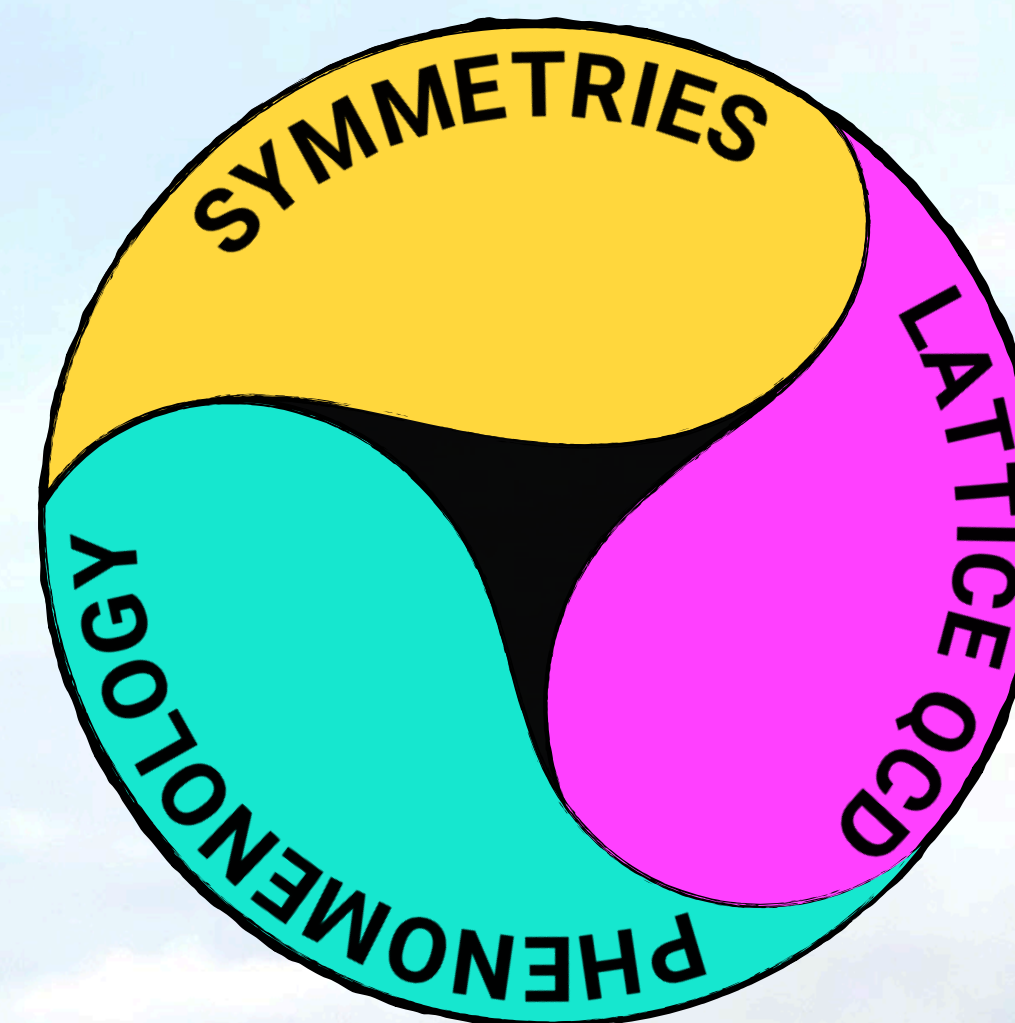


THEORY OF HADRON RESONANCES

HFHF Kolloquium 14.10.2024



MAXIM MAI

UNIVERSITY OF BERN (MAIN)

THE GEORGE WASHINGTON UNIVERSITY

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076

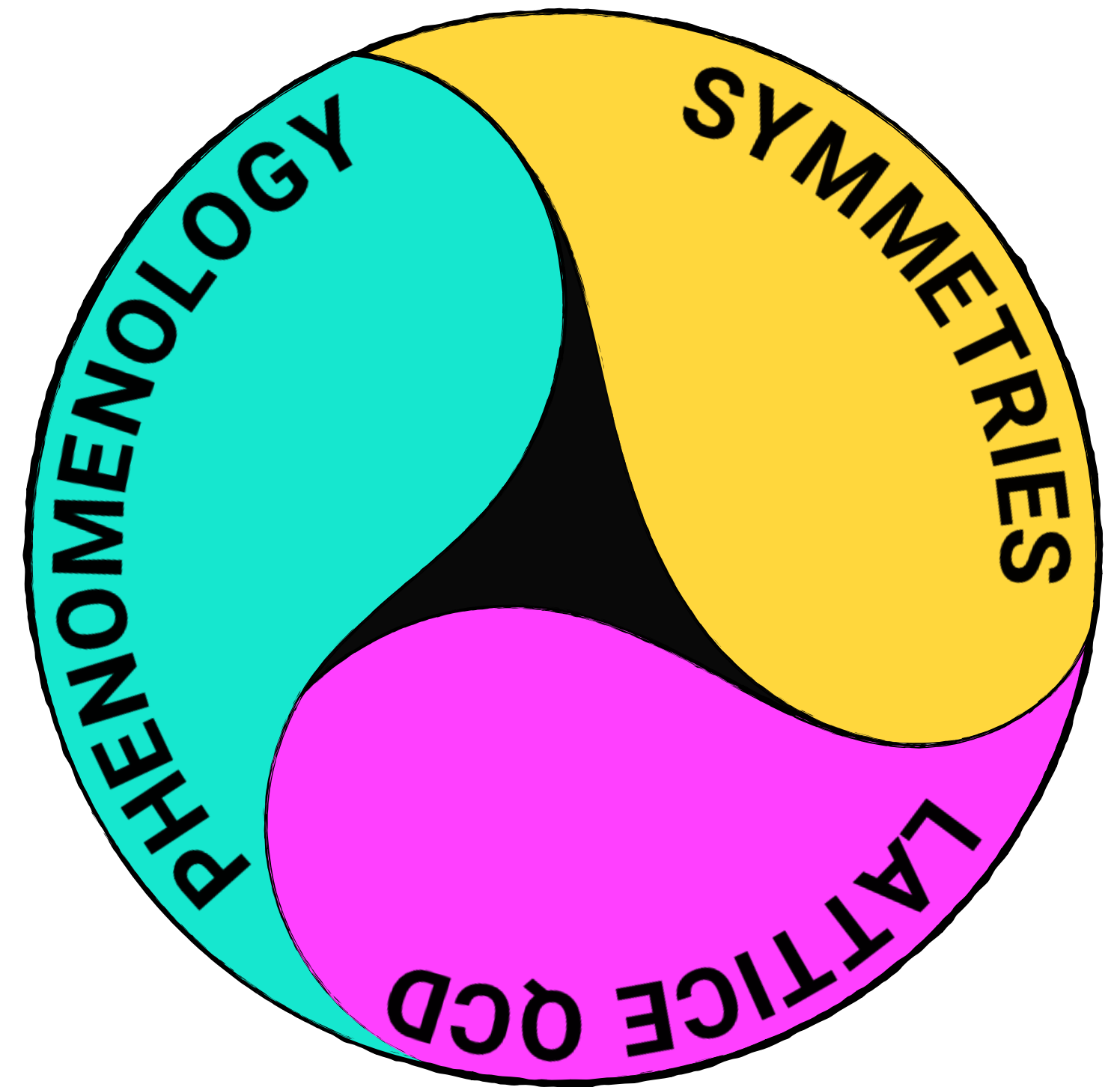
DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907

DFG: Heisenberg Programme (project number: 532635001)

NSF: PHY-2012289



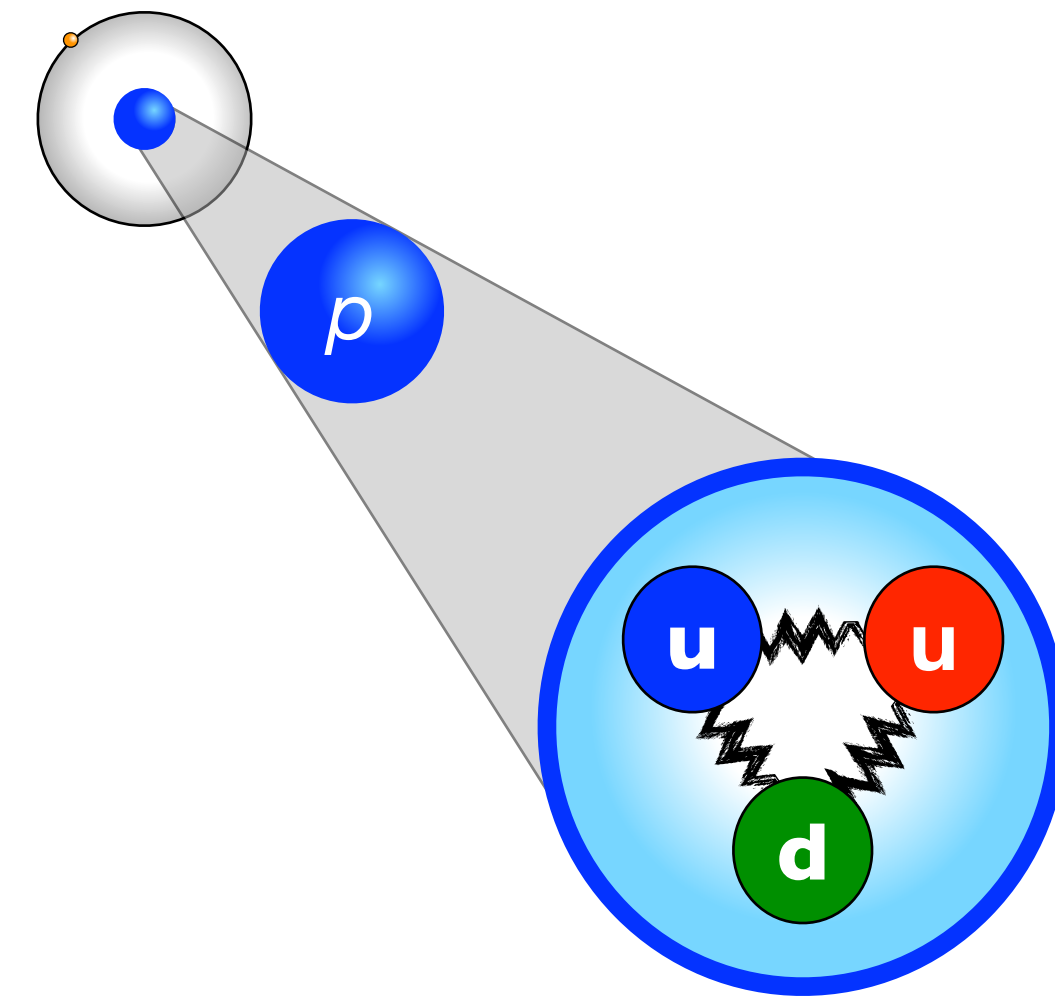
BUMPS AND POLES



HADRONIC SPECTRUM

Protons/neutrons

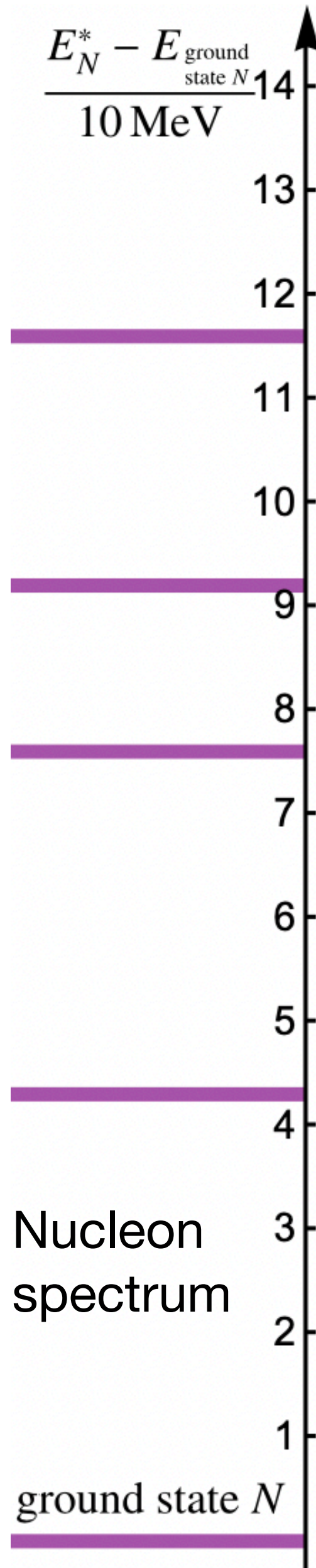
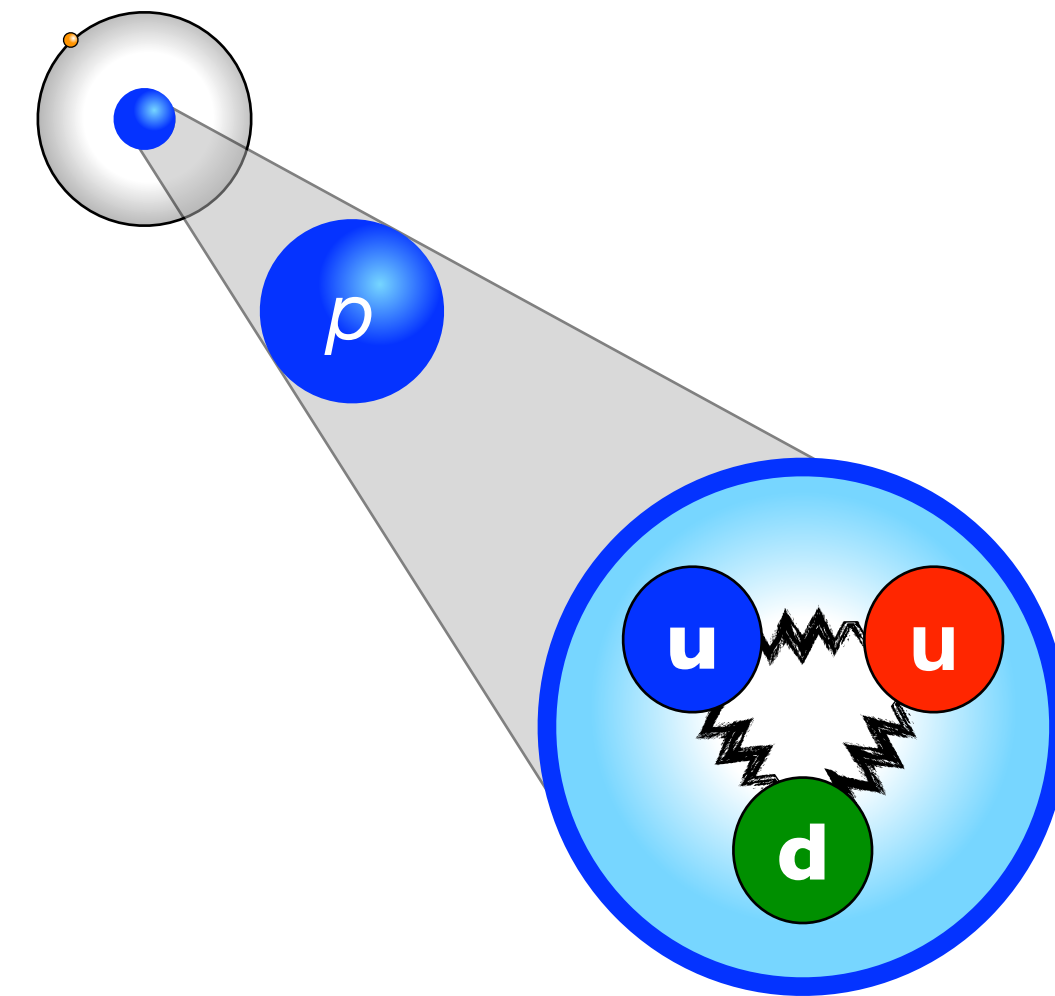
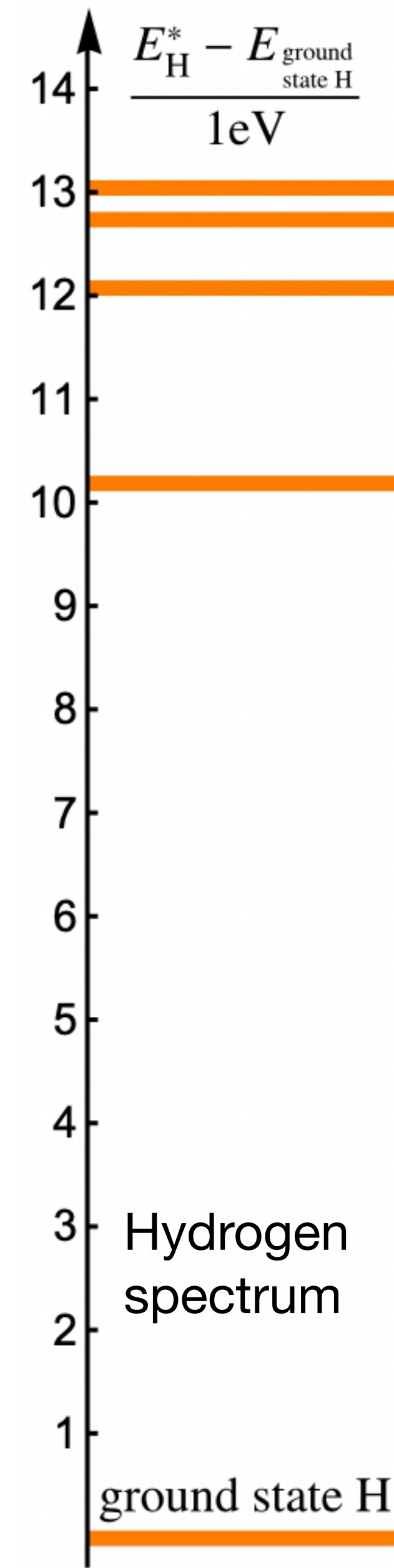
- 99% of the mass of visible matter in the universe
- Building blocks: **quarks & gluons**
- Part of a large class of particles: **hadrons**



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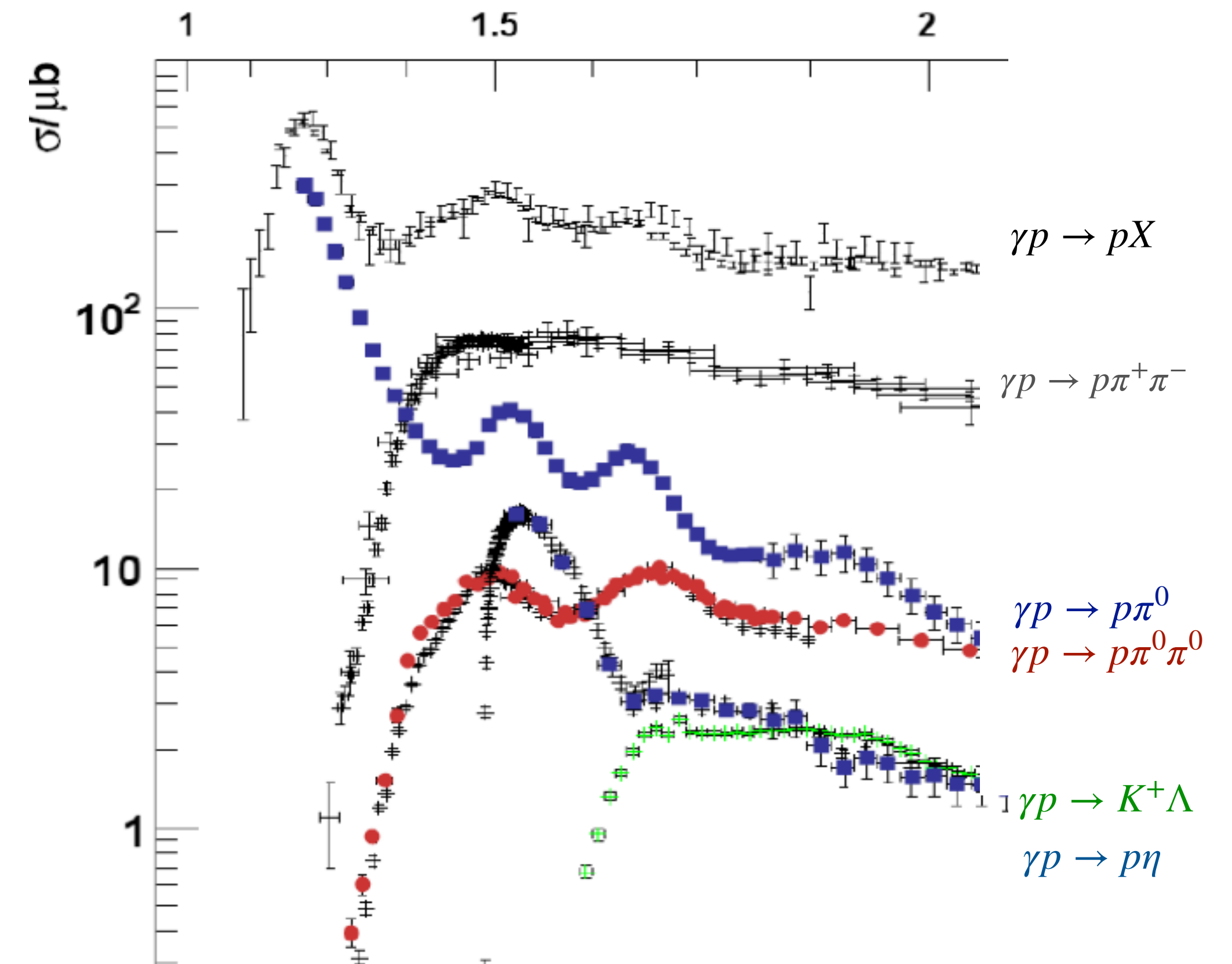
RESONANCE PARAMETER

JLAB, ELSA, MAMI CLAS12, GlueX, ..

Review: Thiel/Afzal/Wunderlich
Prog.Part.Nucl.Phys. 125 (2022) 103949

Interpreting interaction probabilities

- many available data and ongoing experiments
- bumps $\Rightarrow$ resonances?



“If I could remember the names of all these particles,
I would have been a botanist.”

Enrico Fermi

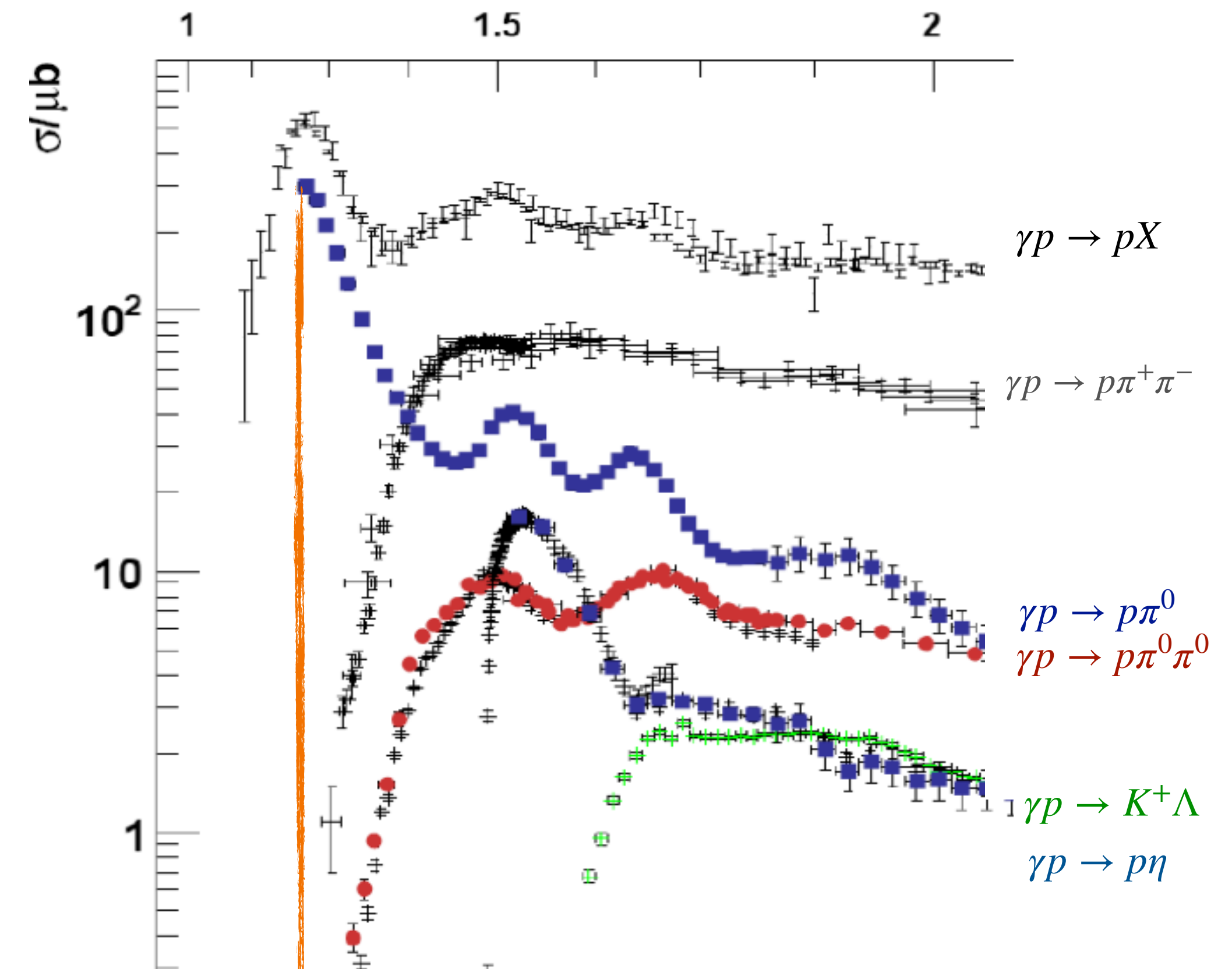
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Botany 101 – $\Delta(1232)$
Anderson/**Fermi**/...PhysRev.85.934

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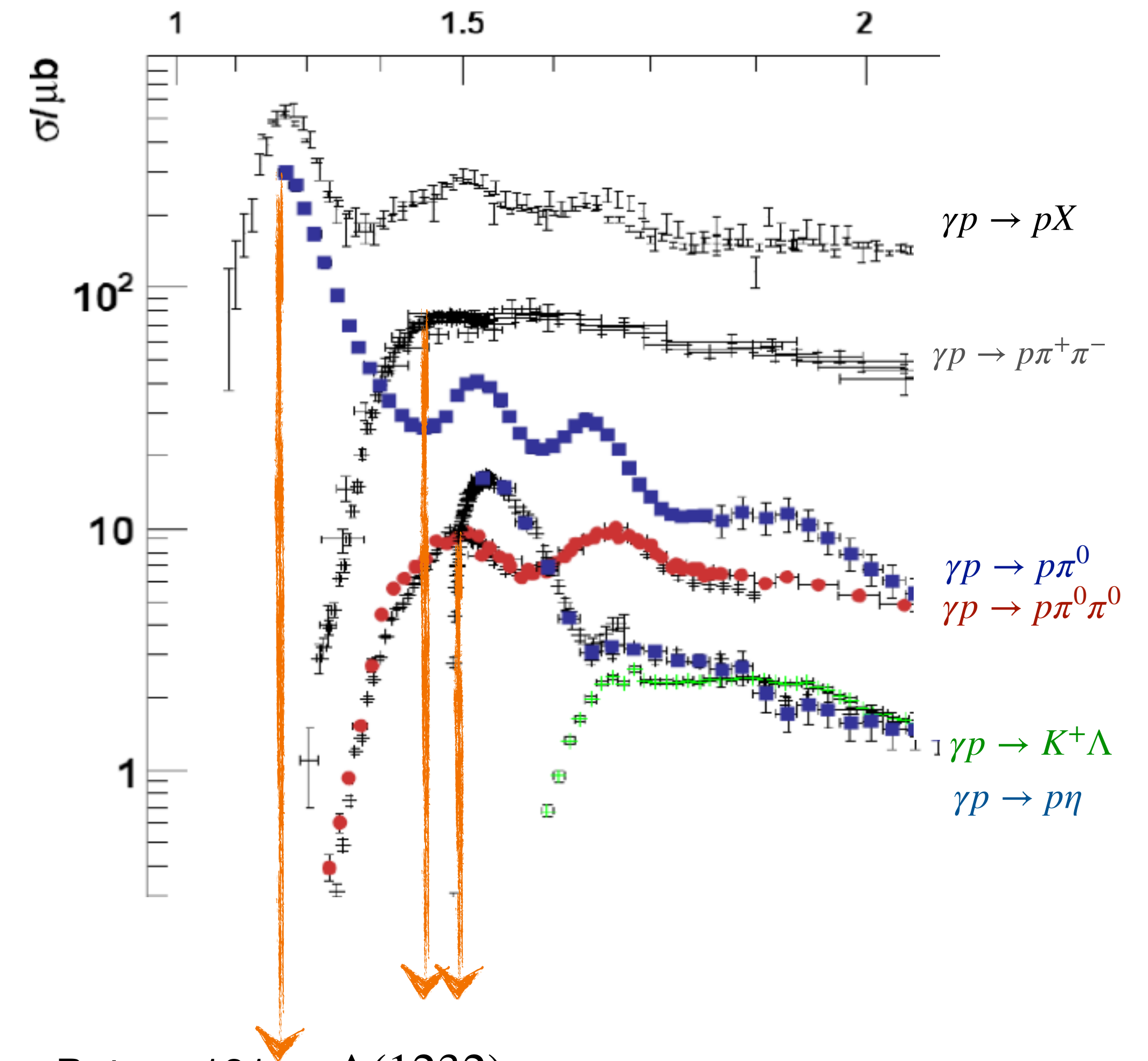
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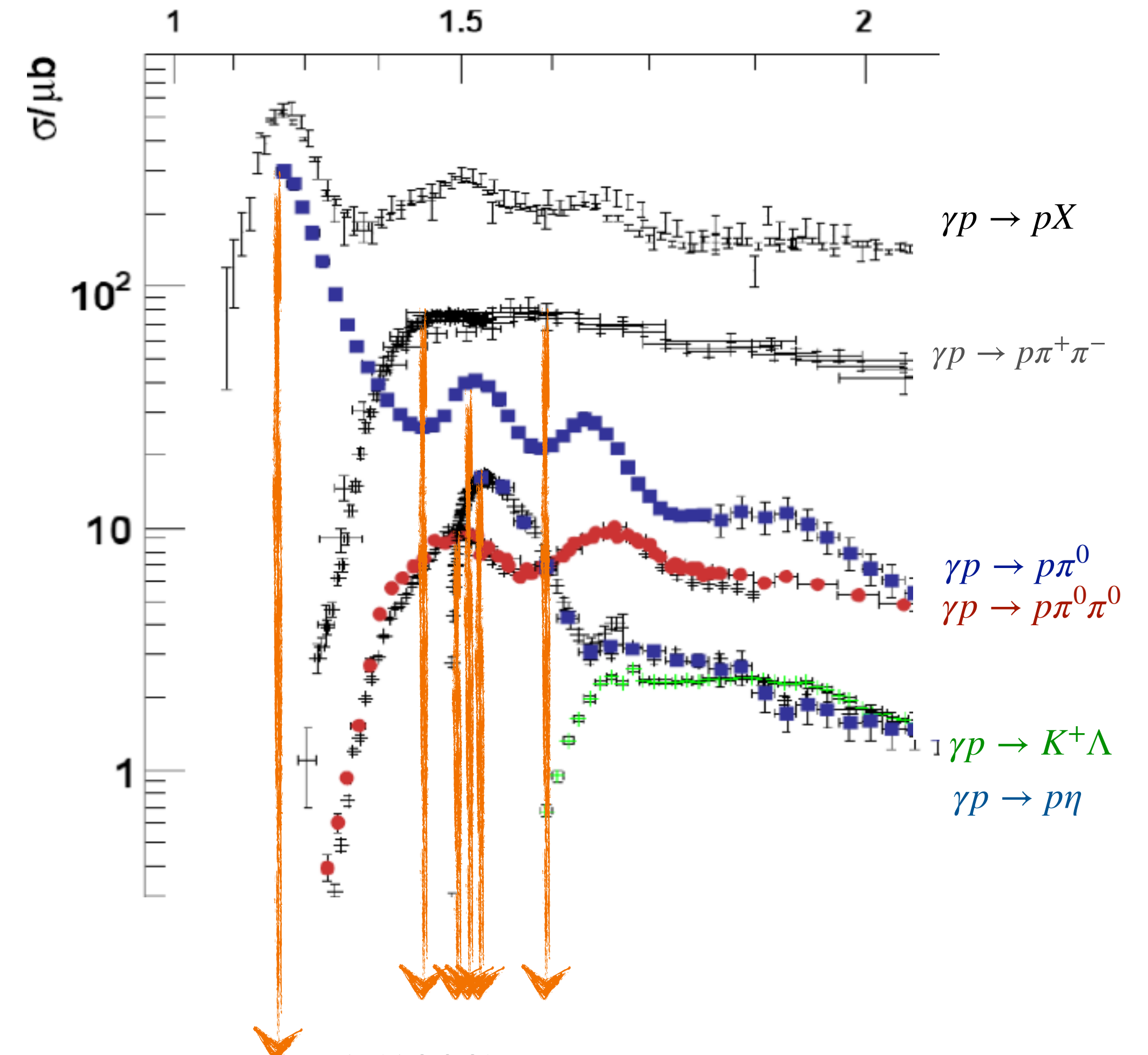
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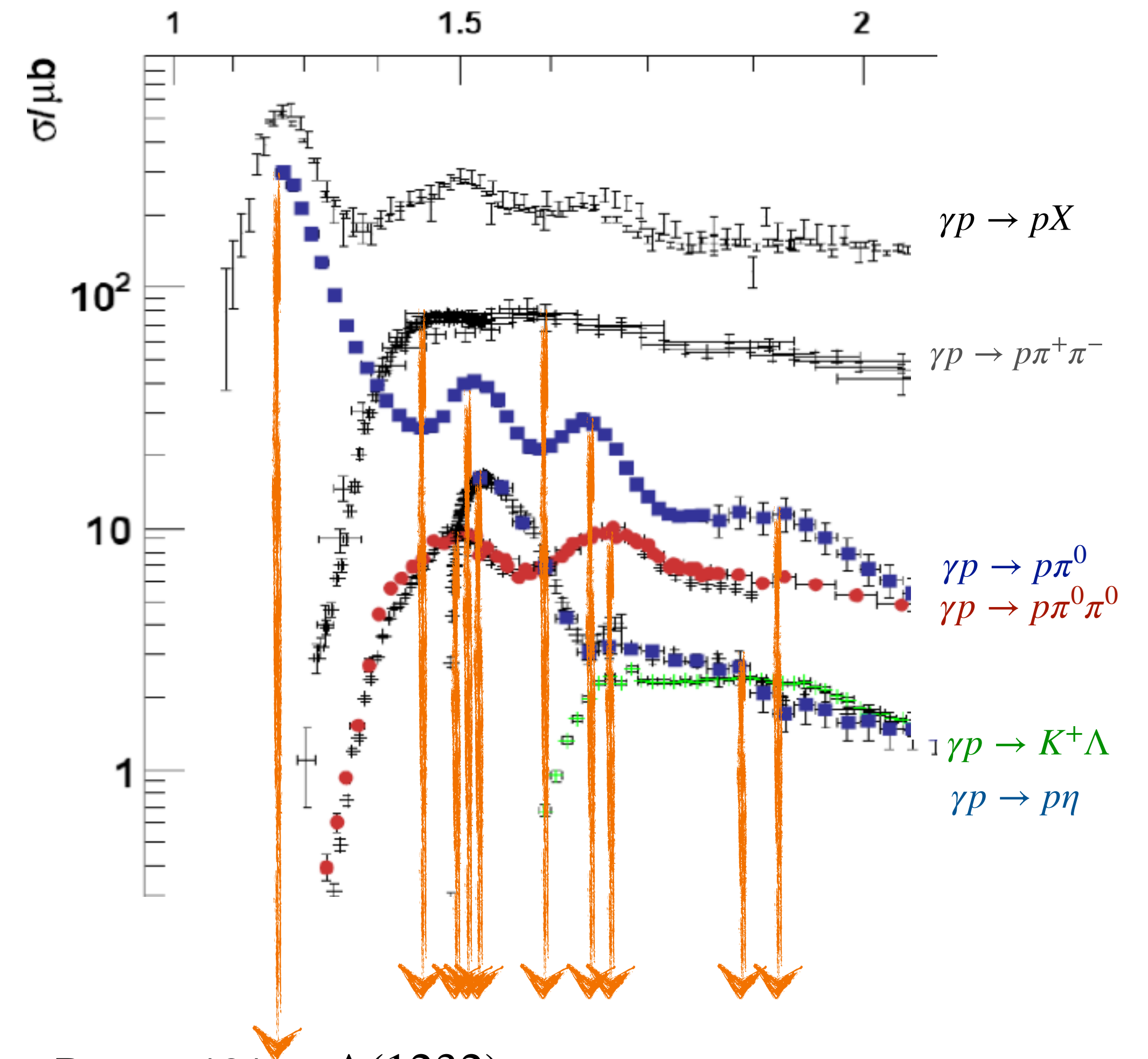
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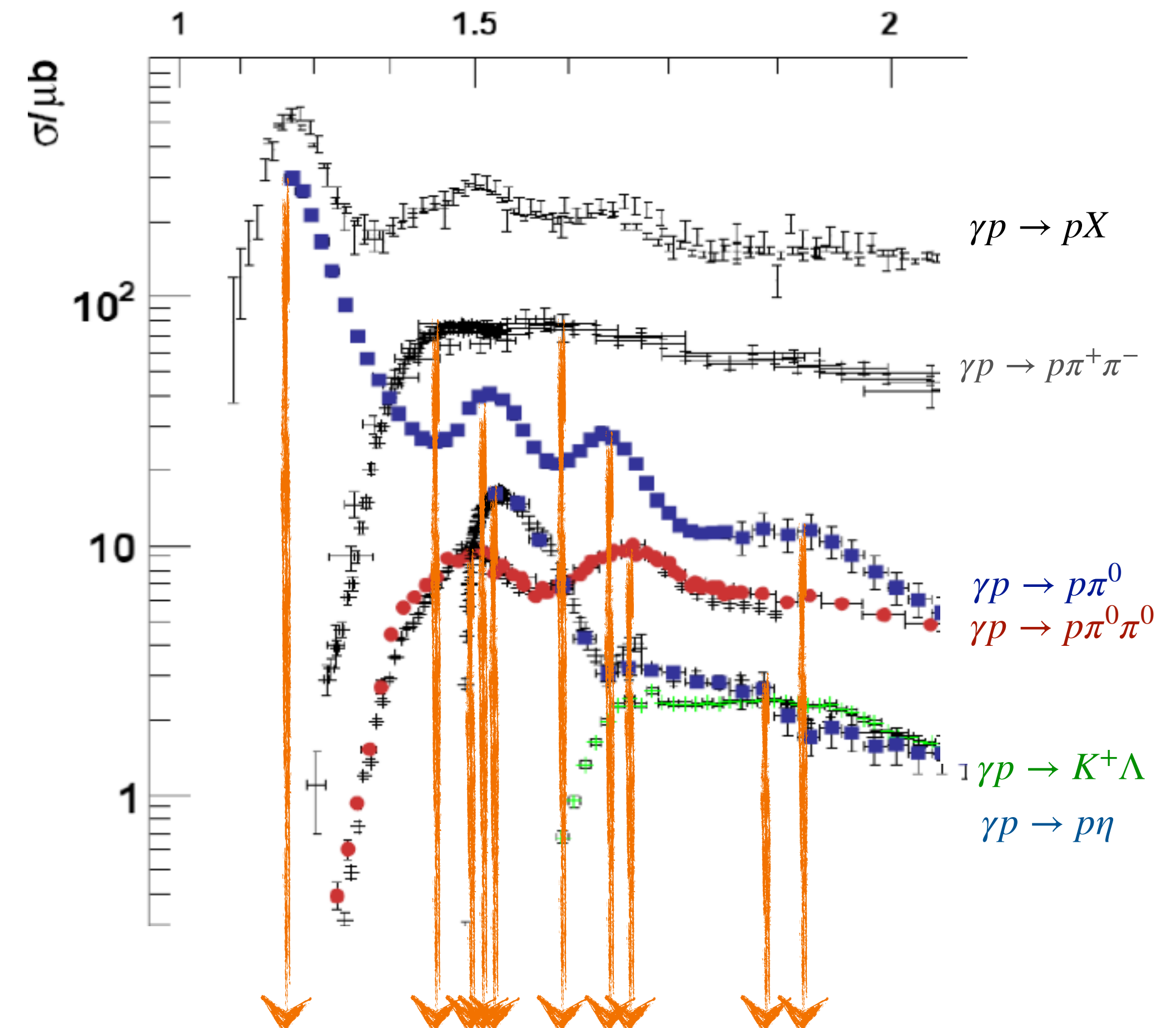
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Interpreting interaction probabilities

- many available data and ongoing experiments
- bumps $\Rightarrow$ resonances?
 - $\rightarrow$ reaction-type dependent



B

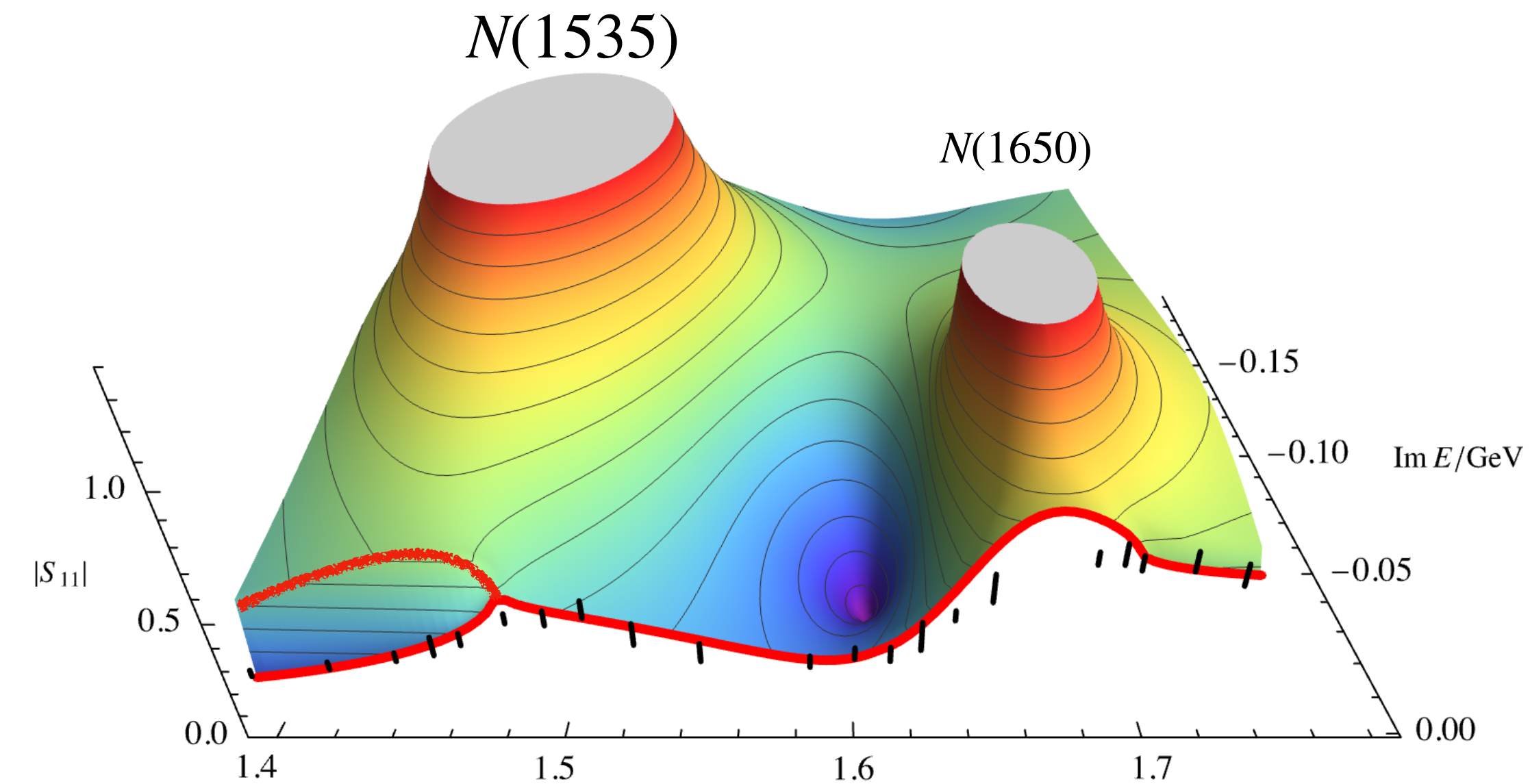
need P ~~D~~ G="particle bump group"

RESONANCE PARAMETER

Transition amplitudes (S-matrix)

- Mathematical constraints: Unitarity/Analyticity/Crossing symmetry
- Reaction-independent methodology
- Universal input:
 - ➡ Lattice QCD
 - ➡ Experiment
 - ➡ Effective Field Theories

Analytic continuation to unphysical
Riemann Sheets



Data: SAID: Phys. Rev. C 74 (2006) 045205
Model: MM et al. Phys.Rev.D 86 (2012) 094033

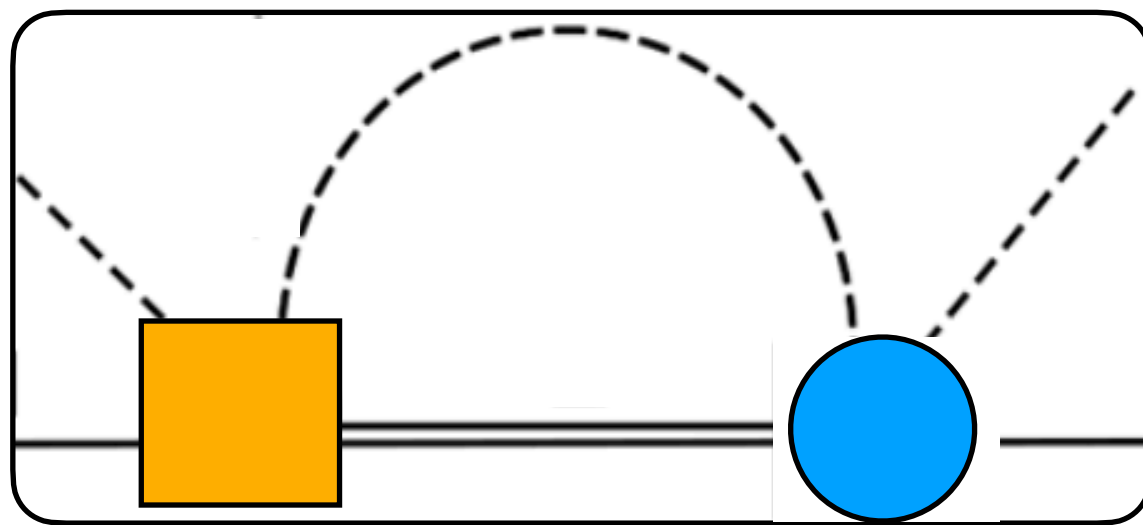
TRANSITION AMPLITUDE [JBW]

symmetries

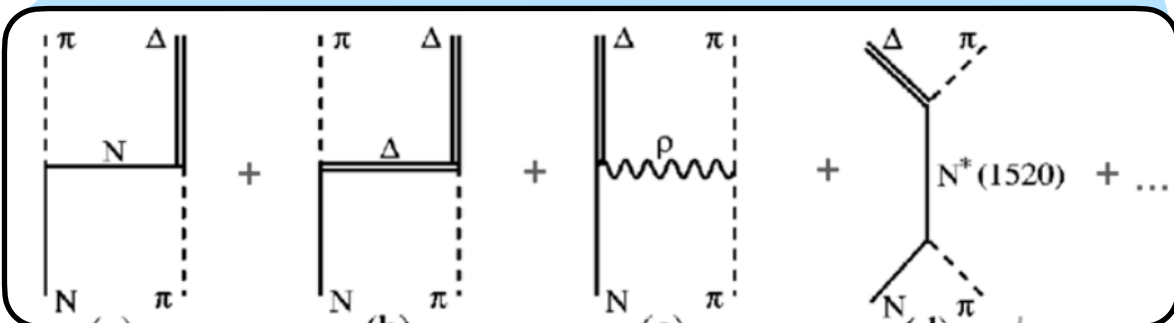
- Phenomenological Lagrangian
- CPT, chiral symmetry ...
- Many unknown parameters

Wess, Zumino, Lagrangian method for chiral symmetries, Phys. Rev. 163 (1967) 1727–1735

TRANSITION AMPLITUDE [JBW]



- Dynamical coupled-channel equation (DCC)
- Two-body unitarity (Lippman-Schwinger Eq.)
- Relativistic effects

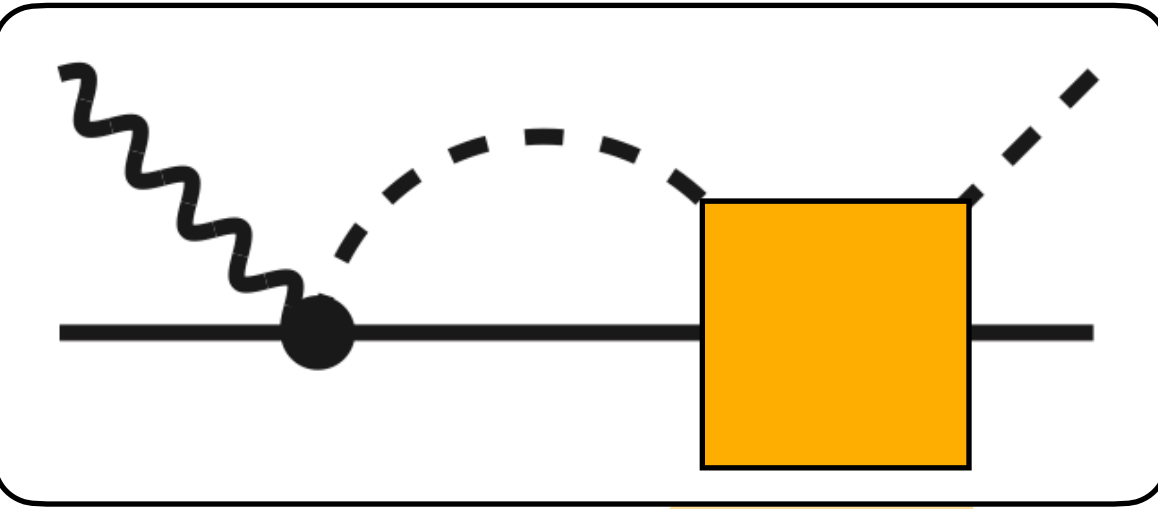


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scattering data ($\# \sim 10^4$)
CrystalBall/CERN/...

symmetries

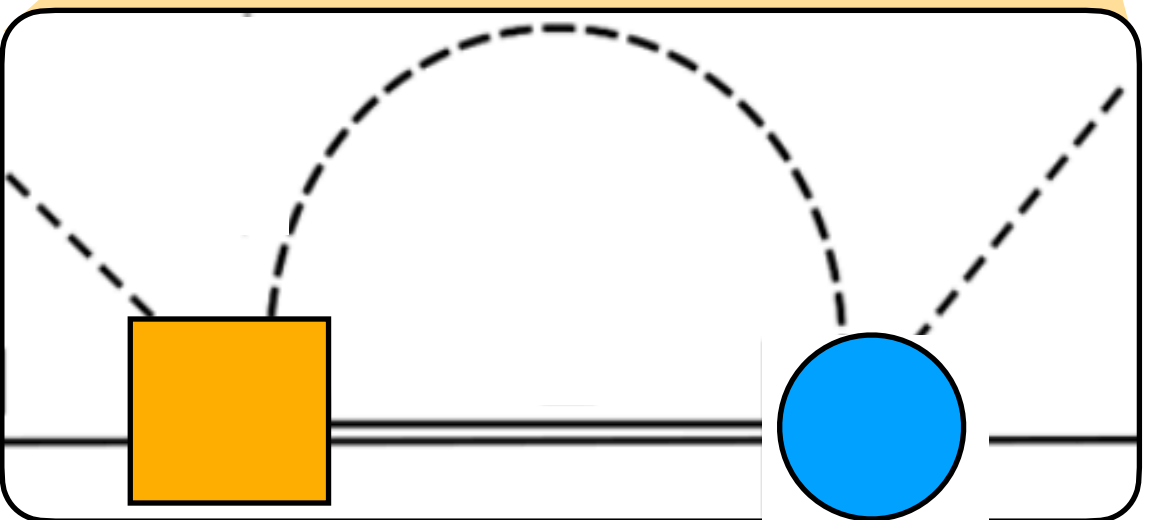
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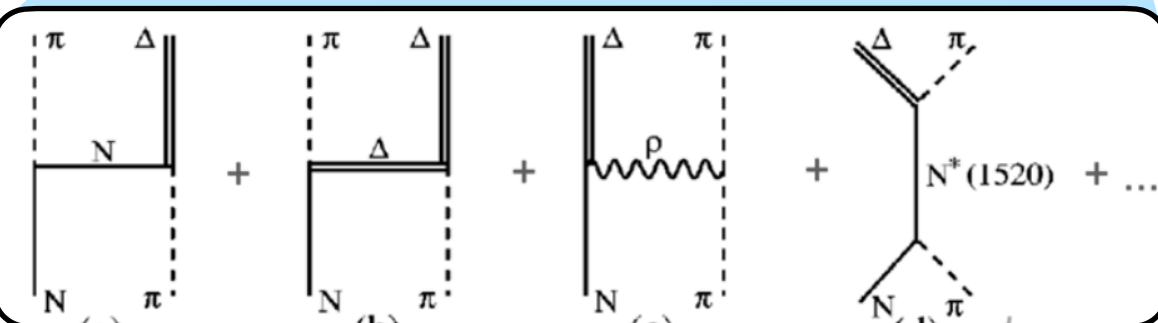
- virtual/real photon coupling
- final state unitarity
- Siegert's theorem etc.. Siegert, PR 5 (1937) Tiator, Few Body Syst. 57(2016)

[JBW] PRC103(2021), PRC106(2022), EPJA59 (2023) PRL133(2024)
Workman/Hergenrather/MM/Döring/Granados/Haberzettl/Meißner/Rönchen/Mart/Wang

[JuBo] Rönchen/Döring/Huang/Haberzettl/Haidenbauer/Hanhart/Krewald/Meißner/Nakayama EPJA 50, 101 (2014)



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Wess, Zumino, Lagrangian method for chiral symmetries, Phys. Rev. 163 (1967) 1727–1735

meson electroproduction data
(# $\sim 10^5$) CLAS/GLUEX/...

⋮

meson photoproduction data
(# $\sim 10^4$) ELSA/MAMI/...

⋮

scattering data (# $\sim 10^4$)
CrystalBall/CERN/...

↑
symmetries

TRANSITION FORM FACTORS

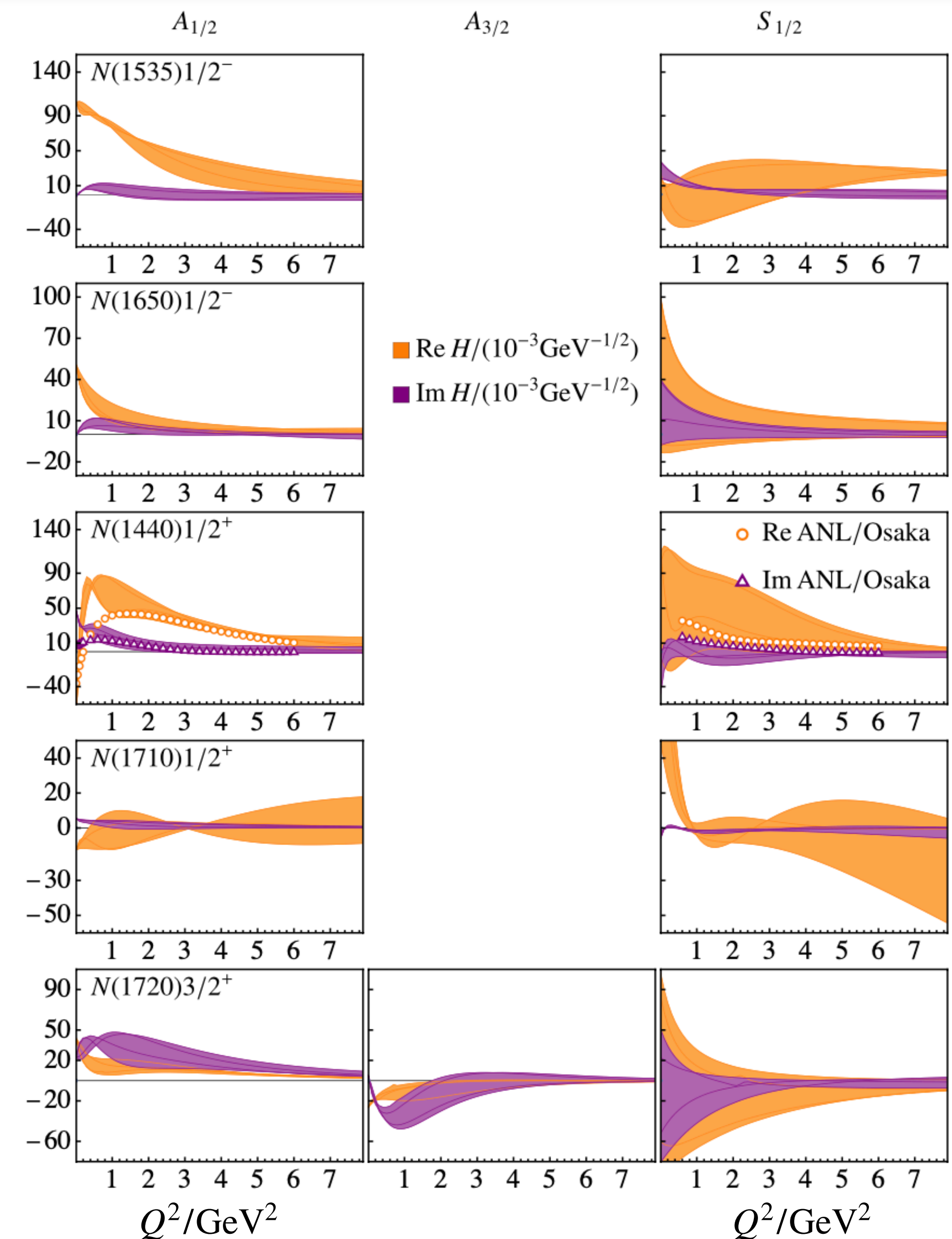
Residuum at the pole position

- Hadron structure probe
- transition between excited and ground state baryons

$$H_h^{l\pm,I}(Q^2) = C_I \sqrt{\frac{p_{\pi N}}{\omega_0} \frac{2\pi(2J+1)z_p}{m_N \tilde{R}^{l\pm,I}}} \tilde{\mathcal{H}}_h^{l\pm,I}(Q^2),$$

- 12 N/Δ states are determined

[JBW] PRC103(2021), PRC106(2022), EPJA59 (2023) PRL133(2024)
Workman/Hergenrather/MM/Döring/Granados/Haberzettl/Meißner/Rönchen/Mart/Wang



...

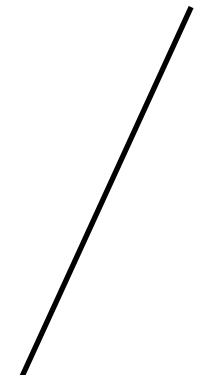
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Residuum at the pole position

- FT in light front reference frame
- Charge distribution for resonance transitions

$$\rho_0^{NN^*}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1^{NN^*}(Q^2),$$

$$aA_{1/2} + bS_{1/2}$$


TRANSITION FORM FACTORS

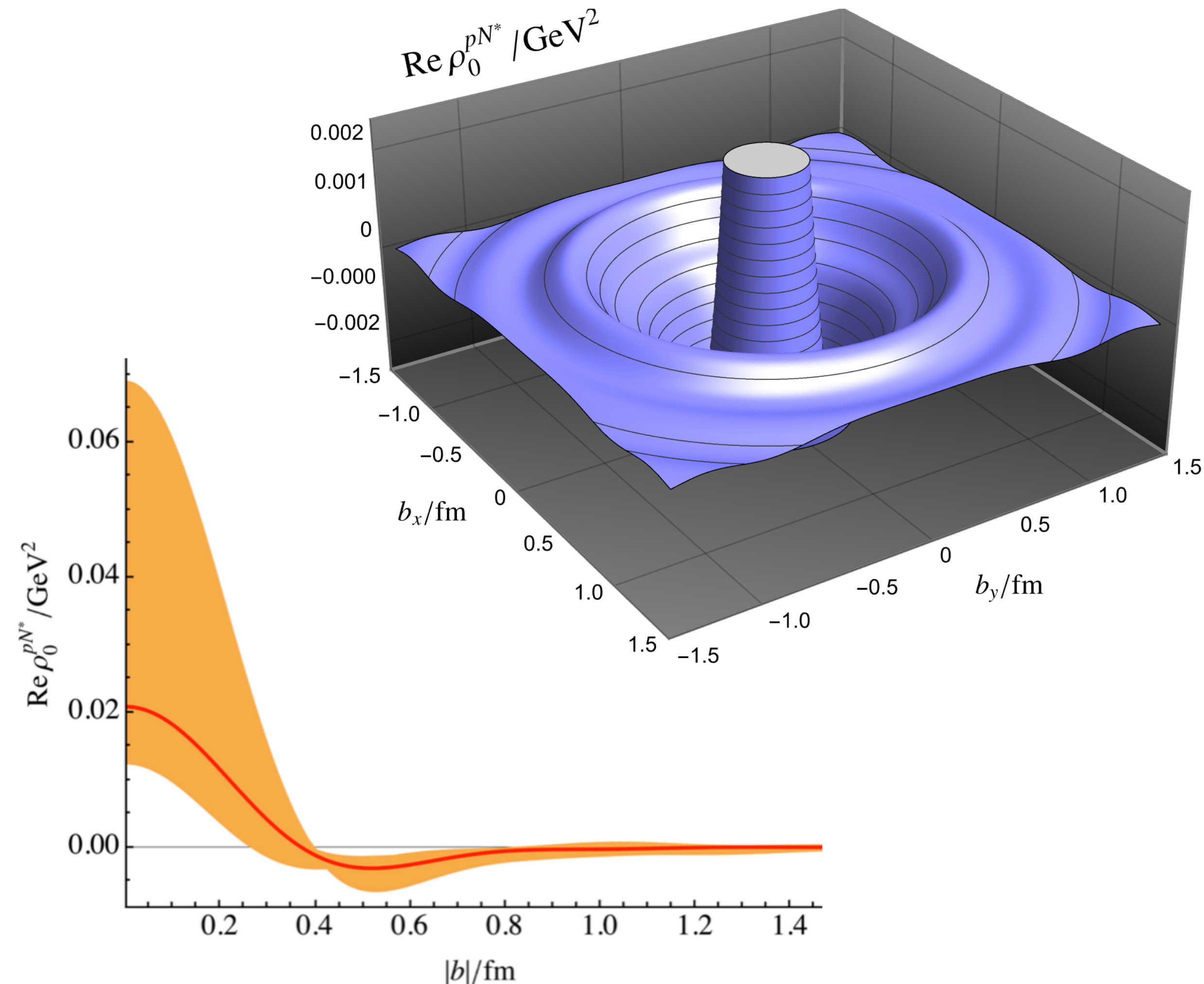
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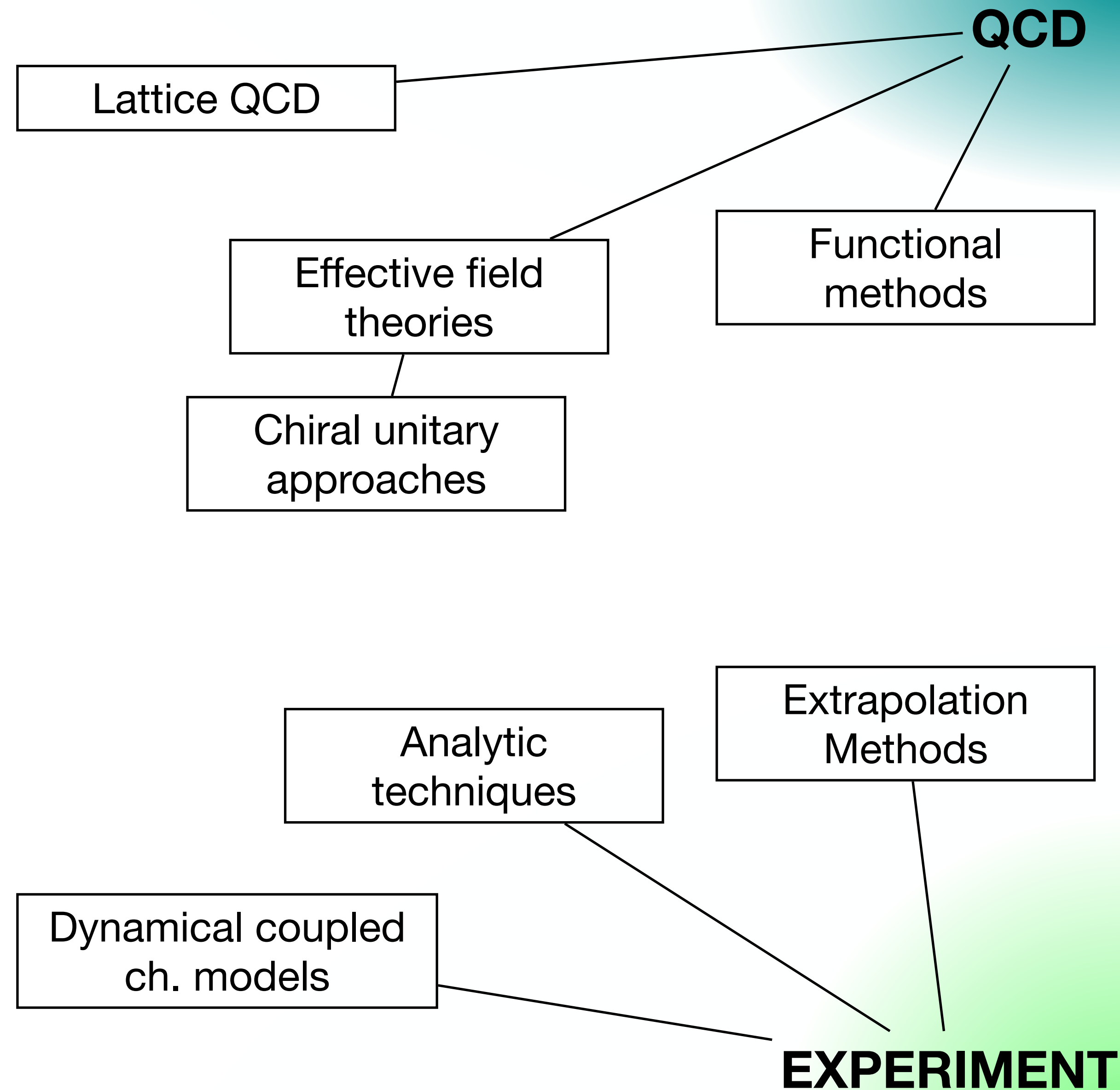
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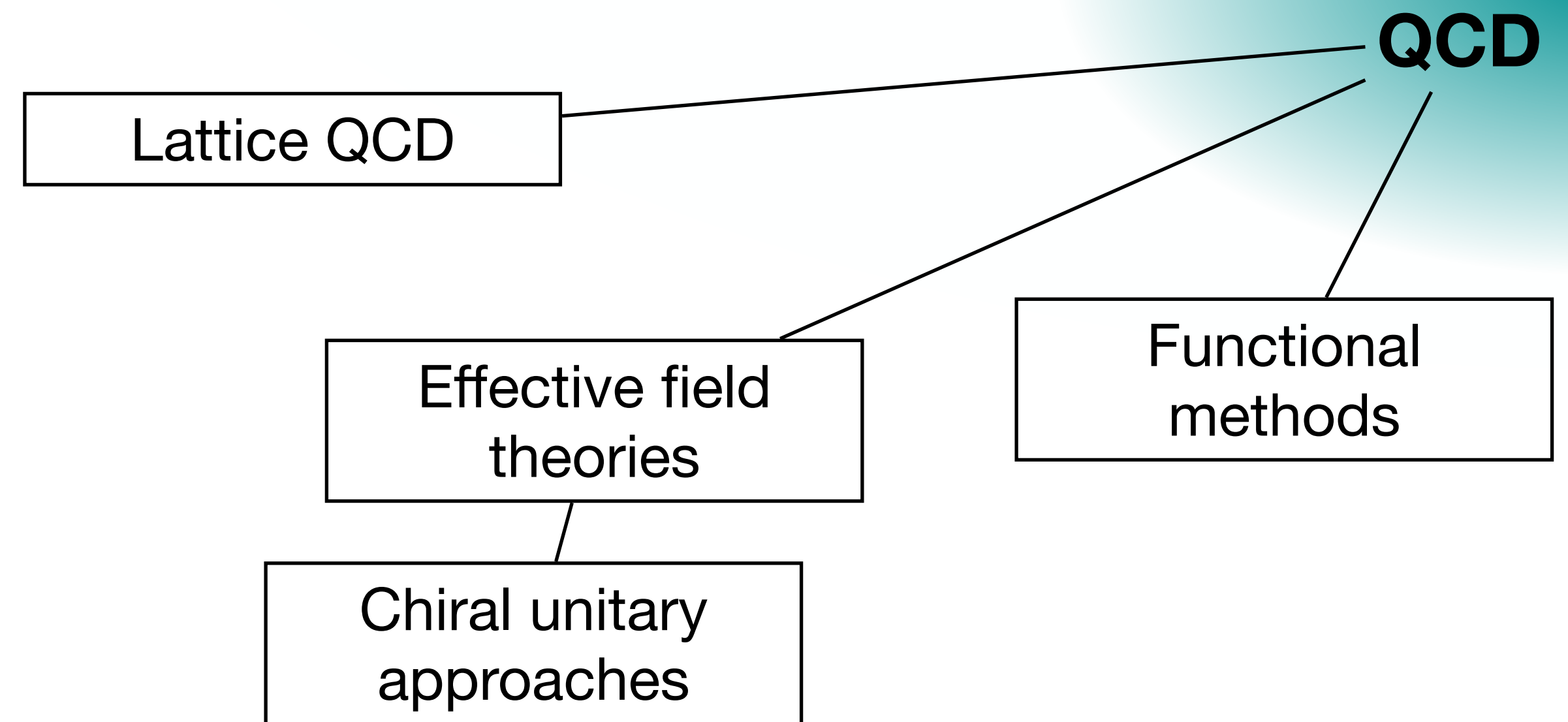
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THEORY LANDSCAPE



THEORY LANDSCAPE



Baryon transition form factors ✓

- data driven DCC approach
- new experimental data
- “charge density of resonances”

[JBW] *Phys.Rev.Lett.* 133 (2024) 10, 101901

Dynamical coupled
ch. models

Analytic
techniques

Extrapolation
Methods

EXPERIMENT

THEORY LANDSCAPE

QCD

Lattice QCD

Effective field theories

Functional methods

Chiral unitary approaches

?

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[JBW] *Phys.Rev.Lett.* 133 (2024) 10, 101901



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Analytic techniques

Extrapolation Methods

EXPERIMENT

THEORY LANDSCAPE

QCD

Lattice QCD

remainder of the talk

$\omega(782)$

- Resonant 3-hadron dynamics
- quark mass dependence
- symmetry tests

Yan et al. 2407.16659 [hep-lat] — In press at PRL

Effective field theories

Functional methods

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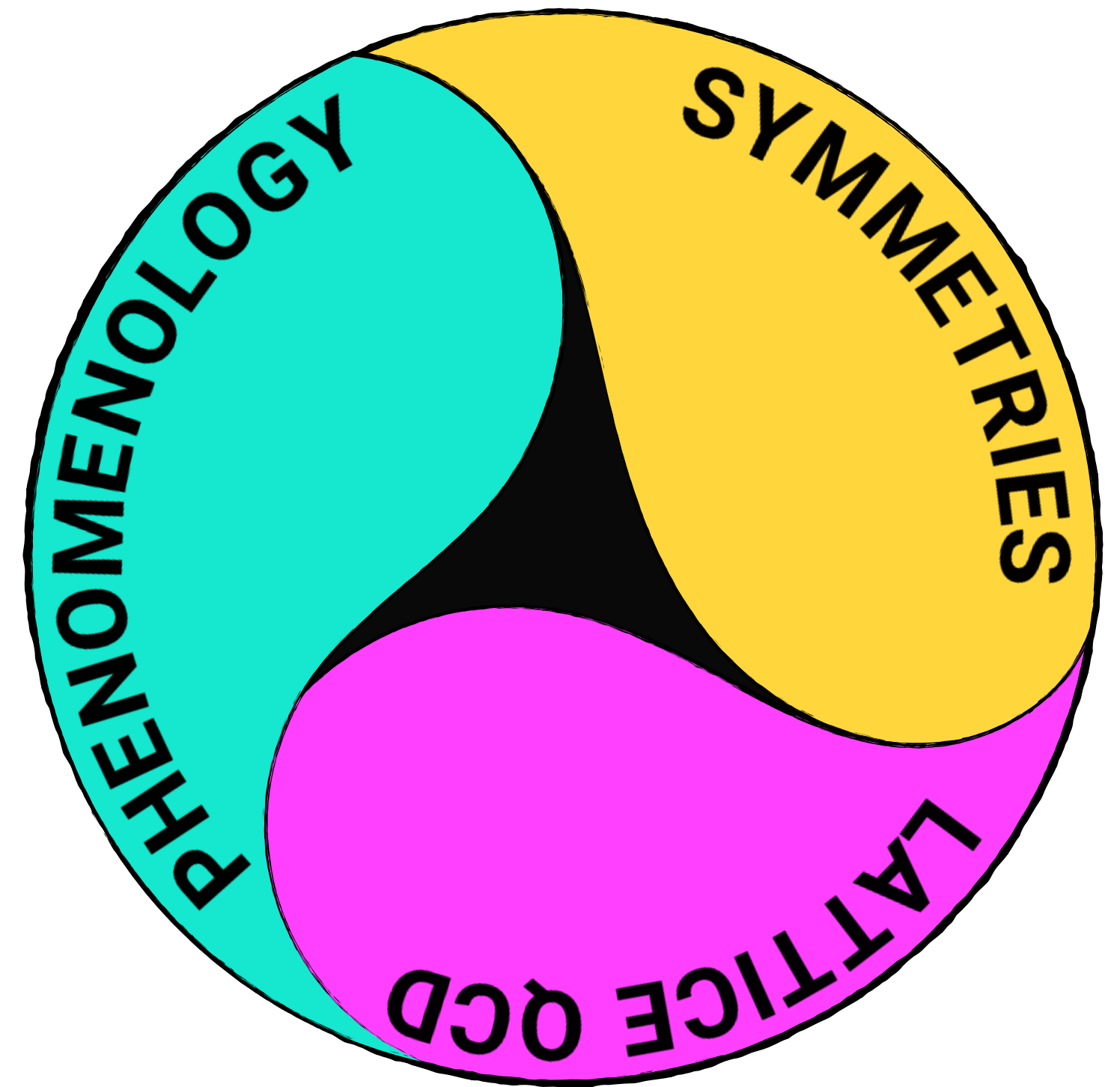
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EXPERIMENT

HADRONIC THREE-BODY PROBLEM



IMPACT

Most known states have large 3-body content

Review: MM/Meißner/Urbach *Phys.Rept.* 1001 (2023) 1-66

- $\omega(782) \rightarrow \pi\pi\pi$
- $a_1(1260) \rightarrow \pi\pi\pi$
- $N(1440) \rightarrow \pi\pi N$
- $X(3872) \rightarrow DD\pi$

Beyond Standard Model/ τ -EDM/...

Belle/Belle2 <https://arxiv.org/pdf/2207.06307>

HLbL contribution?

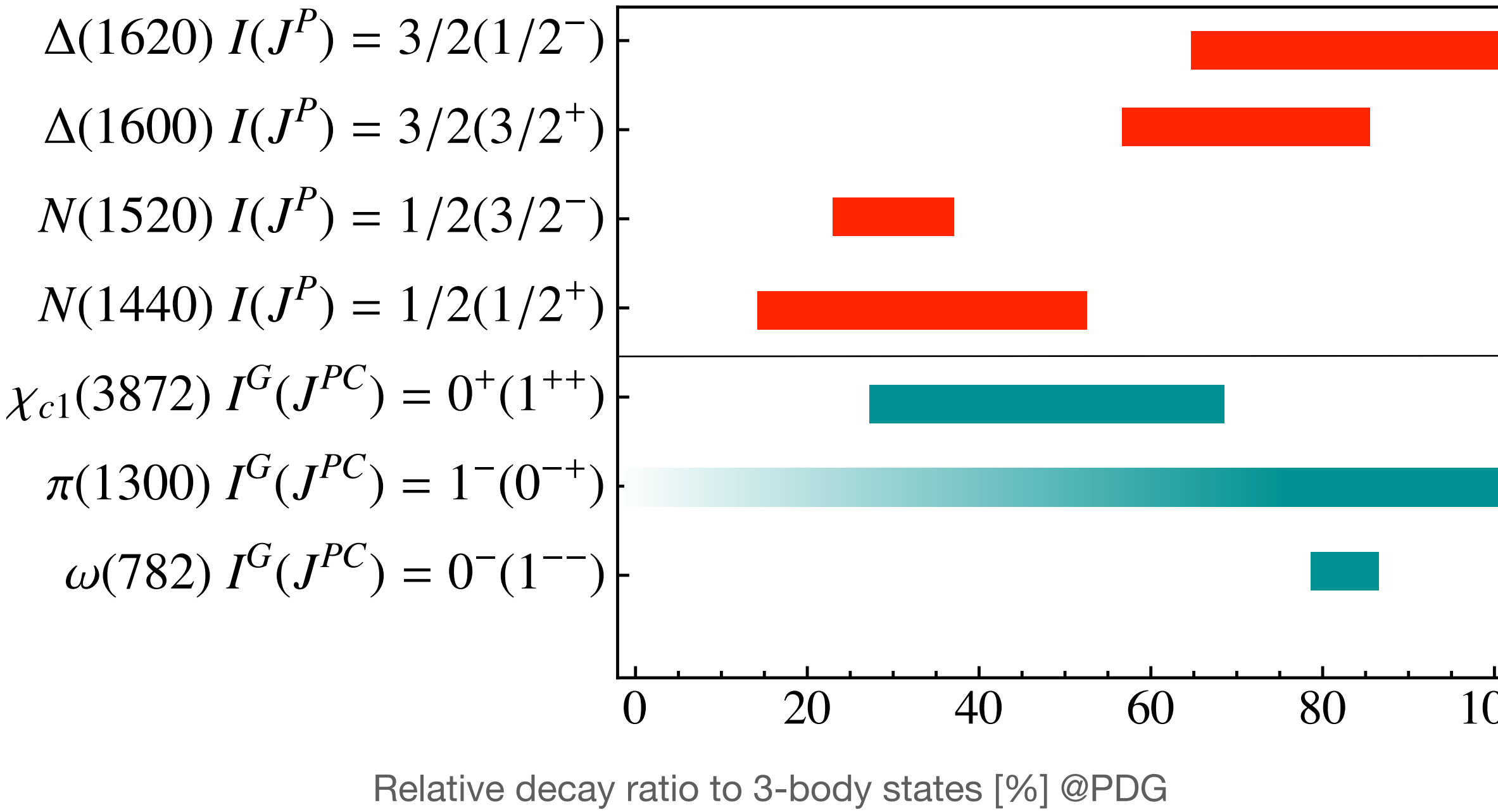
DSE — Eichmann/Fischer/Haeuser/Regenfelder 2411.05652
LQCD — [Mainz 2024] 2411.07969v1

Exotic states of matter

GlueX@JLAB; COMPASS@CERN;

Triangle singularities

Ketzer/Aceti/Dai/Oset/Mikhashenko/Bayar/Guo... Review: FK Guo *Prog.Part.Nucl.Phys.* 112 (2020) 103757



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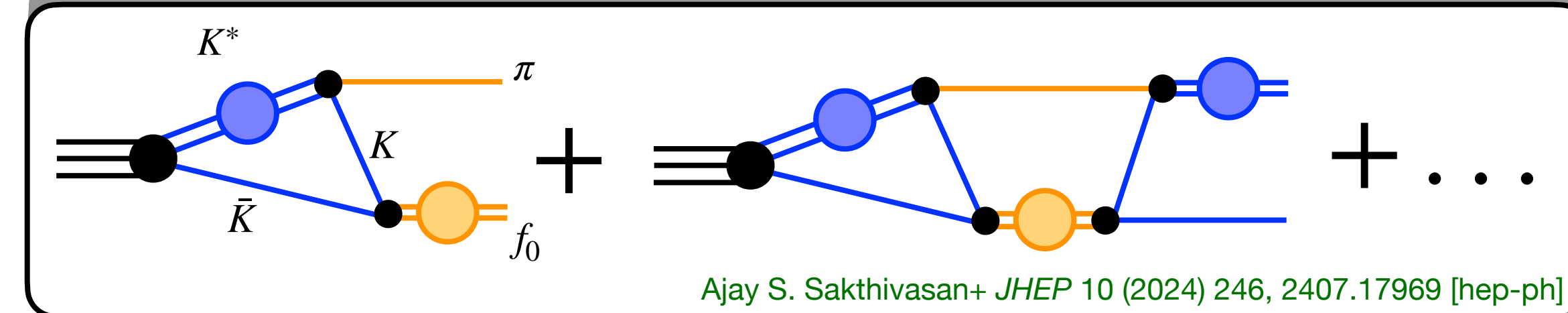
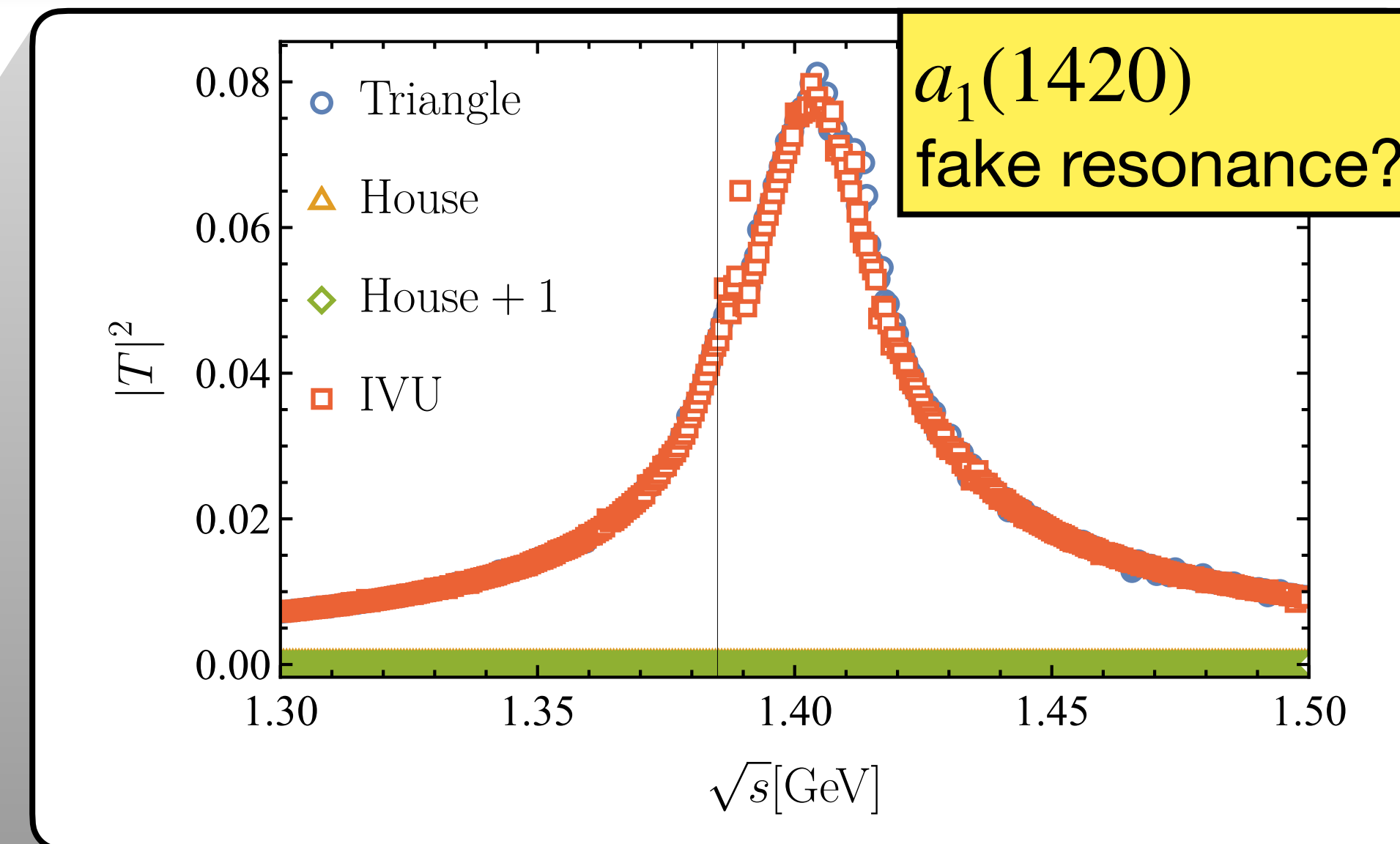
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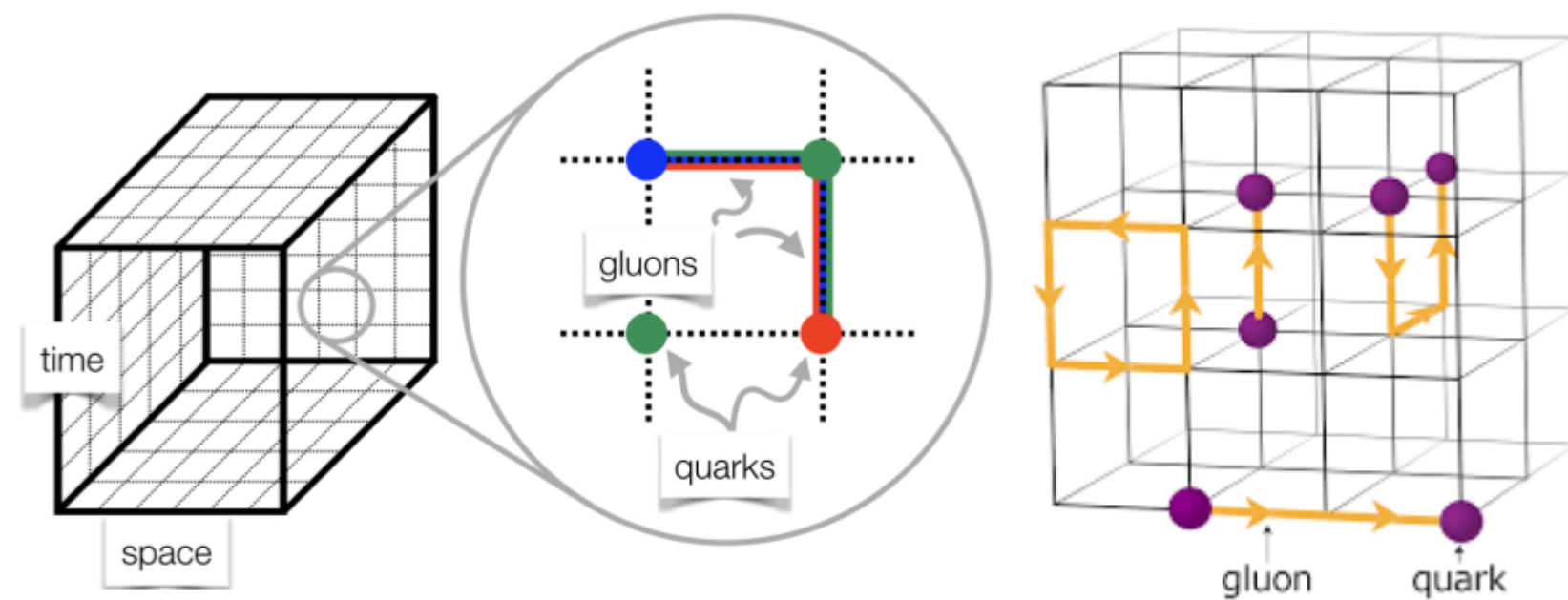
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Lattice QCD



Credit: Ed van Bruggen/edryd

$\omega(782)$ Yan/MM/Garofalo/Meißner/Liu/Lu/Urbach (in press at PRL)

T_{cc}^* : Padmanath/Prelovsek PRL129(2022)

$\pi\pi K, \pi K K$ Draper/Hanlon/Romero-Lopez/Morningstar/Sharpe *JHEP*05 (2023)

$a_1(1260)$ MM/Culver/Brett/Alexandru/Doring/Lee PRL127 (2021)

$\pi\pi\pi(I=3)$ Hansen/Briceño/Edwards/Thomas/Wilson PRL126 (2021)

$\pi\pi\pi(I=3)$ Fischer/Kostrzewa/Liu/Fernando Romero-López/Ueding/Urbach EPJC81 (2021)

$\pi\pi\pi(I=3)$ MM/Culver/Brett/Alexandru/Doring/Lee PRD101 (2020)

$\pi\pi\pi(I=3)$ Blanton/Romero-Lopez/Sharpe PRL124 (2020)

$KKK(I=3/2)$ Alexandru/brett/Culver/MM/Döring PRD102 (2020)

$\pi\pi\pi(I=3)$ Hörz/Hanlon PRL123 (2019)

$\pi\pi\pi(I=3)$ MM/Döring PRL122 (2019)

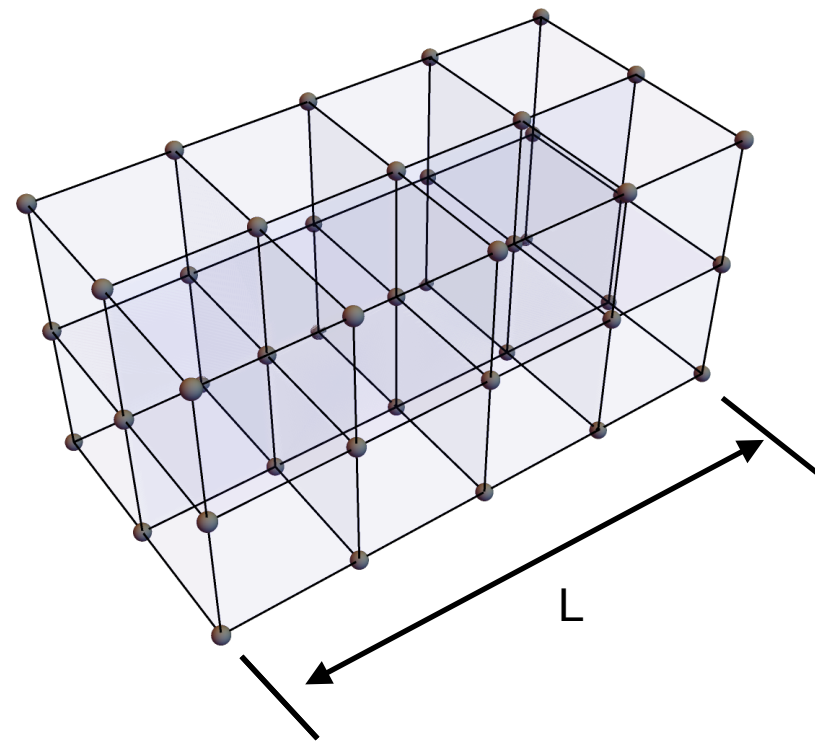
R_{oper}^* : Lang/Leskovec/Padmanath/Prelovsek PRD95 (2017)

X_{3872}^* : Padmanath/Lang/Prelovsek

2024

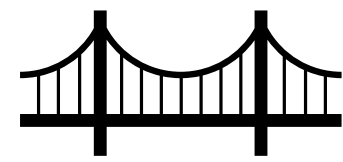
2015

3 HADRONS ON THE LATTICE



Lattice QCD — QCD Green's functions:

- Euclidean space-time
- unphysical pion mass
- **finite-volume**



**Quantization
condition**

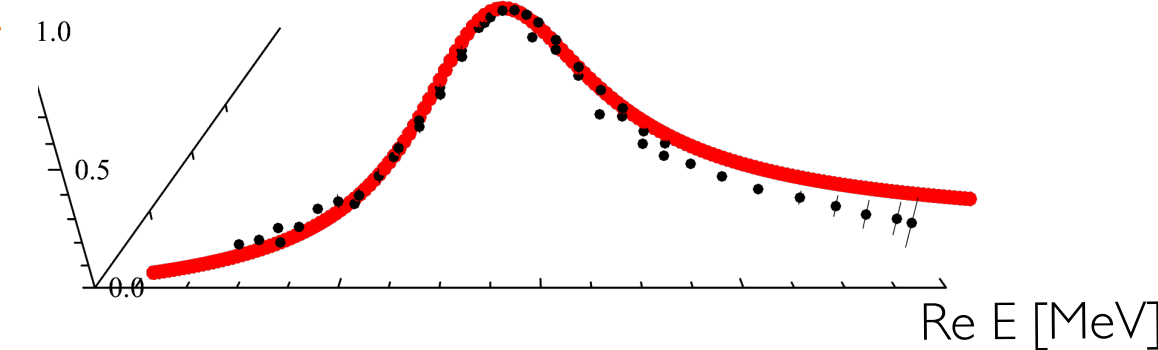
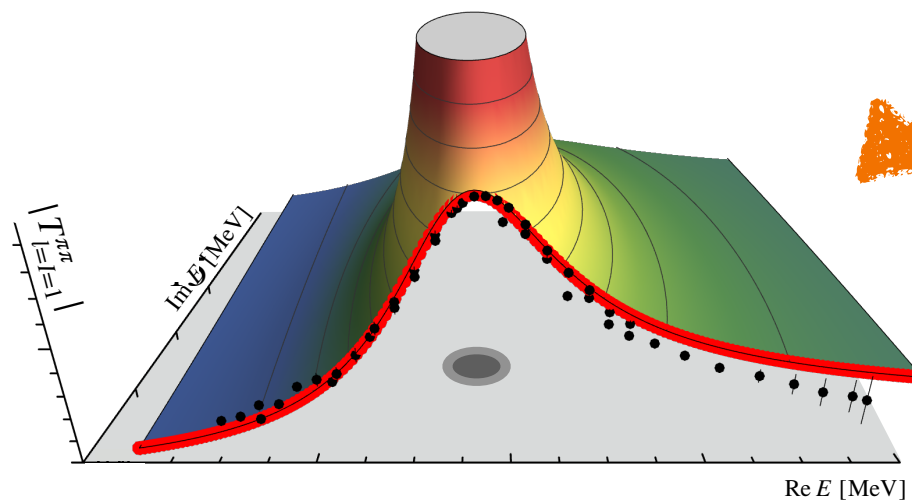
A) on-shell states

- inf. vol.: imaginary part of transition amplitudes
- fin. vol: far propagating states — “feel” the boundaries

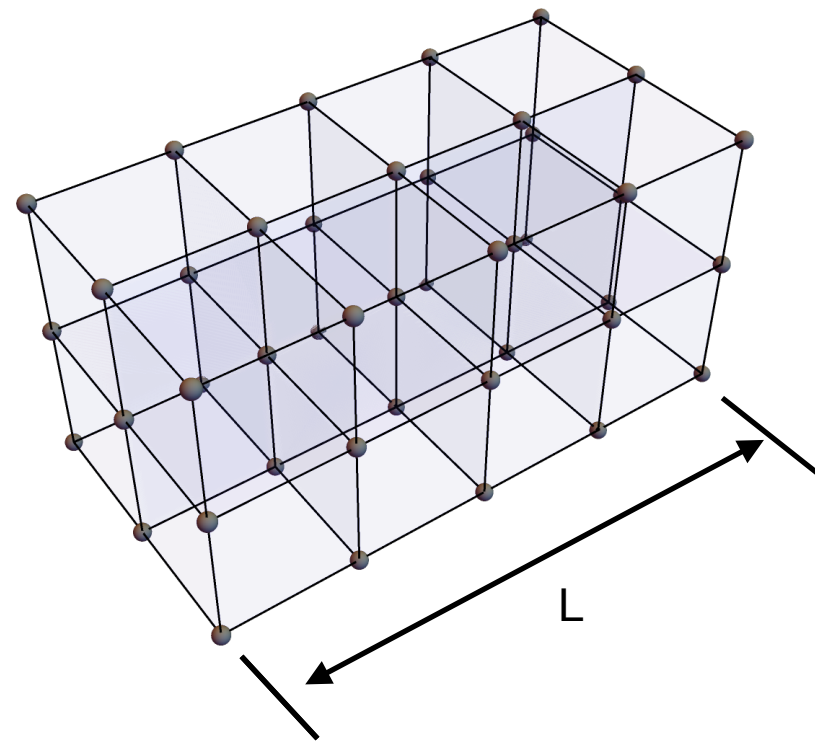
B) off-shell states

- inf. vol.: only real parts
- fin. vol: decay exponentially $\sim e^{-ML}$

⇒ use unitarity (∞ -volume) to separate A from B



3 HADRONS ON THE LATTICE

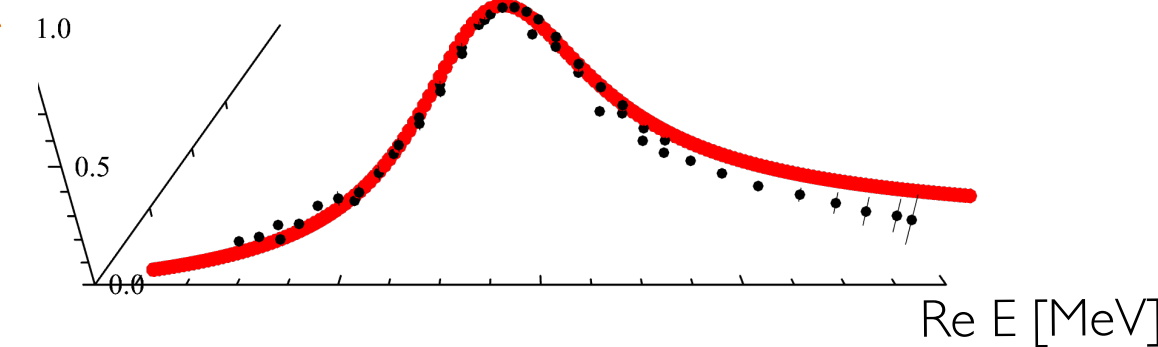
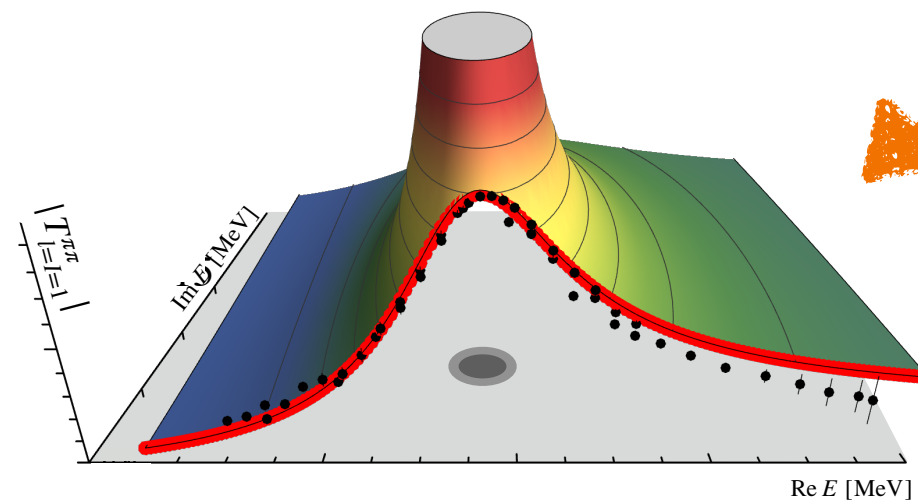


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“Finite Volume Unitarity” FVU

$$\det \left[2L^3 E_p \left(\tilde{K}^{-1} - \Sigma^L \right) - B - C \right]^\Lambda \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017)

Vol.-independent
2-,3-body force

$\tilde{K}, C$

“Infinite Volume Unitarity” IVU

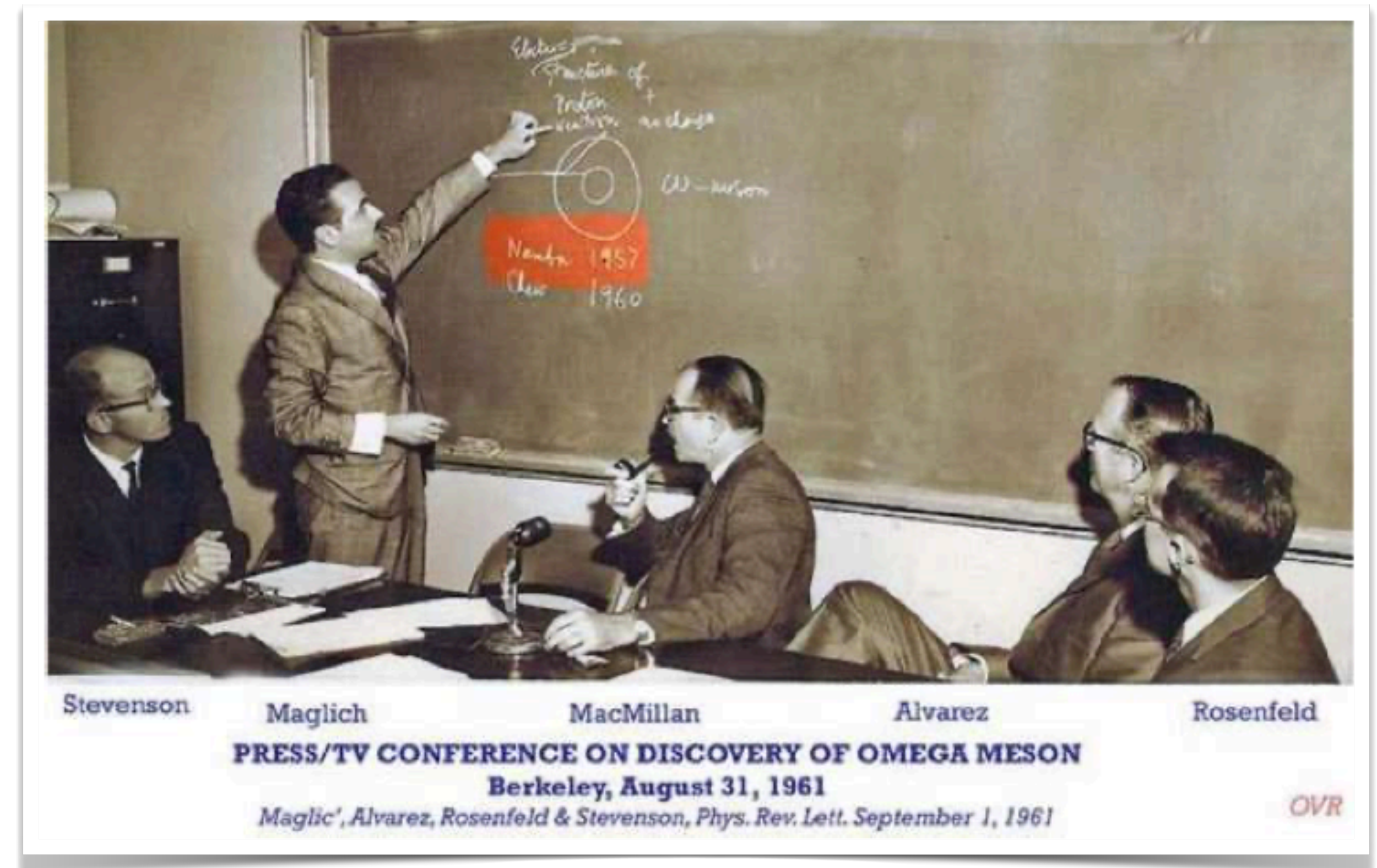
$$T = B + C + \int \frac{d^3 \ell}{(2\pi)^3} \frac{(B + C)}{2E_\ell} \frac{1}{\tilde{K}^{-1} - \Sigma} T$$

MM/Hu/Döring/Pilloni/Szczepaniak
Eur.Phys.J.A 53 (2017)

$\omega(782)$

Impact

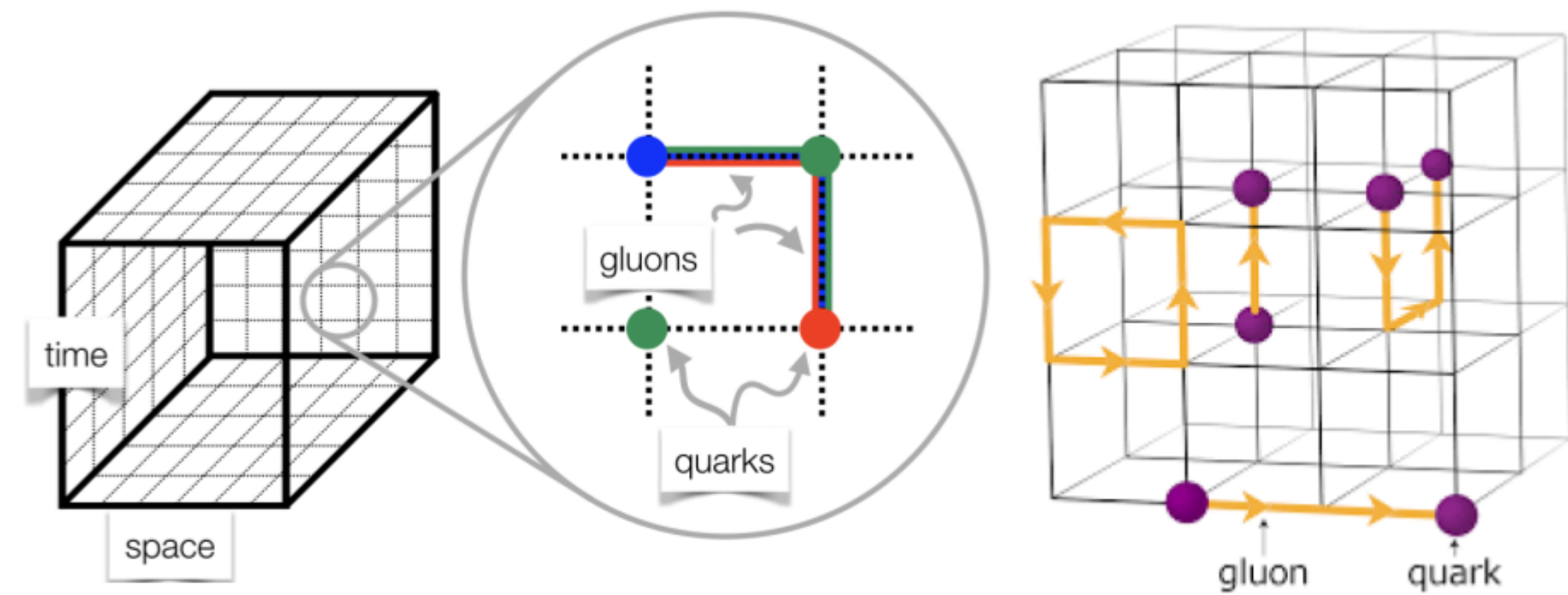
- lightest hadron decaying into 3 particles
- isoscalar response within the VMD of the photon-nucleon interactions
- generates N–N repulsion at < 1 fm in one-boson-exchange picture



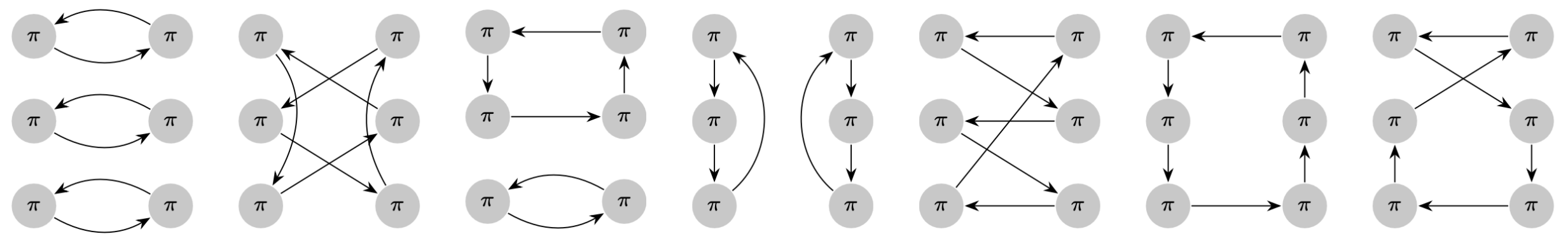
$\omega(782)$ FROM QCD

Lattice QCD setup

- $N_f = 2 + 1$ Clover fermions CLQCD, 2024
- 2/3 particle operators OpTion package @HaoboYan
- 2 pion masses ($\approx 210, 305$ MeV)
- 2 volumes ($L^3 = 32^3, 48^3$)
- Many configurations to include...



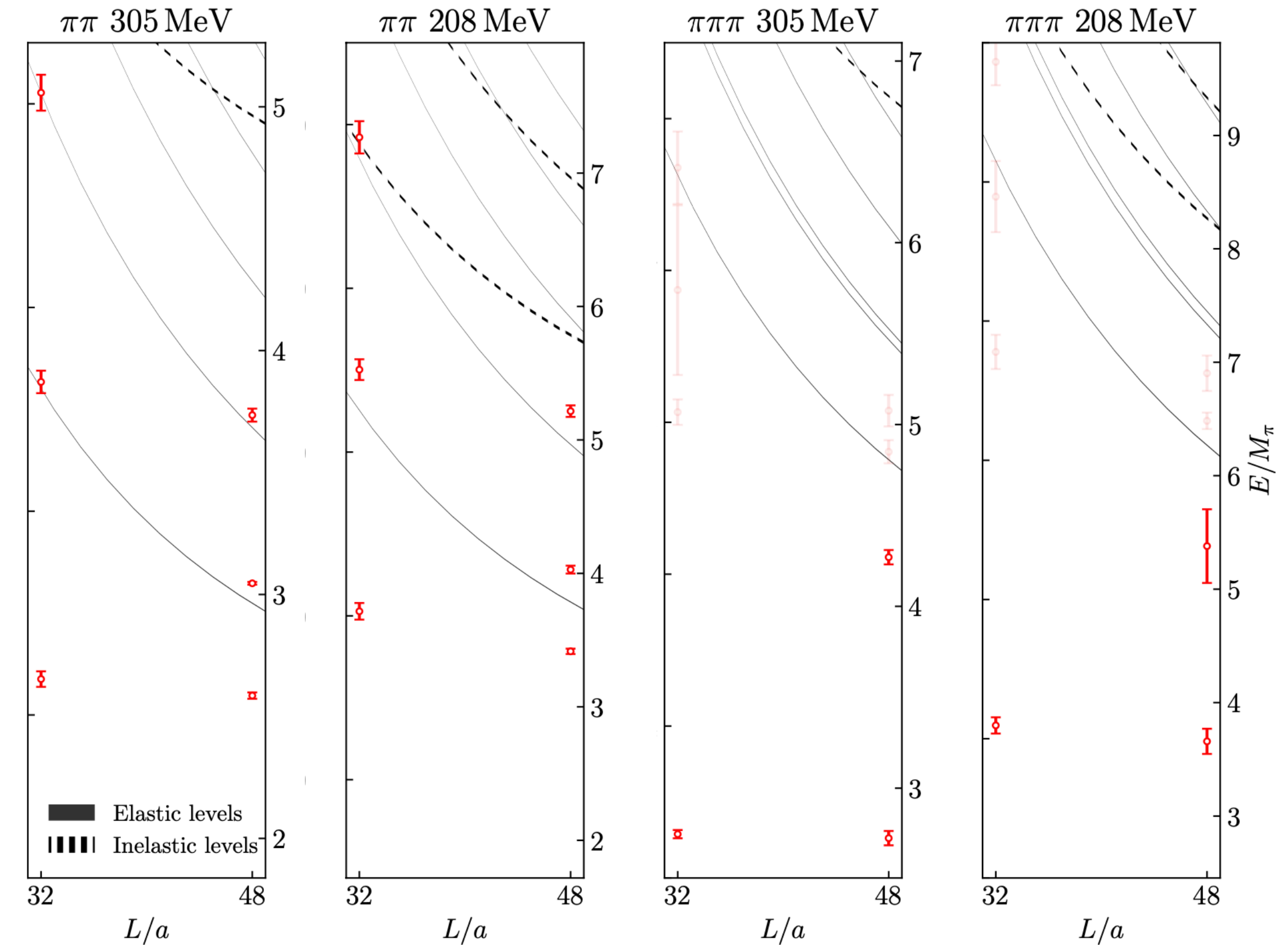
Credit: Ed van Bruggen/edryd



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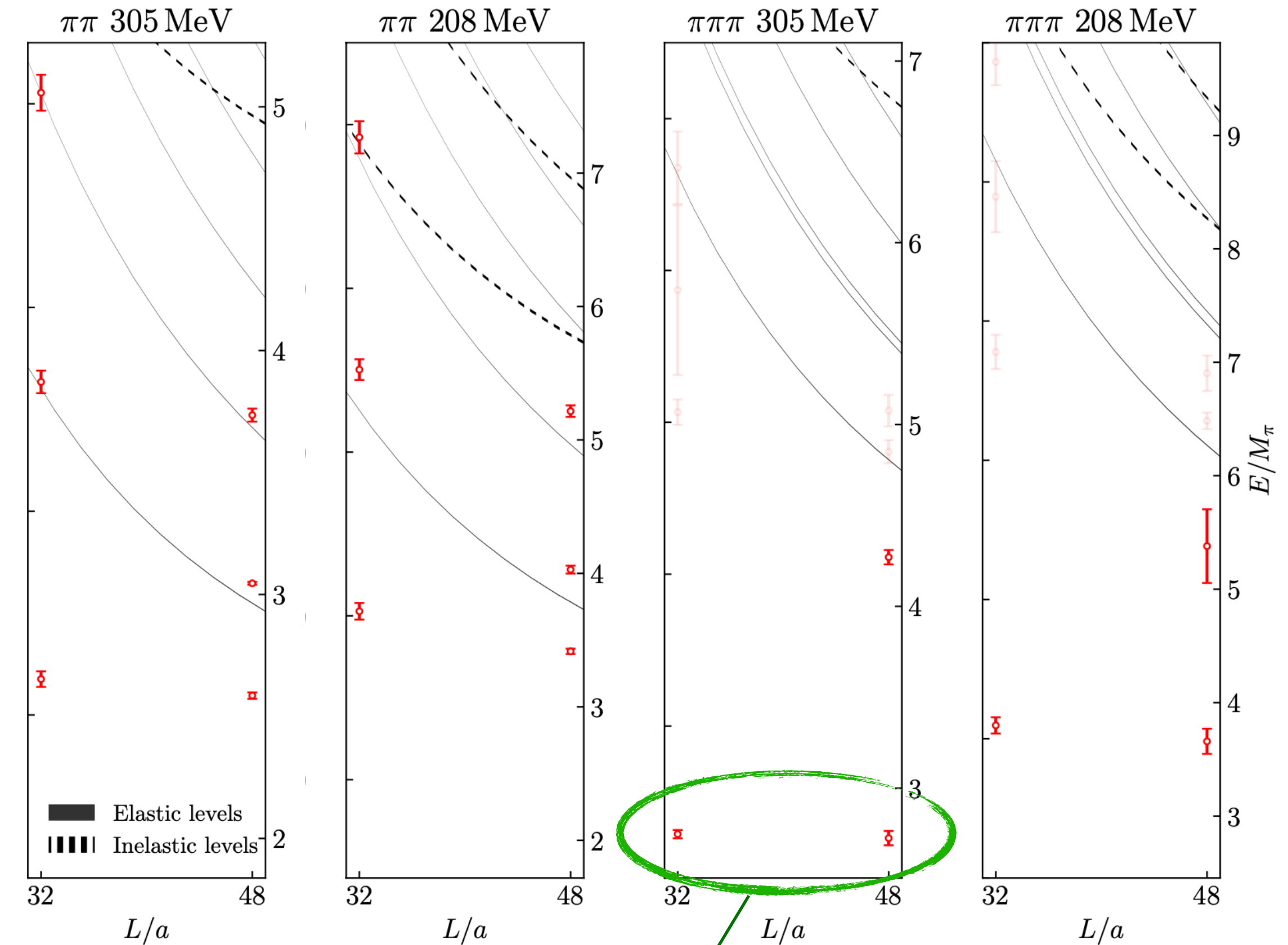
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- **Finite-volume spectrum: 17 Energy eigenvalues**



$\omega(782)$ FROM QCD

Mapping to infinite volume

$$\mathbf{FVU} \det \left[2L^3 E_p \left(\tilde{K}^{-1} - \Sigma^L \right) - B - C \right]^\Lambda \equiv 0$$

MM/Döring *Eur.Phys.J.A* 53 (2017) 12, 240

- known quantities B, Σ^L, E_L
- unknown volume-indep. 2/3-body “force” $\tilde{K}, C$

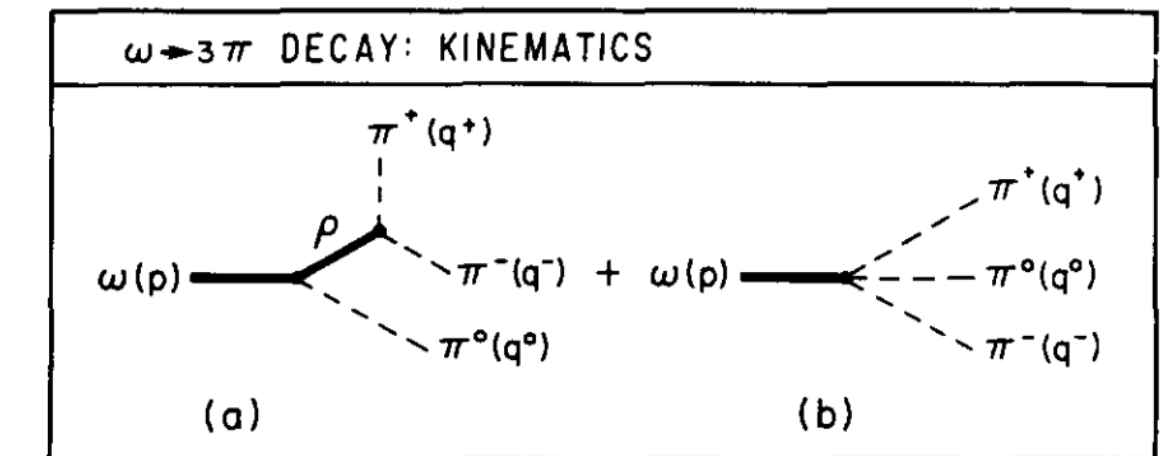
GEN: Generic form

$$\tilde{K}(s)^{-1} = \delta_{\vec{p}'\vec{p}} \delta_{\lambda'\lambda} (a_0 + a_1 \sigma_{\vec{p}}(s))$$

$$c_{11}^{GEN}(c_0, m_\omega, c_1) = \frac{c_0}{s - m_\omega^2} + c_1$$

EFT: Effective field theory form

saturated by s-channel meson-exchange



Gell-Mann/Sharp/Wagner/Fujiwara/Kawarabayashi...
Review: Meißner *Phys.Rept.* 161 (1988) 213

$$\left[\tilde{K}^{-1} \right]_{p' \lambda', p \lambda} = \delta_{\lambda' \lambda} \delta_{p' p} \frac{\sigma_p - M_\rho^2}{2g^2},$$

$$\tilde{c}_{11} = \frac{6s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{64g^2 \pi^3 f_\pi^6 (s - M_\omega^2)},$$

$\omega(782)$ FROM QCD

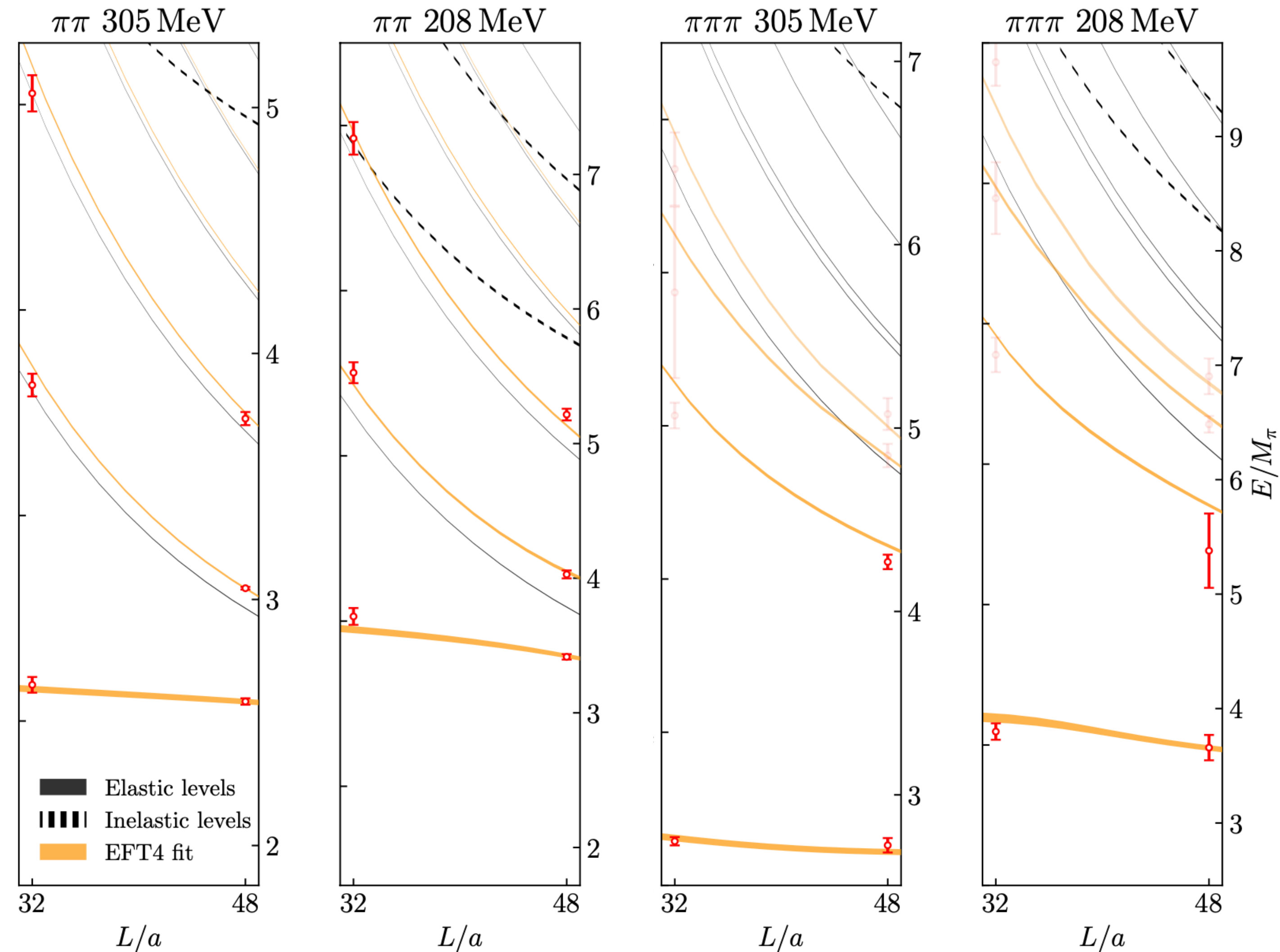
Finite-volume spectrum = Energy eigenvalues

Mapping to infinite volume

$$\text{FVU} \det \left[2L^3 E_p (\tilde{K}^{-1} - \Sigma^L) - B - C \right]^\Lambda \equiv 0$$

MM/Döring *Eur.Phys.J.A* 53 (2017) 12, 240

- known quantities B, Σ^L, E_L
- unknown volume-indep. 2/3-body “force” $\tilde{K}, C$

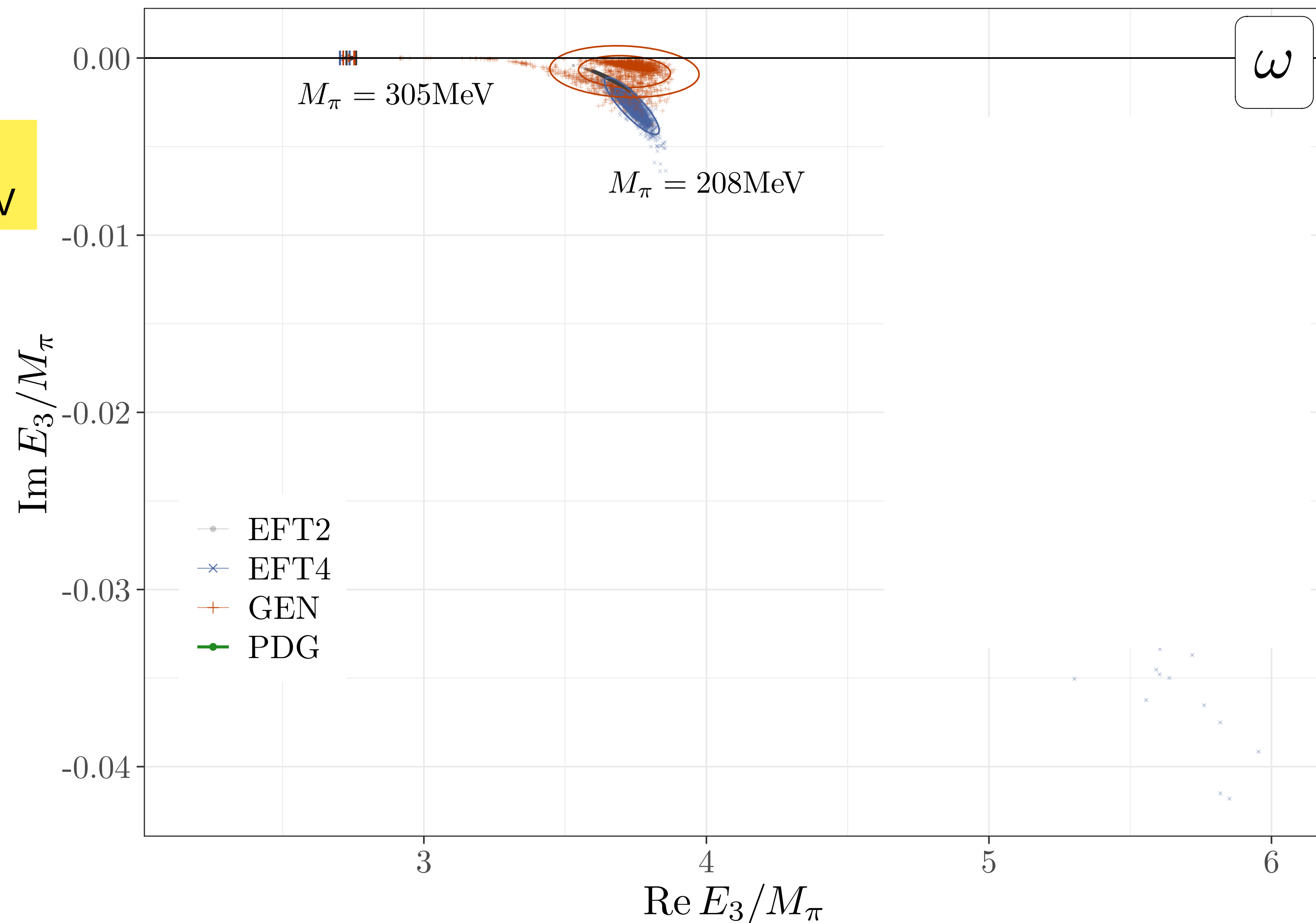


QCD VS EXPERIMENT

Final step: vol.-ind. 2-/3-body force $C, \tilde{K}$ — solve integral equation /analytically continue to the II Riemann Sheet

$\omega(782)$ becomes a bound state at ~300 MeV

GEN/EFT2/EFT4 ansatzes consistent

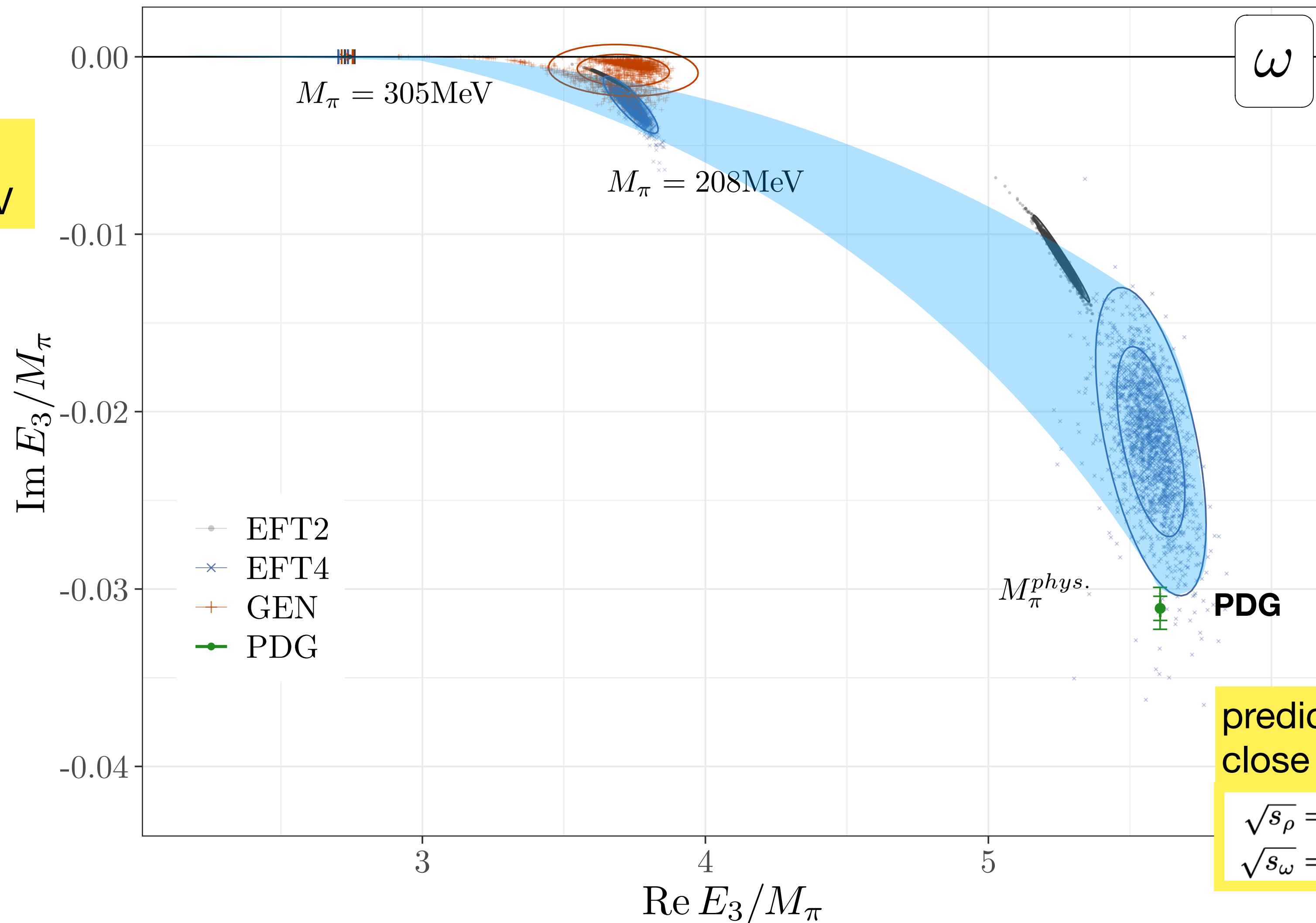


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prediction@physical point — close to the EXP value

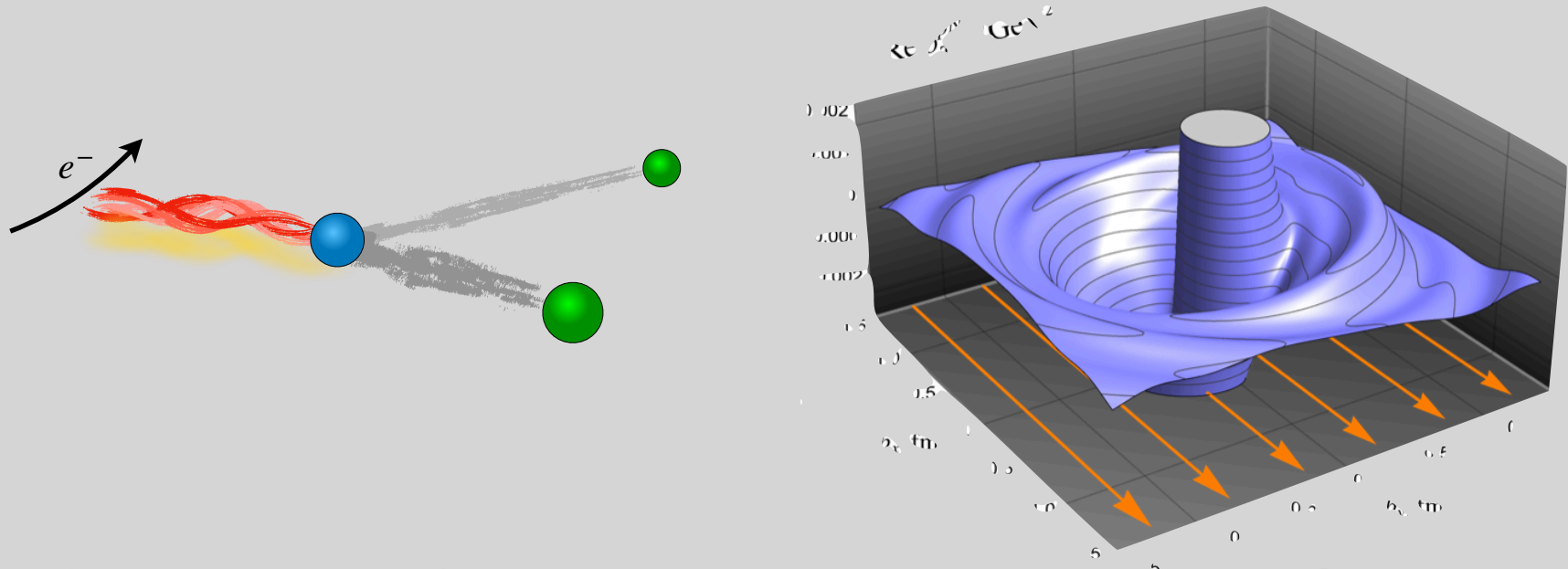
$$\sqrt{s_\rho} = (748.9(10.0) - i 63.5(1.8)) \text{ MeV} ,$$

$$\sqrt{s_\omega} = (778.0(11.2) - i 3.0(5)) \text{ MeV} ,$$

SUMMARY

Photon probe (CLAS/GlueX experiments)

- Transition form factors of resonances
- Charge distribution of excited states (Pole)



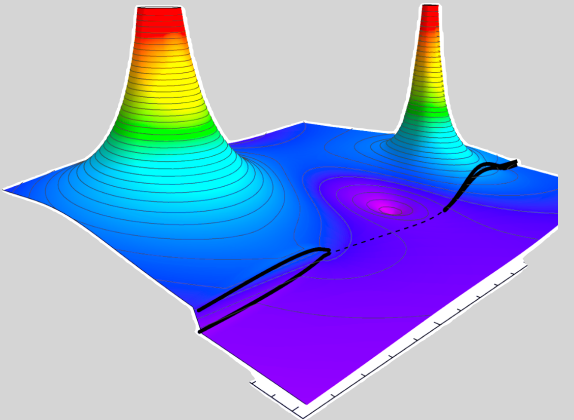
OUTLOOK

- Large Q^2 dependence (Functional methods?)
- $DD\pi - N(1440) - \dots$ spin-exotics? — $a_1(1420)$
- EFT tests — Universality of $\omega \rightarrow 3\pi, \rho \rightarrow 2\pi$ coupling? — KSFR relation? ...
- Cutoff — Gradient flow?
- HLbL contributions?

DSE — Eichmann/Fischer/Haeuser/Regenfelder **2411.05652**
 LQCD — [Mainz 2024] **2411.07969v1**

Hadronic degrees of freedom

- Many resonances generated through hadron-hadron interaction
- S-matrix+EFT guidance



Lattice QCD

- First calculations of 3-body resonances from QCD
- Quark mass dependence

