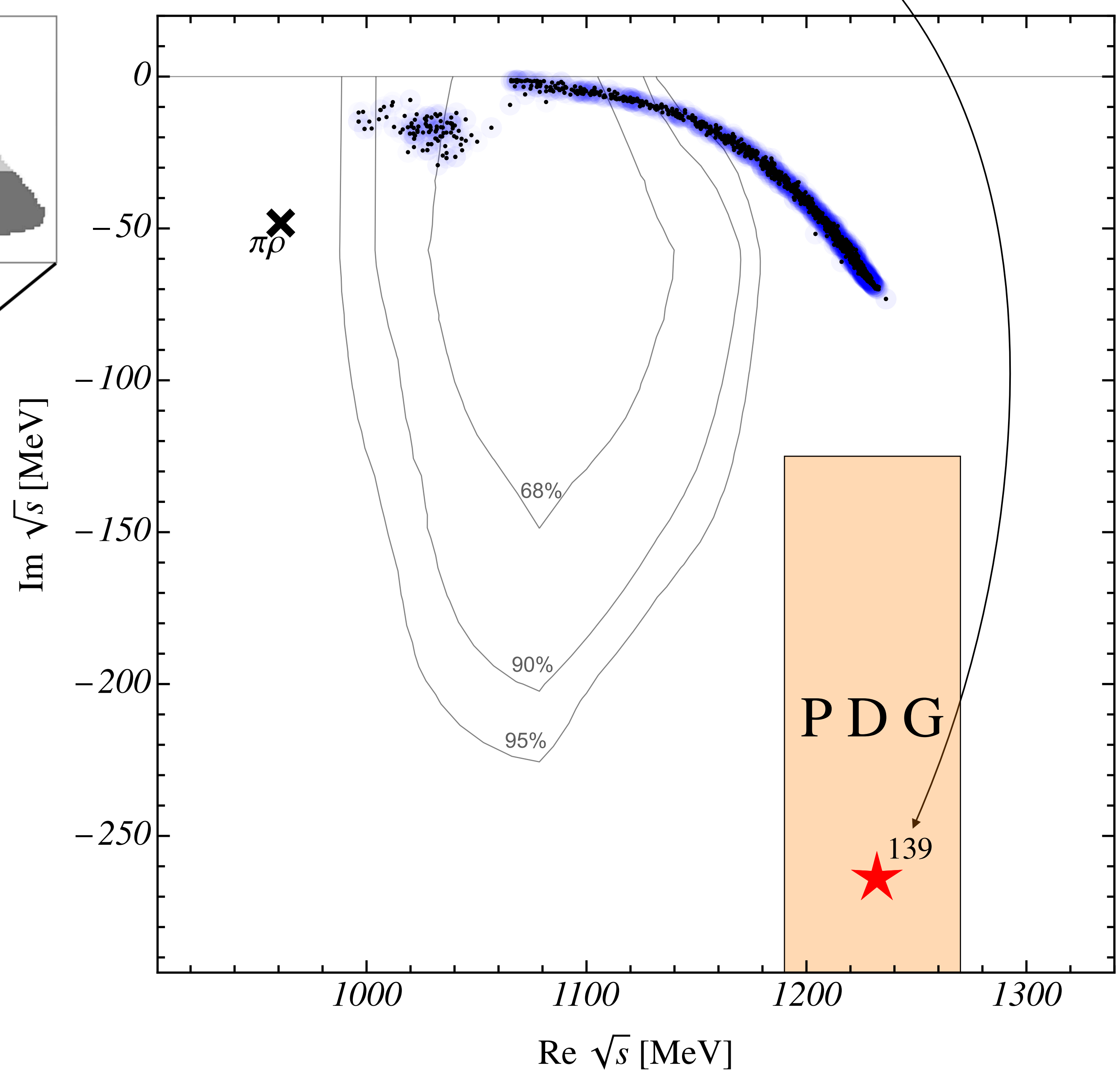




cf. Sadasivan/
MM/Döring(2020)



What does phenomenology says?

- $\tau \to (\pi\pi\pi)\nu_\tau$ from ALEPH@CERN
- fit to line shape to fix C

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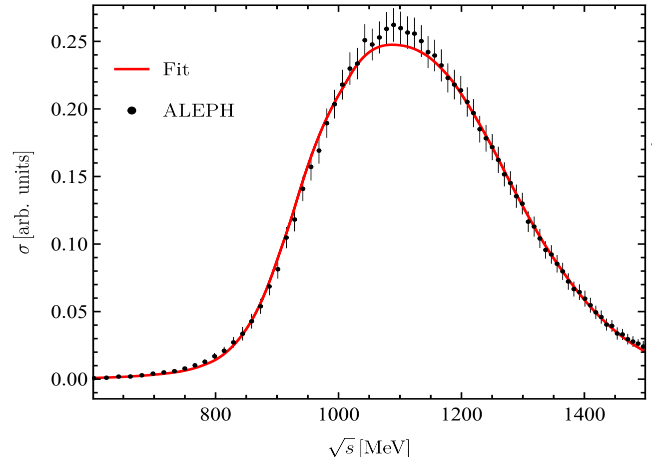

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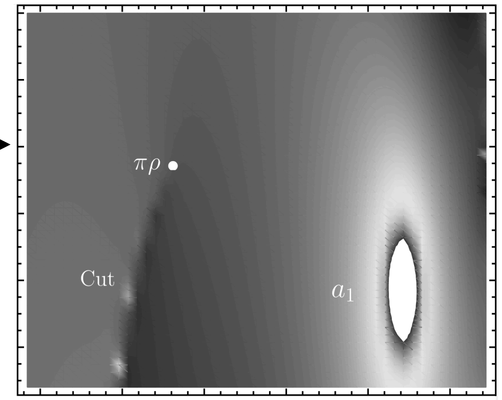

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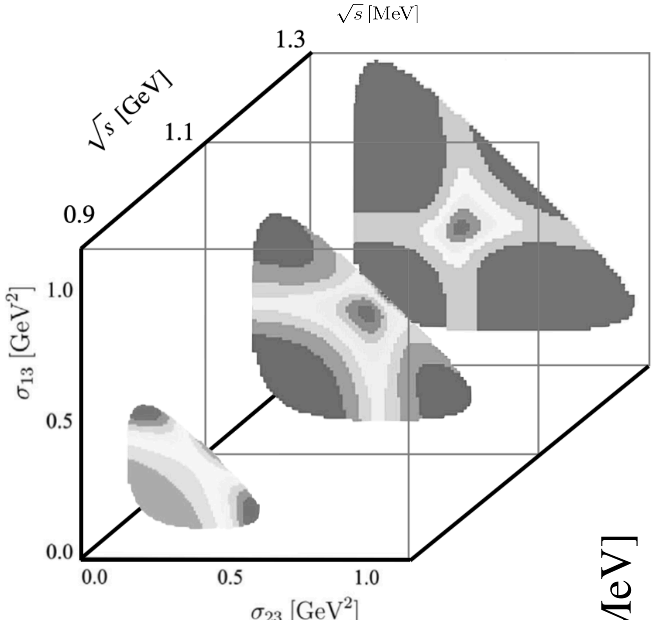

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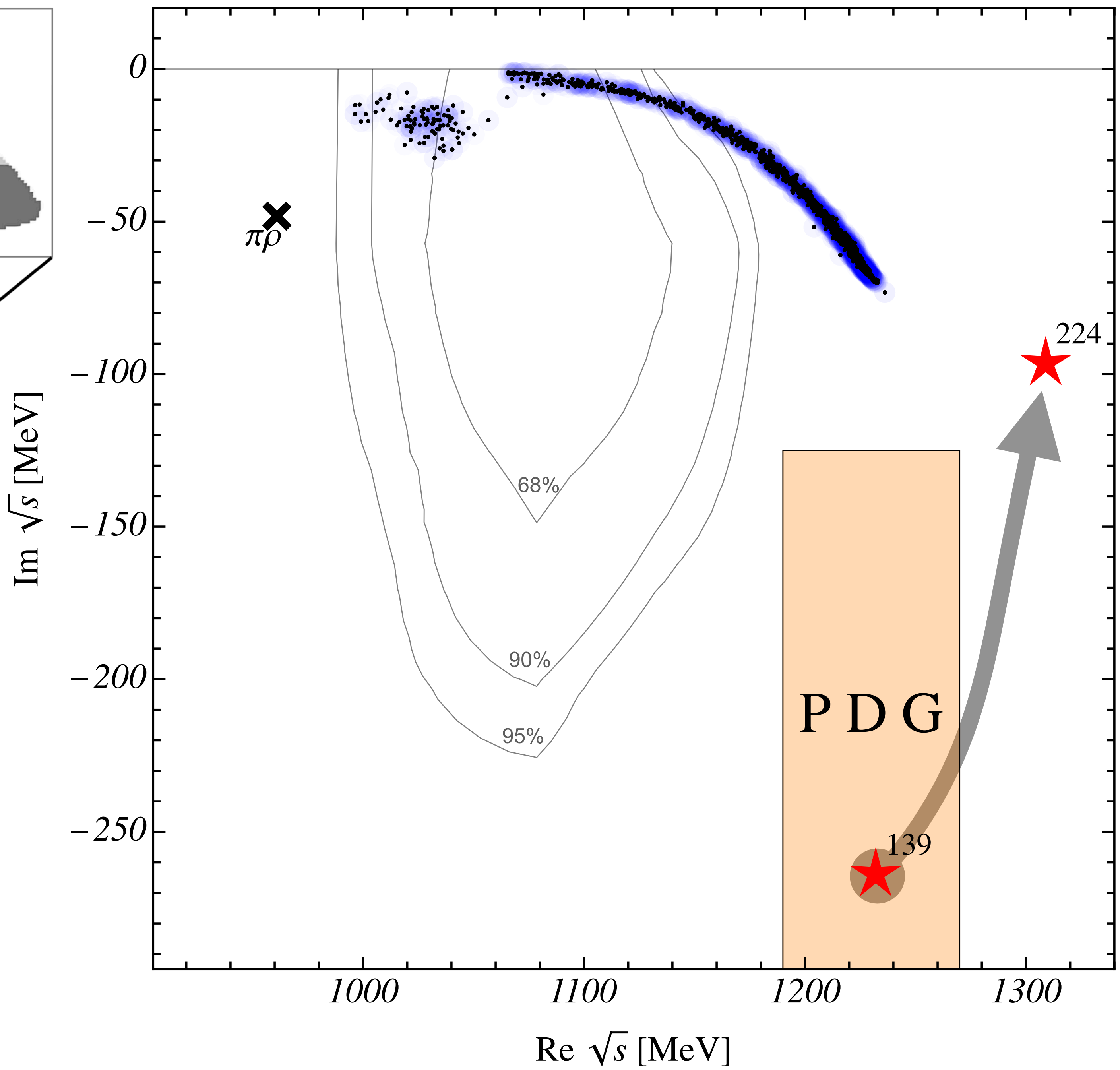


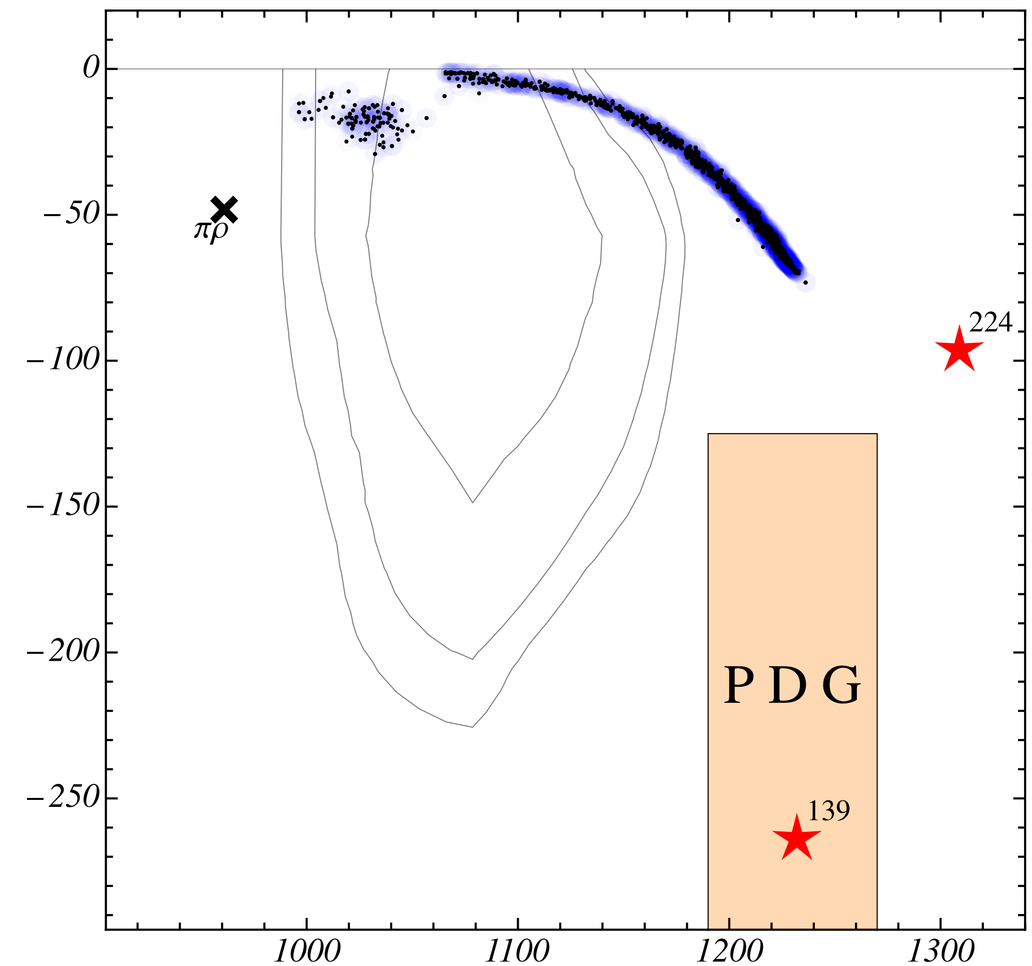
(naive) ***chiral extrapolation*** *confirms expectations:*

- *a_1 becomes heavier*
- *... and narrower*

SUMMARY/OUTLOOK

First-ever determination of $a_1(1260)$ parameters from lattice QCD

- Lattice QCD levels with 1/2/3-meson operators calculated
- 3-body quantization condition extended to spin-1 systems
- unitary infinite-volume amplitude solved with complex integration contour and for complex energies
- Pole positions and couplings are determined



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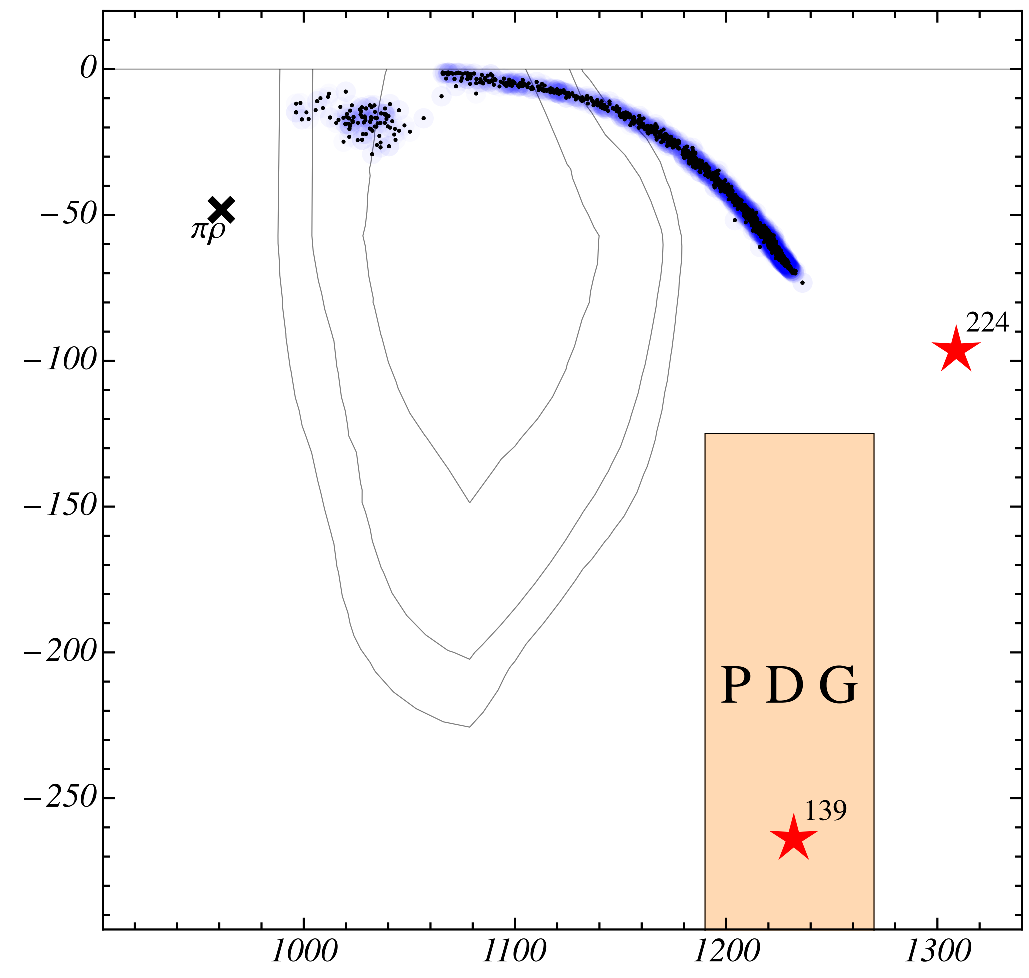
First-ever determination of $a_1(1260)$ parameters from lattice QCD

- Lattice QCD levels with 1/2/3-meson operators calculated
- 3-body quantization condition extended to spin-1 systems
- unitary infinite-volume amplitude solved with complex integration contour and for complex energies
- Pole positions and couplings are determined



Outlook

- More lattice levels (another setup: m_π, L):
  - precise determination of a_1 parameters
  - importance of the $\sigma\pi$ channels
  - chiral trajectory of three-body term C

THANK YOU