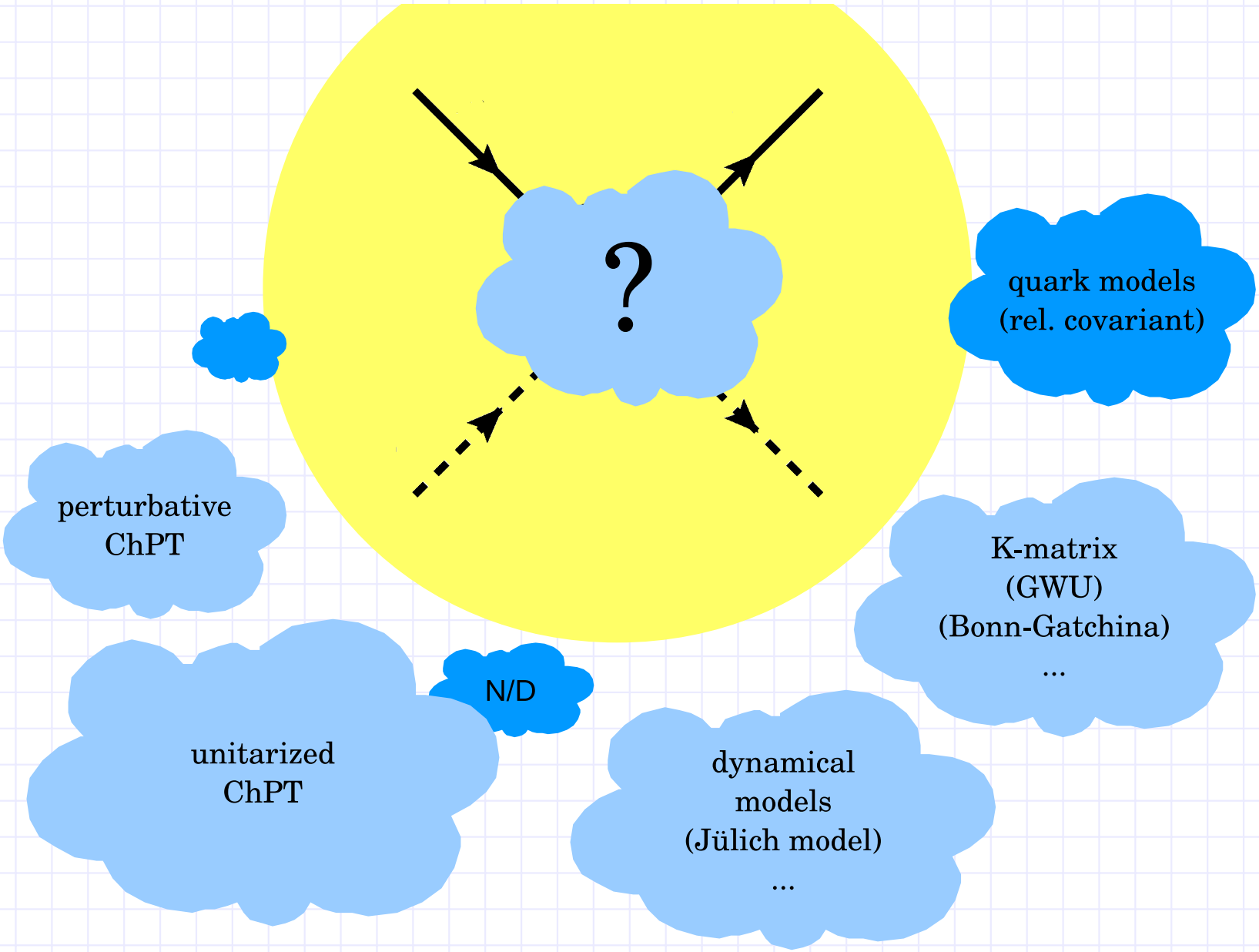
Coupled-channel Bethe-Salpeter

approach to pion-nucleon scattering

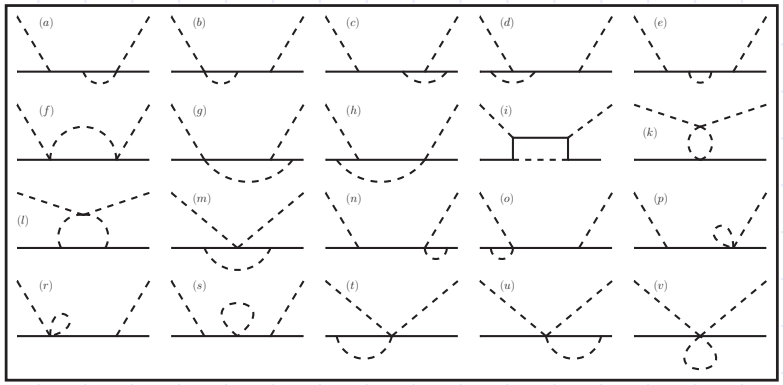
* * *

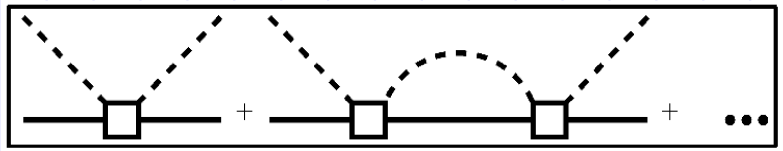
Maxim Mai

P. C. Bruns, MM and U.- G. Meißner
"*Chiral dynamics of the S11(1535) and S11(1650) resonances revisited*"
Phys. Lett. B **697** (2011) 254


?
quark models
(rel. covariant)
perturbative
ChPT
K-matrix
(GWU)
(Bonn-Gatchina)
...
N/D
unitarized
ChPT
dynamical
models
(Jülich model)
...

χPT $\Longleftrightarrow$ $U\chi PT$

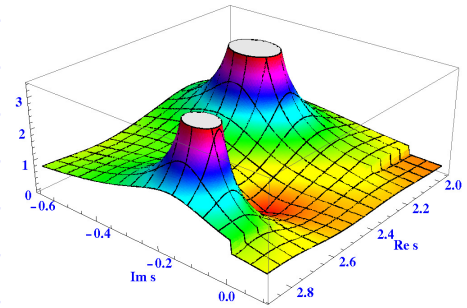




- \+ chiral symmetry
- \+ crossing symmetry
- \+ counterterm renormalization
- − low-energy (ρ, Δ)
- − perturbative unitarity
- − fails in vicinity of resonances
  e.g. $\Lambda(1405)$
  (e.g Kaiser 2001/M.M. et. al. 2009)

- \+ chiral symmetry
- \+ energy range extended $\sim 1650 MeV$
- \+ elastic unitarity
- − no crossing symmetry
- ? renormalization difficult

$\longrightarrow$ threshold behaviour



Method

- LO chiral Lagrangian:

$$\mathcal{L}^{(1)}_{\phi B} = \langle \bar{B}(i\gamma_\mu D^\mu - m_0)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$

  - WT–contact term (contributes to the s-wave)
  - $u-$ and $s-$channel Born graphs (to be omitted)

- sizable NLO corrections - chiral Lagrangian:

M.M., Bruns, Kubis, Meißner 2009

$$\begin{aligned}
\mathcal{L}^{(2)}_{\phi B} =& \mathbf{b}_{D/F}\langle \bar{B}[\chi_+, B]_\pm\rangle + \mathbf{b}_0\langle \bar{B}B\rangle\langle\chi_+\rangle + \mathbf{b}_{1/2}\langle \bar{B}\Big[u_\mu, [u^\mu, B]_\mp\Big]\rangle + \mathbf{b}_3\langle \bar{B}\Big\{u_\mu, \{u^\mu, B\}\Big\}\rangle \\
&+ \mathbf{b}_4\langle \bar{B}B\rangle\langle u_\mu u^\mu\rangle + i\mathbf{b}_{5/6}\langle \bar{B}\sigma^{\mu\nu}\Big[[u_\mu, u_\nu], B\Big]_\mp\rangle + i\mathbf{b}_7\langle \bar{B}\sigma^{\mu\nu}u_\mu\rangle\langle u_\nu B\rangle \\
&+ \frac{i\,\mathbf{b}_{8/9}}{2m_0}\Big(\langle \bar{B}\gamma^\mu\Big[u_\mu, [u_\nu, [D^\nu, B]]_\mp\Big]\rangle + \langle \bar{B}\gamma^\mu\Big[D_\nu, [u^\nu, [u_\mu, B]]_\mp\Big]\rangle\Big) \\
&+ \frac{i\,\mathbf{b}_{10}}{2m_0}\Big(\langle \bar{B}\gamma^\mu\Big\{u_\mu, \{u_\nu, [D^\nu, B]\}\Big\}\rangle + \langle \bar{B}\gamma^\mu\Big[D_\nu, \{u^\nu, \{u_\mu, B\}\}\Big]\rangle\Big) \\
&+ \frac{i\,\mathbf{b}_{11}}{2m_0}\Big(2\langle \bar{B}\gamma^\mu[D_\nu, B]\rangle\langle u_\mu u^\nu\rangle + \langle \bar{B}\gamma^\mu B\rangle\langle [D_\nu, u_\mu]u^\nu + u_\mu[D_\nu, u^\nu]\rangle\Big)
\end{aligned}$$

Method

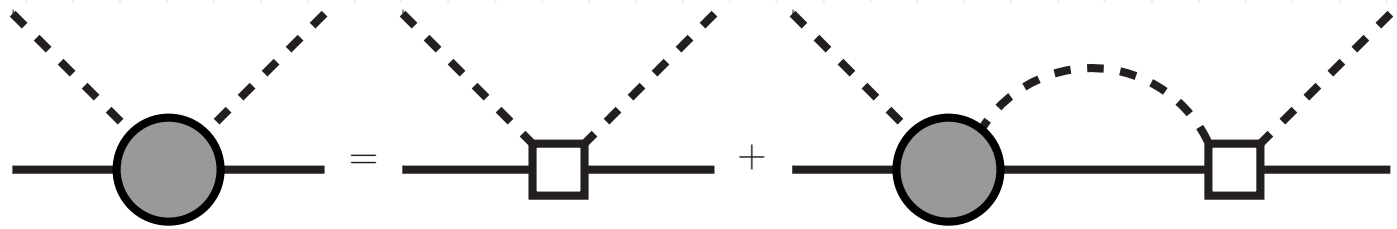
- chiral potential for $\phi(q_i)B(p-q_i) \longrightarrow \phi(q_j)B(p-q_j)$:

$$V(\not{q}_2, \not{q}_1; p) = A_{WT}(\not{q}_1 + \not{q}_2) + A_{14}(q_1 \cdot q_2) + A_{57}[\not{q}_1, \not{q}_2] + A_M + A_{811}\left(\not{q}_2(q_1 \cdot p) + \not{q}_1(q_2 \cdot p)\right)$$

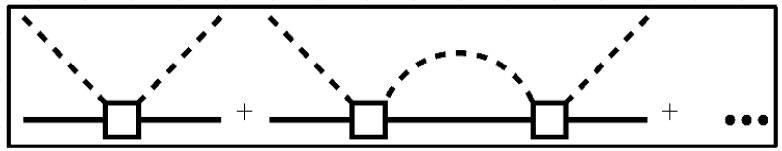
  - $\phi B \in \{p\pi^0,\ n\pi^+,\ p\eta,\ \Lambda K^+,\ \Sigma^0 K^+,\ \Sigma^+ K^0\}$
  - p-wave contributions via A_{14}, A_{57}, A_{811}
  - 14 LECS to be fixed

- Bethe–Salpeter equation:

$$T(\not{q}_2, \not{q}_1; p) = V(\not{q}_2, \not{q}_1; p) + i \int \frac{d^d l}{(2\pi)^d} V(\not{q}_2, \not{l}; p) \frac{1}{\not{p} - \not{l} - m + i\epsilon} \frac{1}{l^2 - M^2 + i\epsilon} T(\not{l}, \not{q}_1; p)$$



Method (renormalization)

- loop integration $\longrightarrow$ **UV**–divergencies:
  - strict chiral expansion - counterterm renormalization (order by order)
  - 
    iterated bubble sum - ∞ many counterterms ! ! !

shift loop divergencies to the kernel: $V_\delta = V + \delta V$

Nieves, Arriola 1999
Borasoy, Bruns, Meißner, Nißler 2007

but some scale dependence remains...

$\Rightarrow$ model parameters: 14 LECs and 3 SCs

Fit

- compare πN channel:

  Arndt, Briscoe, Strakovsky, Workman 2006

  - $Re(S_{11})$, $Im(S_{11})$, $Re(S_{31})$ and $Im(S_{31})$
  - 49 energy values, $1080 < s < 1560$ MeV
  - errors: $\Delta f(\sqrt{s} < 1280\ \text{MeV}) = 0.005$

    $\Delta f(\sqrt{s} > 1280\ \text{MeV}) = 0.030 \quad (\pi\pi N)$

- χ^2_{dof} minimization by **MINUIT**

$log(\mu_\pi/\text{GeV})$	$+0.974$	$+0.929$	$+0.930$	$+0.932$	$+0.924$
$log(\mu_\eta/\text{GeV})$	$+0.544$	$+0.600$	$+0.350$	$+0.401$	$+0.581$
$log(\mu_K/\text{GeV})$	-0.196	-0.217	-0.089	-0.142	-0.218
$b_1 \cdot GeV$	-0.053	-0.049	-0.026	-0.050	-0.082
...	...	...	...	...	...
$b_D \cdot GeV$	$+0.489$	$+0.543$	$+0.233$	$+0.760$	$+0.641$
$b_F \cdot GeV$	-0.141	-0.156	-0.274	-0.100	-0.098
χ^2_{dof}	1.201	1.189	1.380	1.246	1.232
$\chi^2_{\text{dof}}(S_{11})$	0.773	0.696	1.265	0.910	0.594

natural size!

Fit (result (zoom in))

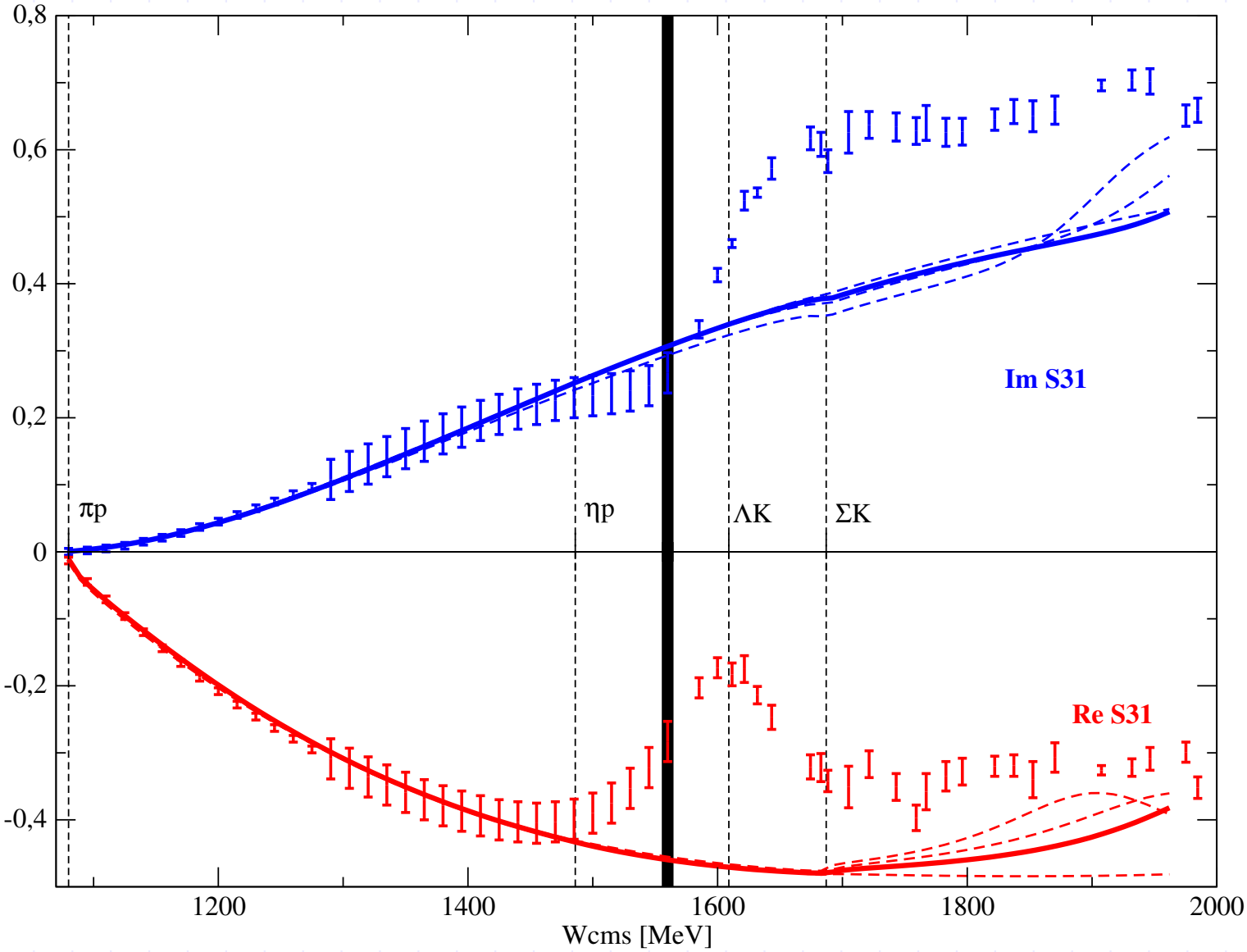
- S_{11} fits better than S_{31}
- s-wave scattering lengths agree roughly with those of direct GWU extraction:

$a_{1/2} \cdot 10^3 M_{\pi^+}$	$+0.139$	$+0.145$	$+0.143$	$+0.165$	$+0.145$
$a_{3/2} \cdot 10^3 M_{\pi^+}$	-0.093	-0.093	-0.100	-0.101	-0.091

$$a_{1/2} = (+0.1747 \pm 0.0022) \times 10^{-3}/M_{\pi^+}$$
$$a_{3/2} = (-0.0894 \pm 0.0017) \times 10^{-3}/M_{\pi^+}$$

- high energy behaviour ...

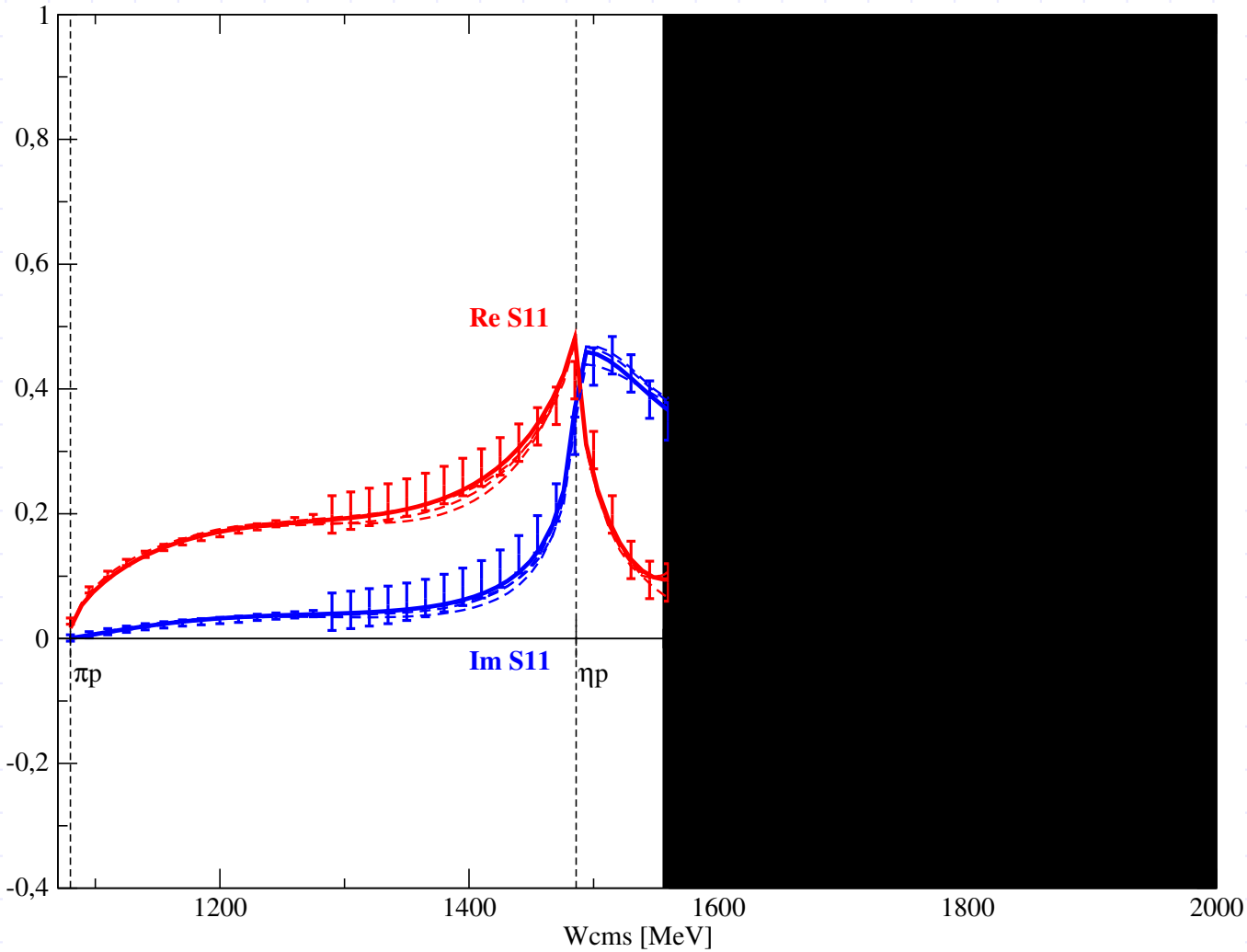
Fit S_{31}



$S_{31}(1620)$ is **not** dynamically generated !

Fit S_{11}

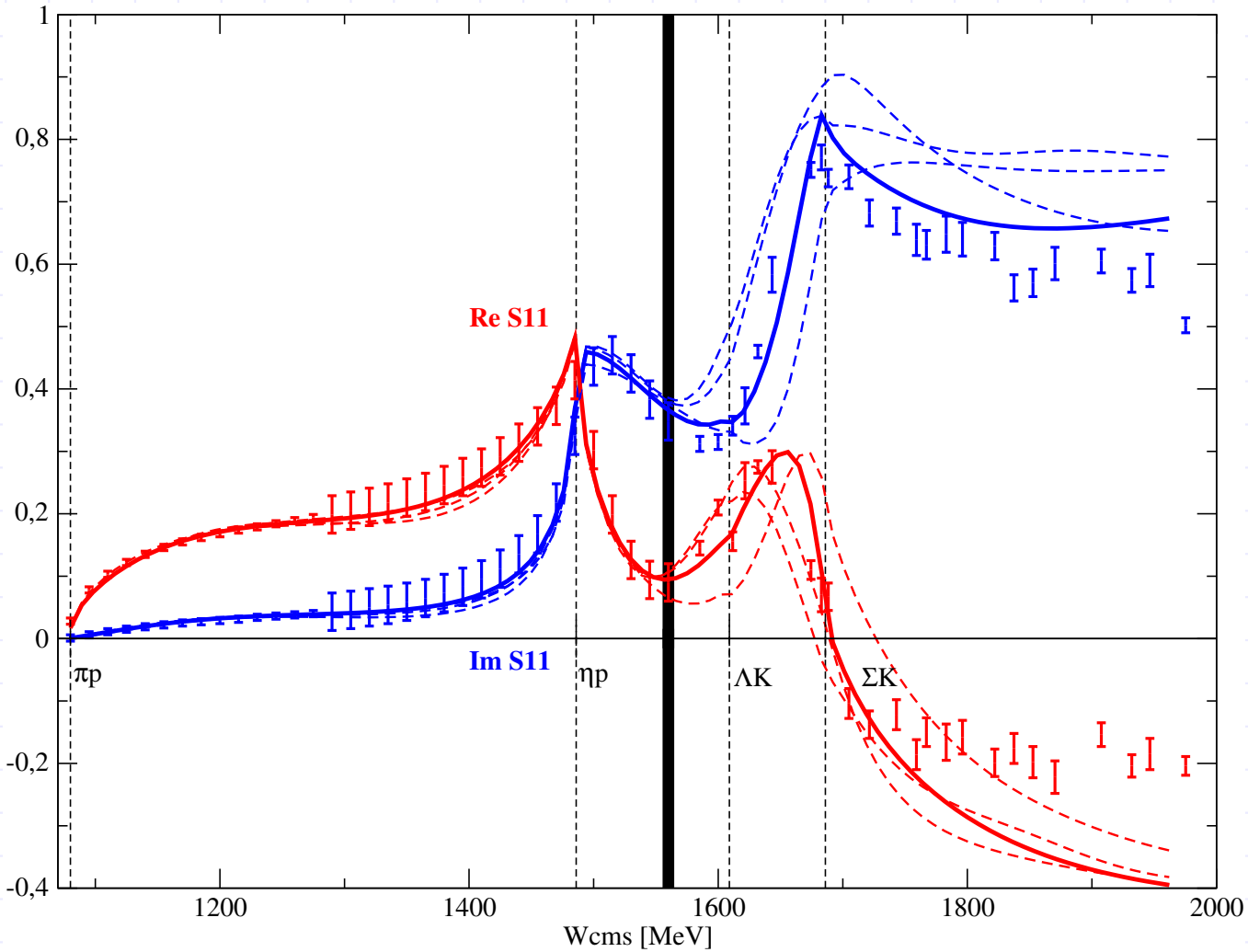
$S_{11}(1535)$ is reproduced nicely



what's about higher energies?

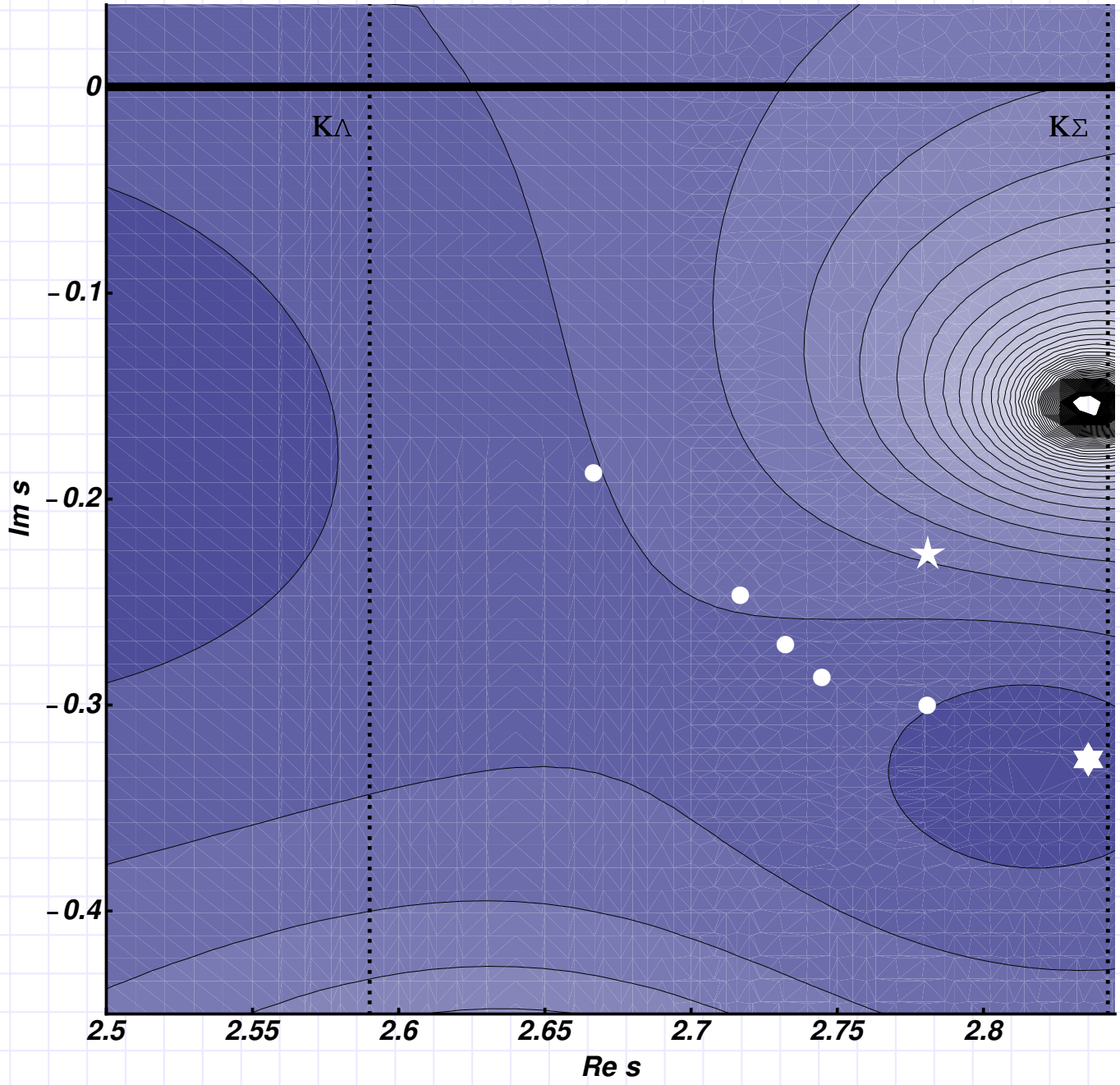
Fit S_{11}

$S_{11}(1535)$ is reproduced nicely



a second structure is obtained $\rightarrow$ $S_{11}(1650)$ **!?**

S_{11}: 2. Riemann sheet



Pole structure - analysis

- sheet (222-111): $W_{1535} = (1.506 - 0.140i)$ GeV
- sheet (2222-11): $W_{1650} = (1.682 - 0.042i)$ GeV

Nieves(2001)	*Doering*(2009)
$1.496 - 0.041i$	$1.519 - 0.064i$
$1.686 - 0.096i$	$1.669 - 0.068i$

Arndt(2006)	*Cutkosky*(1980)
$1.547 - 0.094i$	$1.55 - 0.12i$
$1.634 - 0.058i$	$1.65 - 0.075i$

Hoehler(1979)	*Manley*(1992)
$1.526 - 0.06i$	$1.534 - 0.076i$
$1.670 - 0.09i$	$1.659 - 0.086i$

- structure of resonances: $T_{ij}^{ON}(s) \simeq \frac{g_i g_j}{s - s_R}$

$S_{11}(1535)$: $|g_{\Lambda K^+}|^2 > |g_{p\eta}|^2 > |g_{\Sigma^+ K^0}|^2 \simeq |g_{n\pi^+}|^2 > |g_{\Sigma^0 K^+}|^2 \simeq |g_{p\pi^0}|^2$

$S_{11}(1650)$: $|g_{\Sigma^+ K^0}|^2 > |g_{p\eta}|^2 > |g_{\Sigma^0 K^+}|^2 \simeq |g_{n\pi^+}|^2 > |g_{p\pi^0}|^2 \gg |g_{\Lambda K^+}|^2$

Summary and outlook

✓ s-wave πN scattering analysed in CC UChPT

✓ BSE solved with the full off-shell dependence

.. with **all** local terms of second chiral order

$\Rightarrow$ $S_{31}(1620)$ **is not** generated dynamically

$\Rightarrow$ $S_{11}(1535)$ **is** generated dynamically

$\Rightarrow$ $S_{11}(1650)$ at right position without(**!**) fitting

★ include data for $K\Sigma$ and also $\eta N/\pi\pi N$ channels

★ improve error analysis

... outlook

★ reanalyse the $(\pi)K$-photoproduction in gauge invariant framework ...

Borasoy, Bruns, Meißner, Nißler 2007