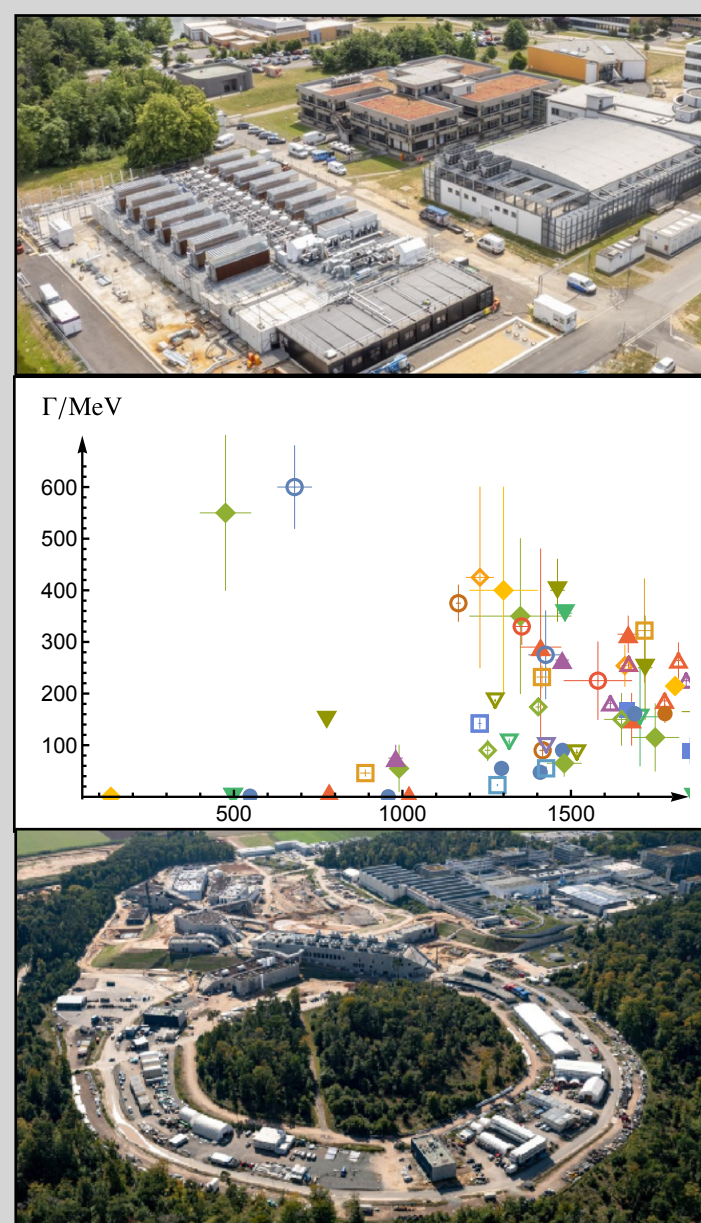
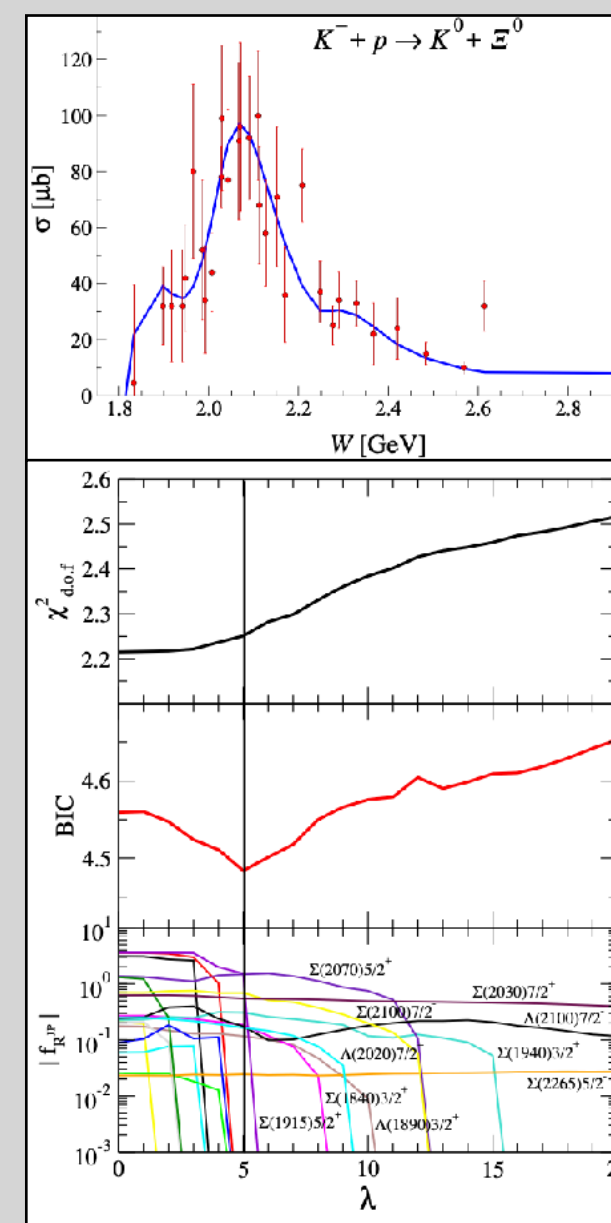


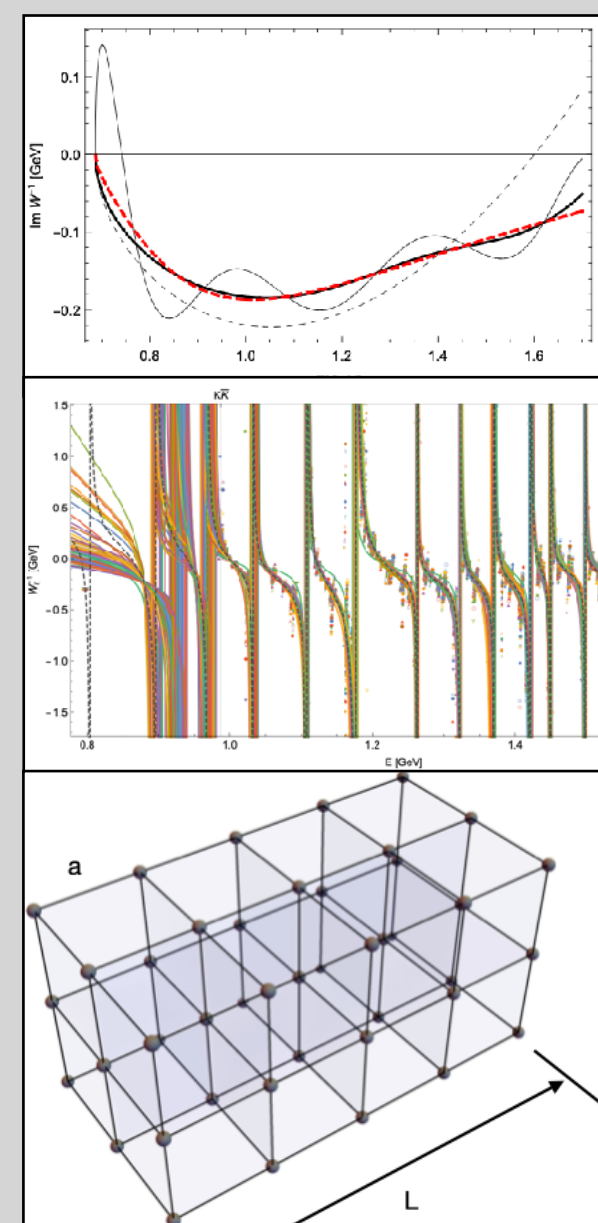
# DENOISING AND SHARPENING THE HADRON SPECTRUM THROUGH MACHINE LEARNING



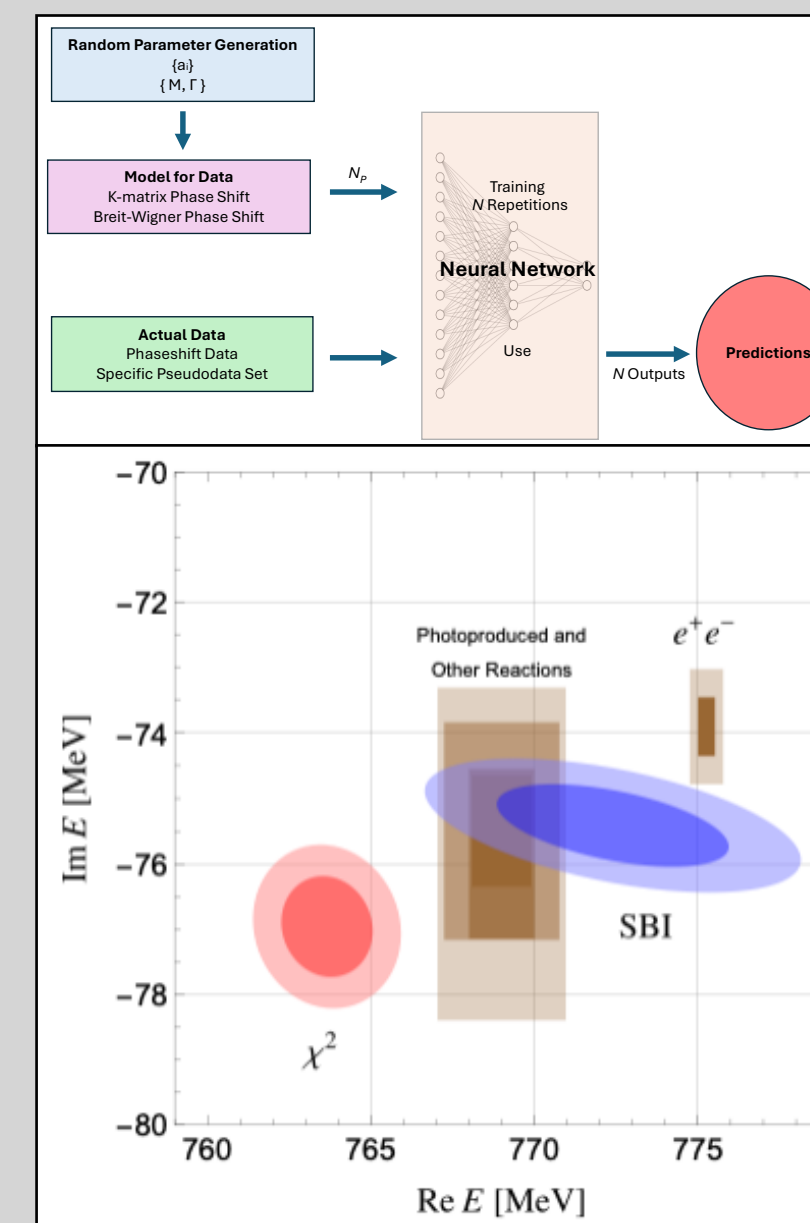
introduction



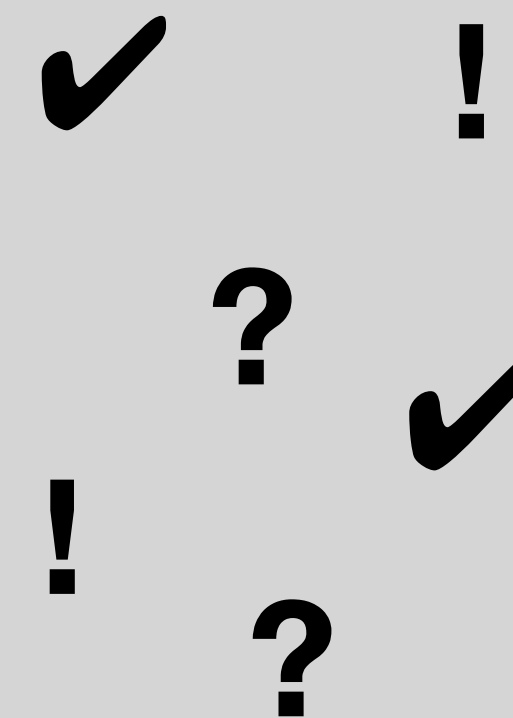
application 1



application 2



application 3



summary/outlook/questions

MAXIM MAI (U. BERN & GWU)

# HADRON SPECTRUM

---

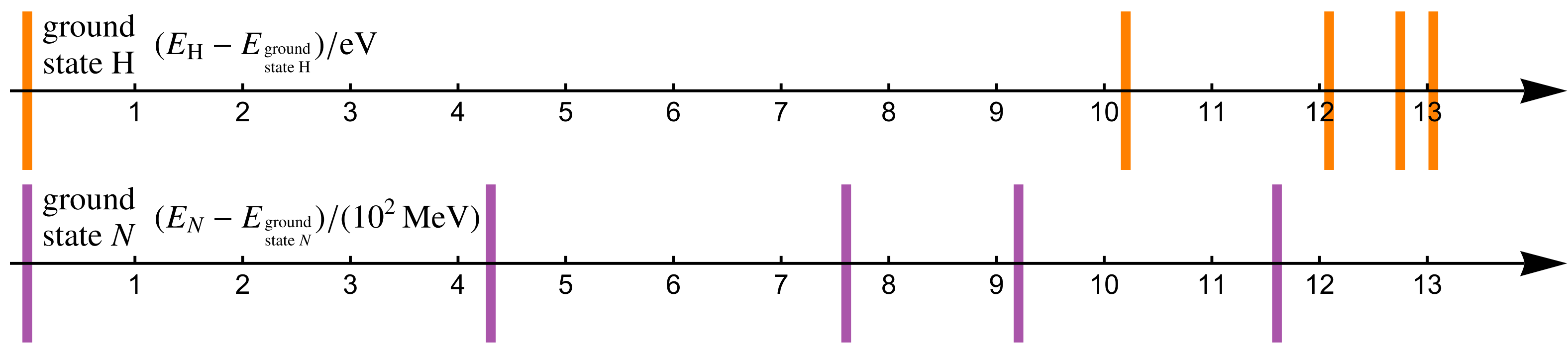
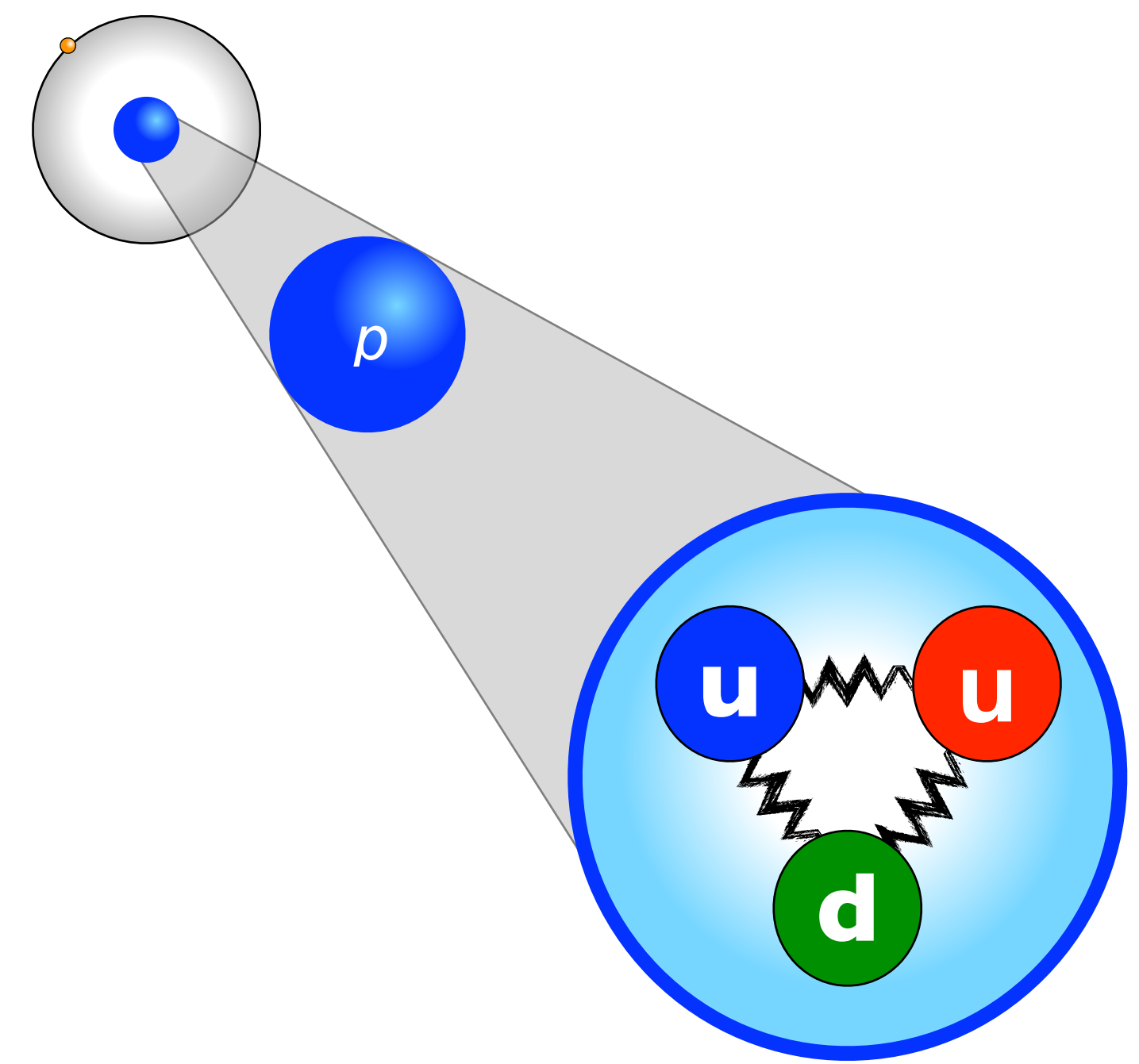
## BOUNDARY OF UNDERSTANDING

# BIG PICTURE

MM Theory of resonances [Encyclopedia of Particle Physics] — (simple introduction)  
<https://doi.org/10.1016/B978-0-443-26598-3.00001-8> e-Print: 2502.02654 [hep-ph]

## Protons/neutrons

- 99% of the mass of visible matter in the universe
- Building blocks: **quarks & gluons (strong force)**
- Part of a large class of particles: **hadrons**



Hydrogen spectrum (✓)

Proton spectrum (?)

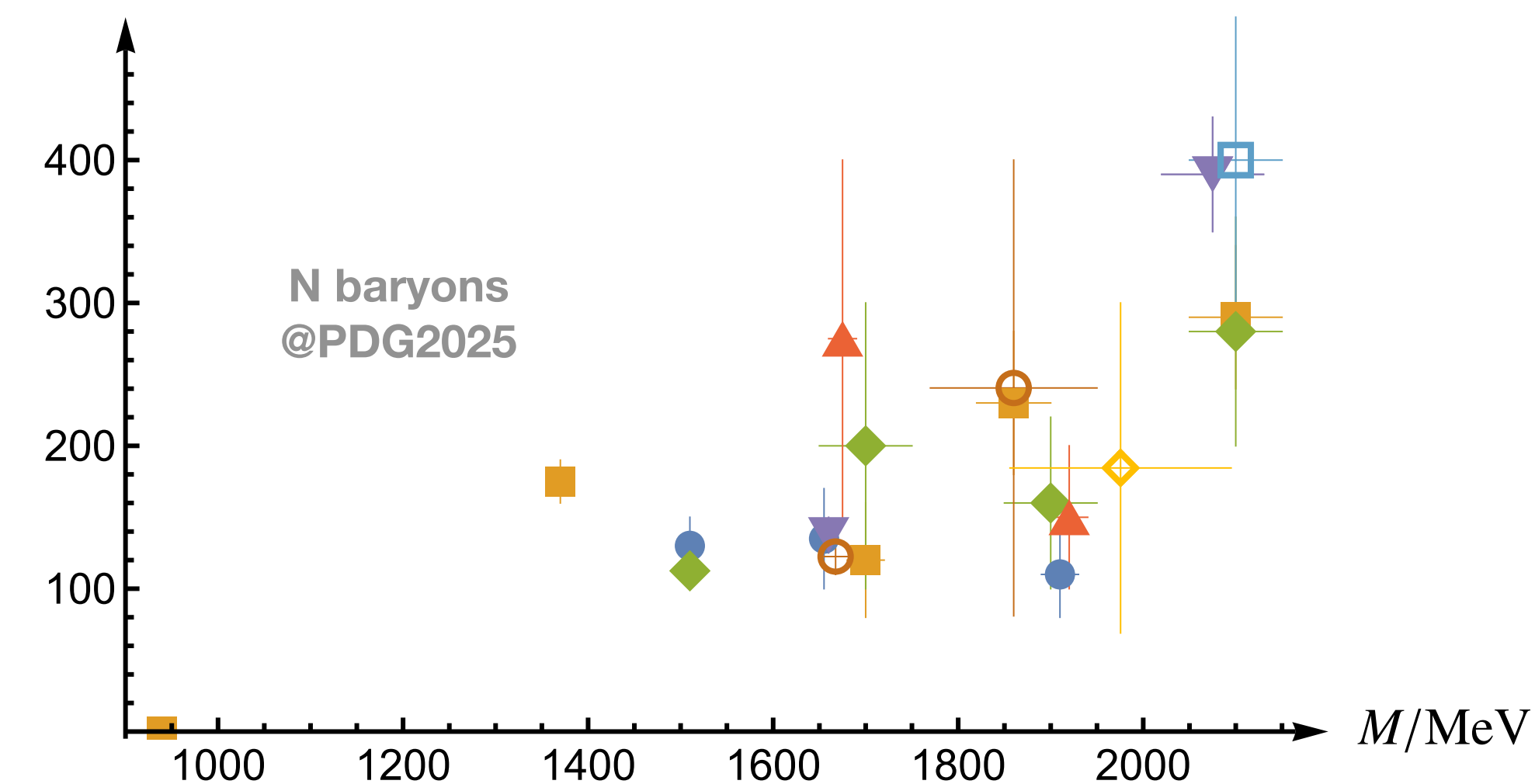
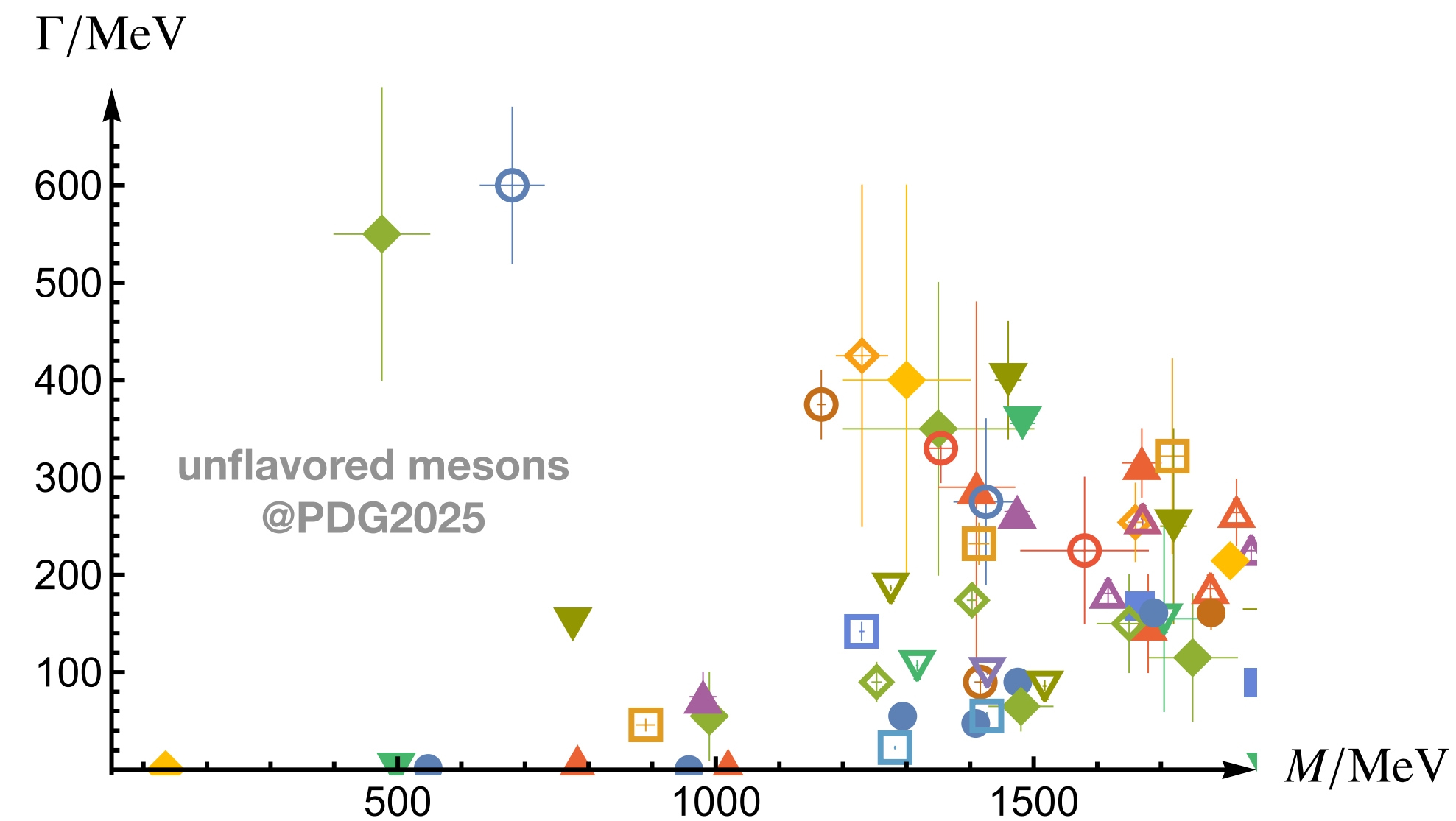
# HADRON SPECTRUM

## Experimental progress

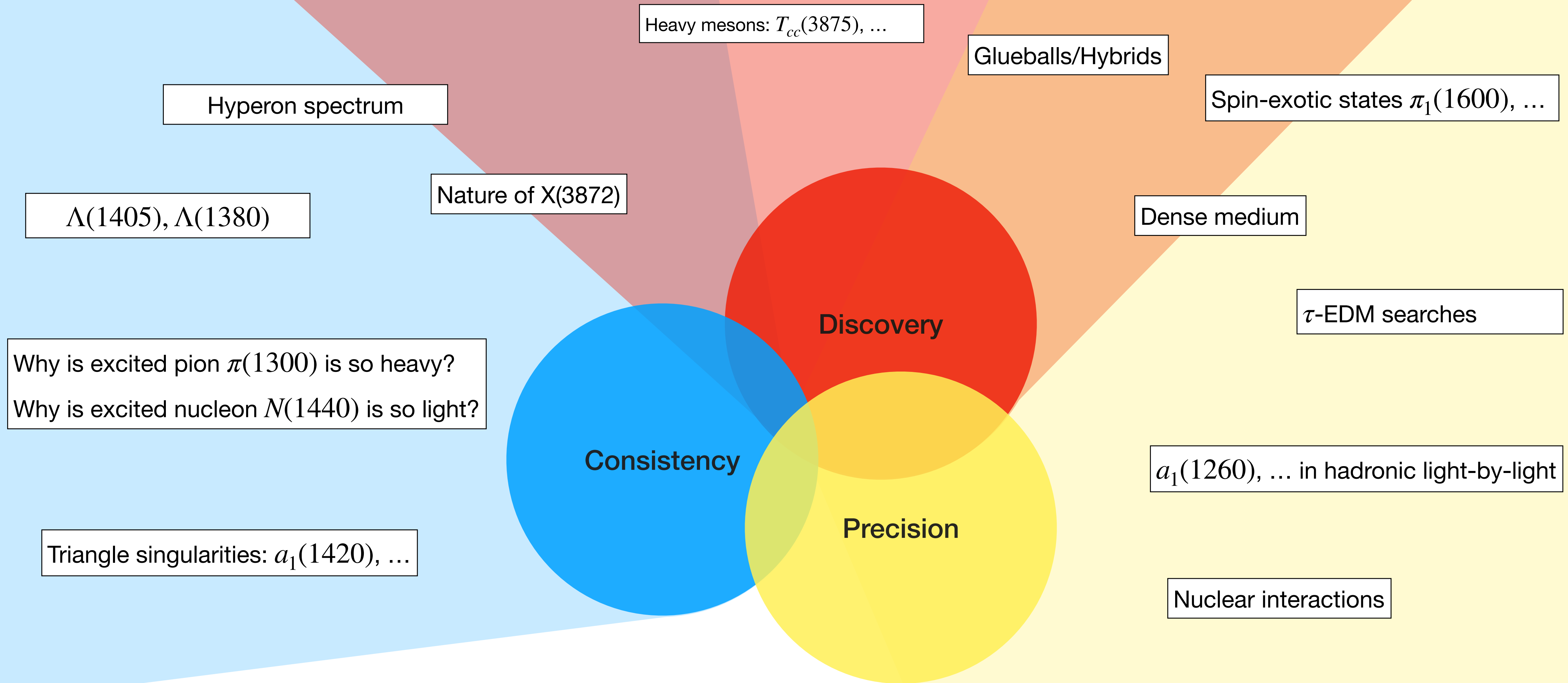
- ▶ 70y research ( $\Delta(1232)$ ,  $\rho(770)$ ,  $\omega(782)$ , ...)
- ▶ mostly excited states
  - $\approx 100$  mesons + 50 baryons (\*\*\*)
- ▶ ongoing experiments @CERN, GSI, JLAB, BES...

## New techniques

- ▶ Lattice QCD
- ▶ Effective Field Theories
- ▶ Machine Learning

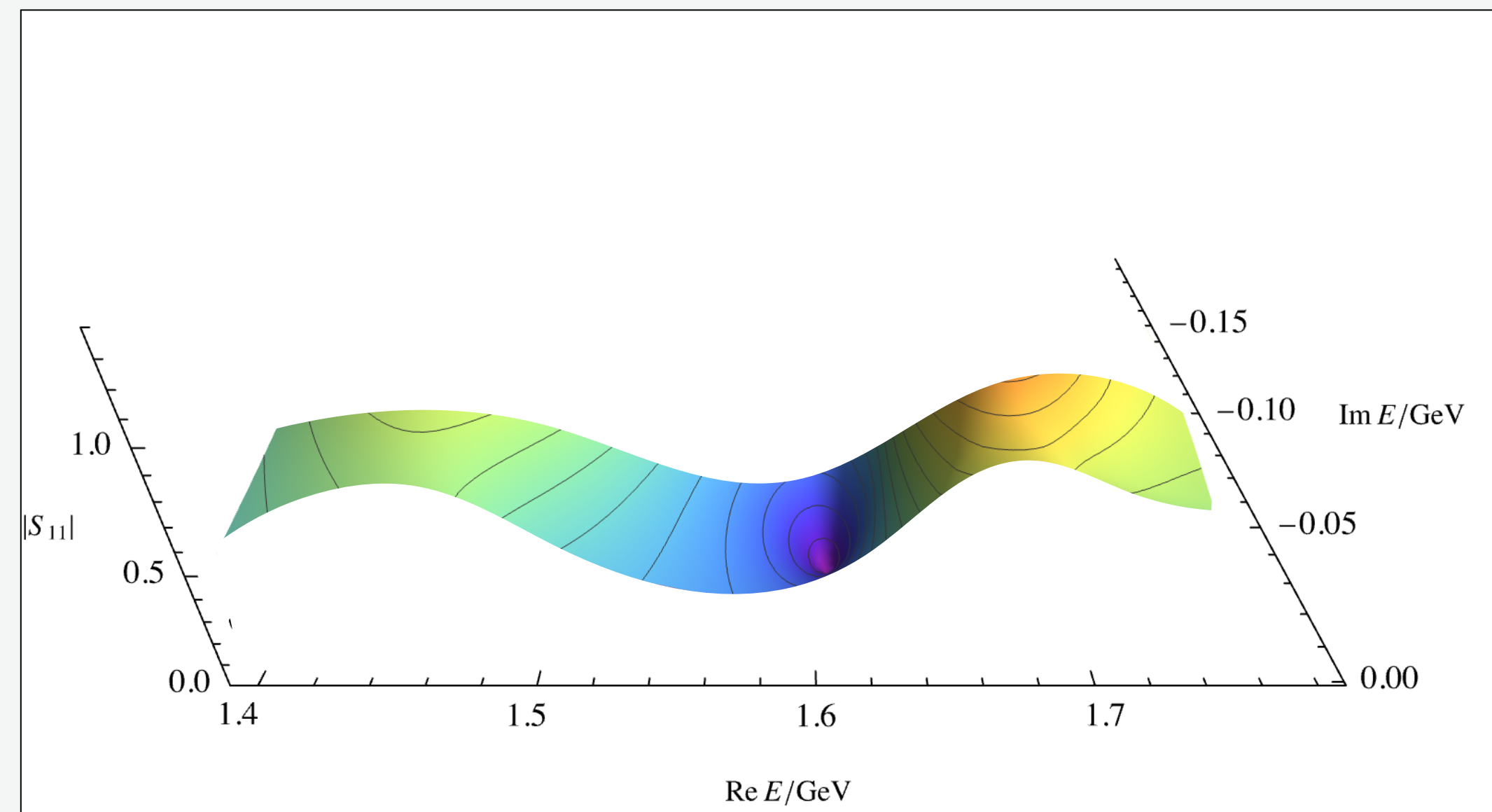


# BROADER IMPACT



# S-MATRIX

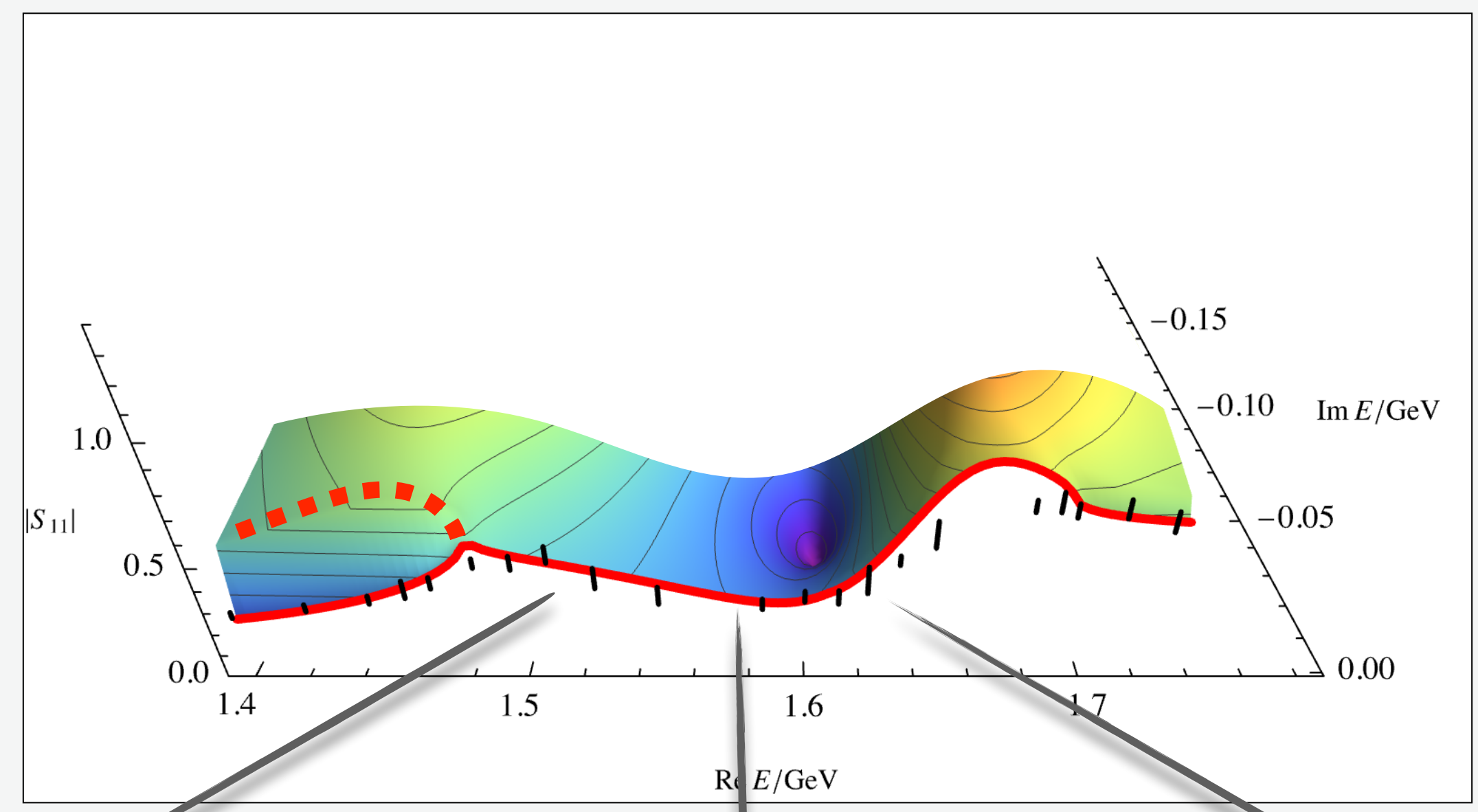
- \* Crossing symmetry (particle/antiparticle)
- \* Unitarity (probability conservation)
- \* Analyticity (causality)



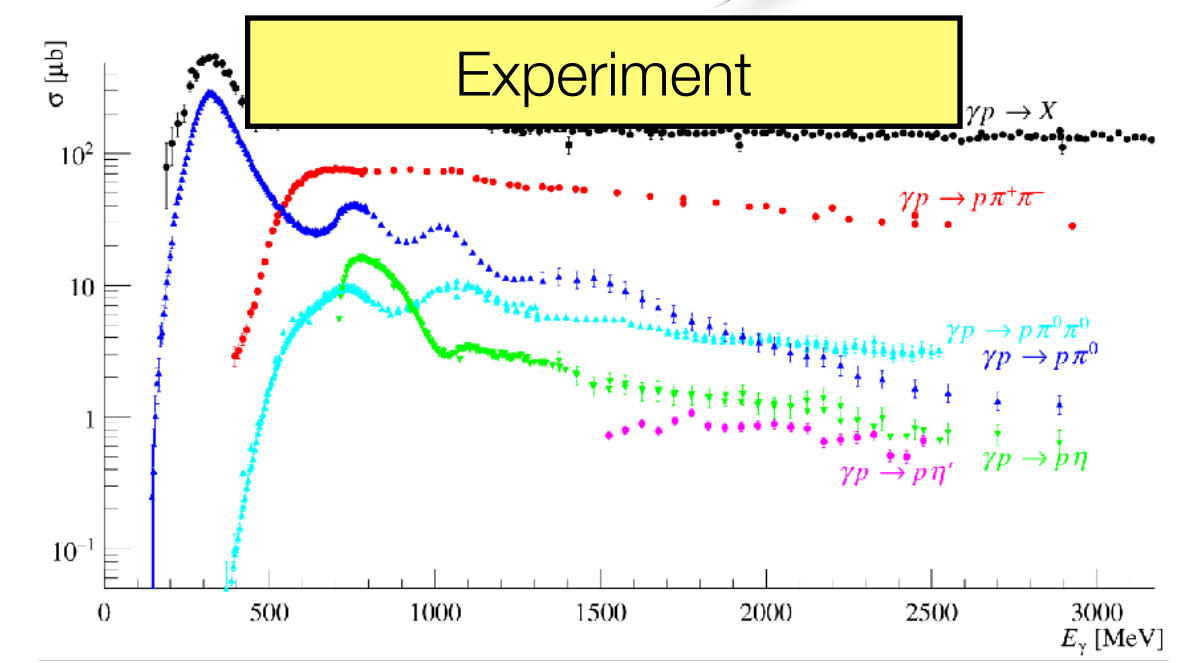
- MM Theory of resonances [Encyclopedia of Particle Physics] – (simple introduction) <https://doi.org/10.1016/B978-0-443-26598-3.00001-8> e-Print: 2502.02654 [hep-ph]
- Scott Willenbrock *Eur.Phys.J.Plus* 139 (2024) 6, 523
- ...

# S-MATRIX

- \* Crossing symmetry (particle/antiparticle)
- \* Unitarity (probability conservation)
- \* Analyticity (causality)



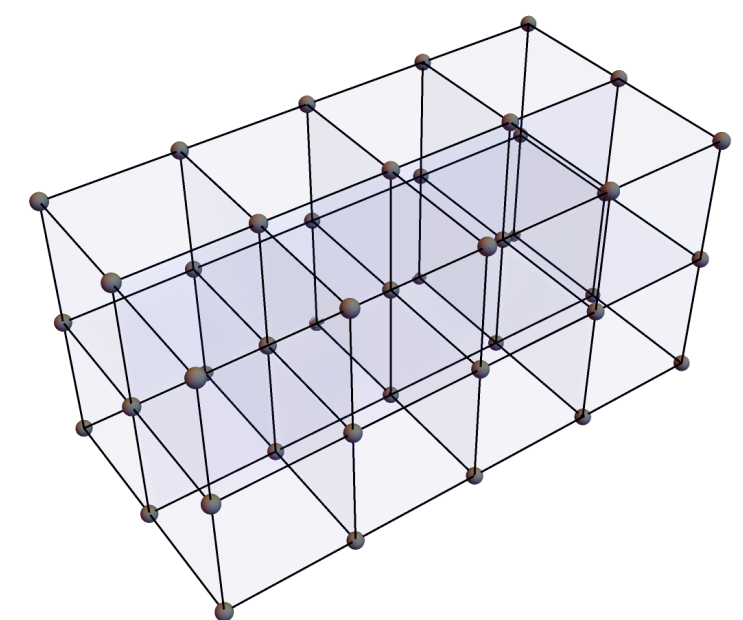
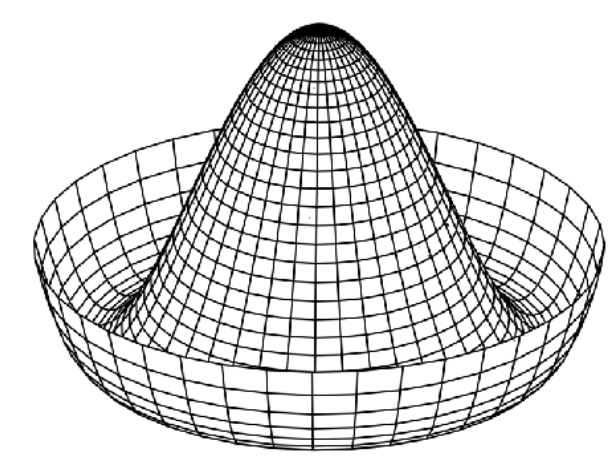
- MM Theory of resonances [Encyclopedia of Particle Physics] – (simple introduction) <https://doi.org/10.1016/B978-0-443-26598-3.00001-8> e-Print: 2502.02654 [hep-ph]
- Scott Willenbrock *Eur.Phys.J.Plus* 139 (2024) 6, 523
- ...



Experiment

Effective field theories

Lattice QCD

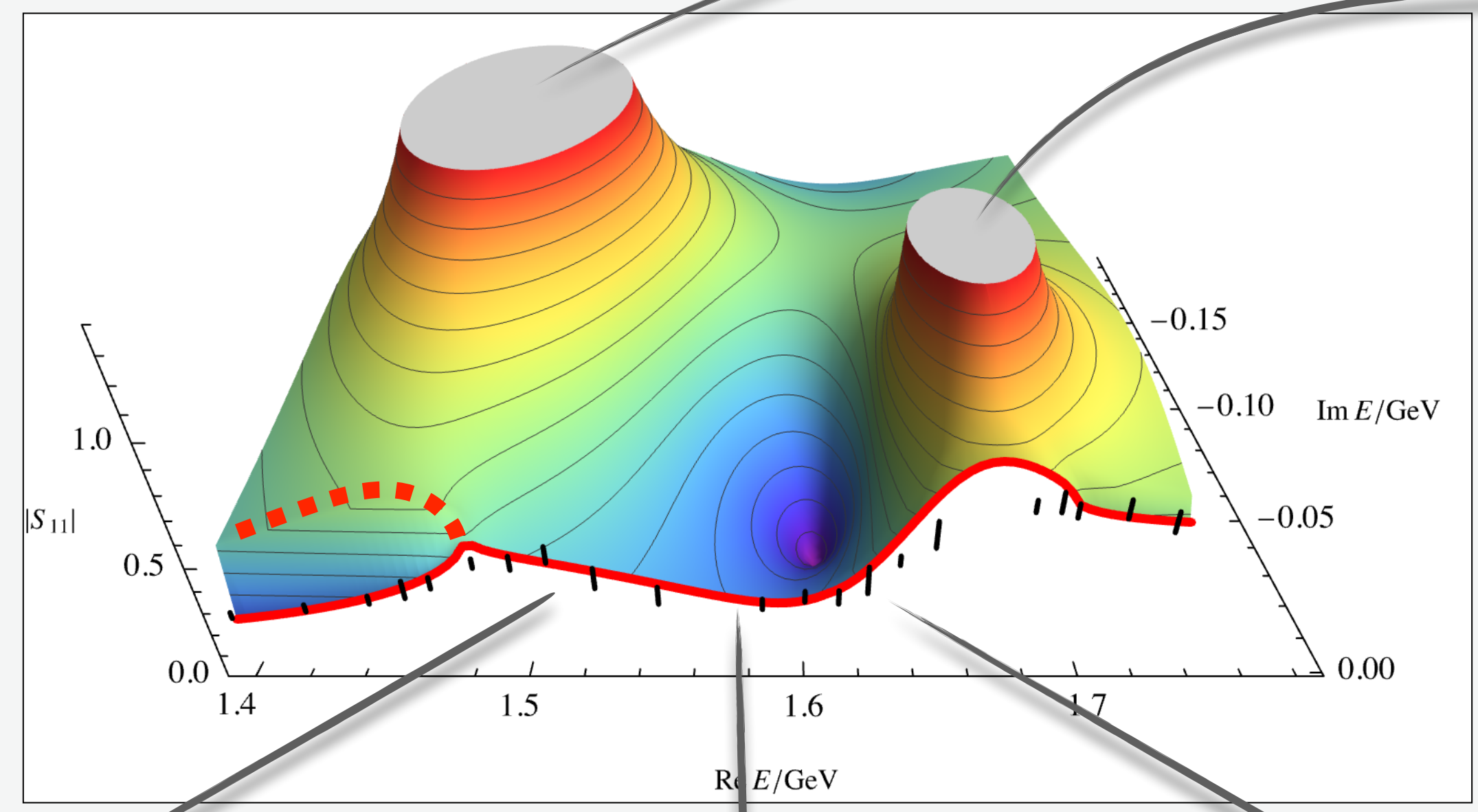


Data: JLAB, ELSA, MAMI CLAS12, GlueX, ...  
Plot: Thiel+ PNP 125 (2022) 103949

Data: SAID: PRC 74 (2006) 045205  
Model: MM+ PRD 86 (2012) 094033

# S-MATRIX

- \* Crossing symmetry (particle/antiparticle)
- \* Unitarity (probability conservation)
- \* Analyticity (causality)

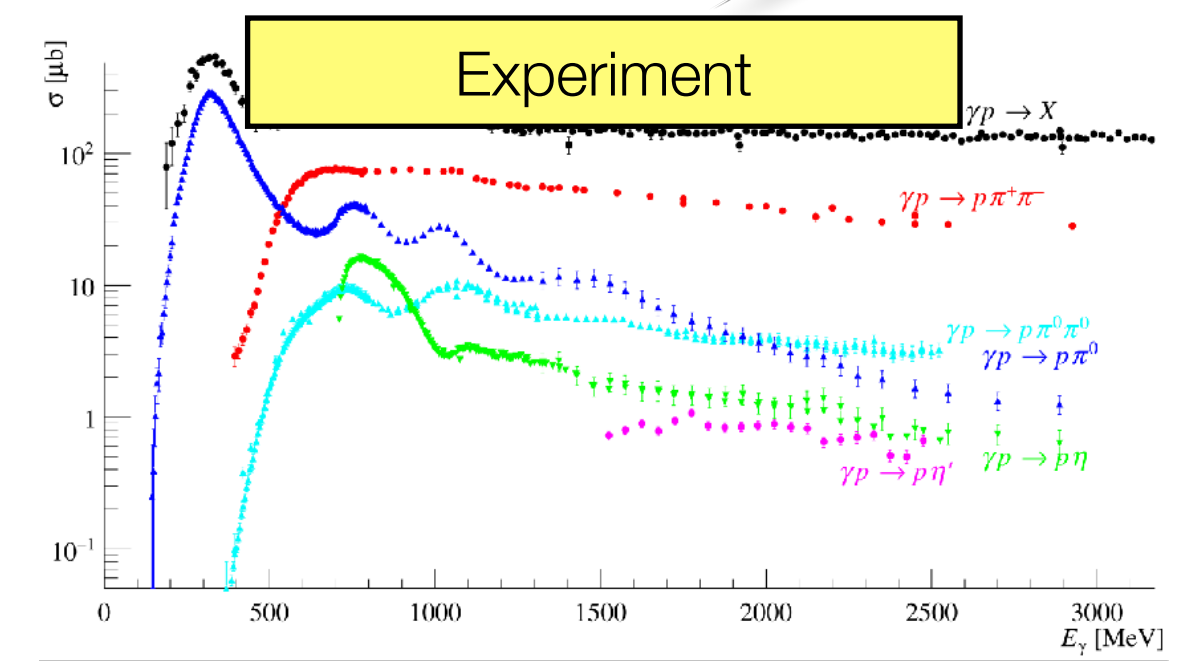


## Poles on unphysical Riemann Sheets

- Universal resonance parameter
- Singularities (cuts, ...) included
- Hadron mass  $M = \text{Re } E_{\text{pole}}$
- Decay width  $\Gamma = -2 \text{Im } E_{\text{pole}}$



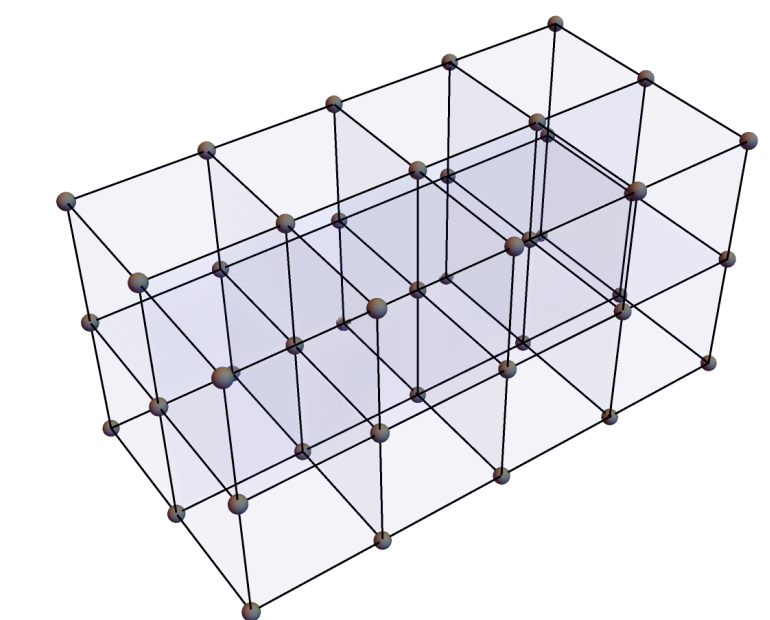
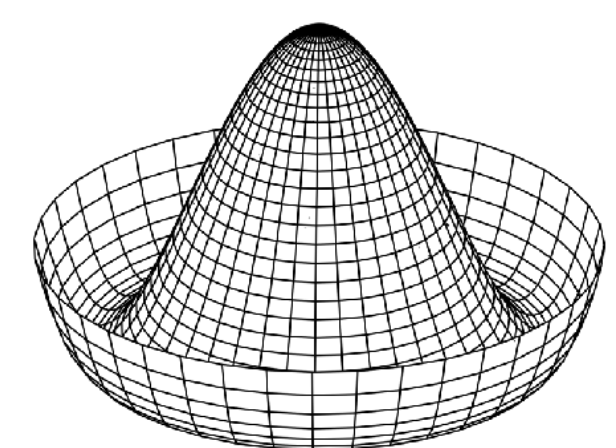
- MM Theory of resonances [Encyclopedia of Particle Physics] – (simple introduction) <https://doi.org/10.1016/B978-0-443-26598-3.00001-8> e-Print: 2502.02654 [hep-ph]
- Scott Willenbrock *Eur.Phys.J.Plus* 139 (2024) 6, 523
- ...



Experiment

Effective field theories

Lattice QCD



Data: SAID: PRC 74 (2006) 045205  
Model: MM+ PRD 86 (2012) 094033

Data: JLAB, ELSA, MAMI CLAS12, GlueX, ...  
Plot: Thiel+ PNP 125 (2022) 103949

# APPLICATION 1

---

## MINIMAL HADRON SPECTRUM

Justin Landay

Maxim Mai

Michael Döring

Helmut Haberzettl

Kanzo Nakayama

**Towards the minimal spectrum of excited baryons**

PHYSICAL REVIEW D 99, 016001 (2019)

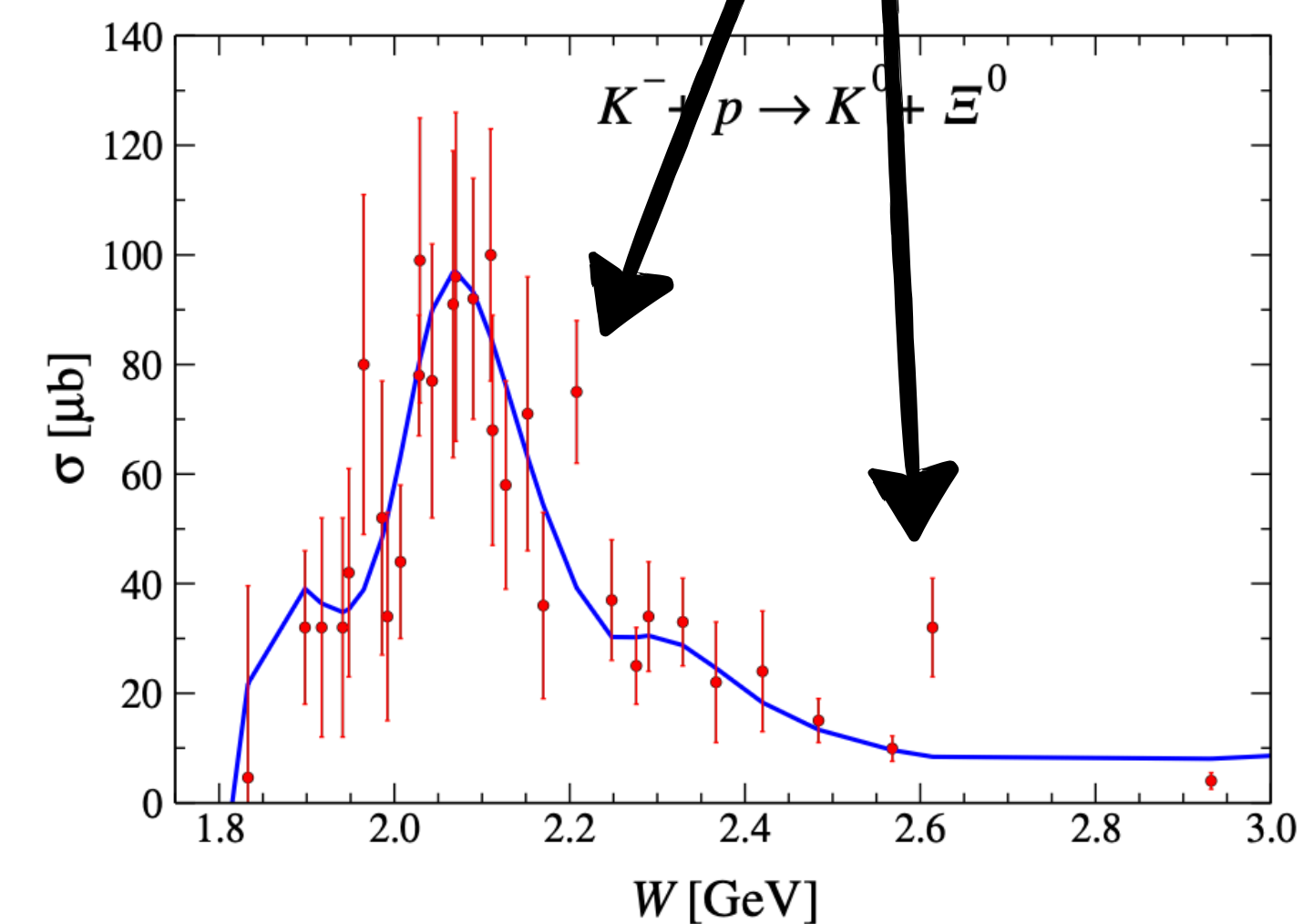
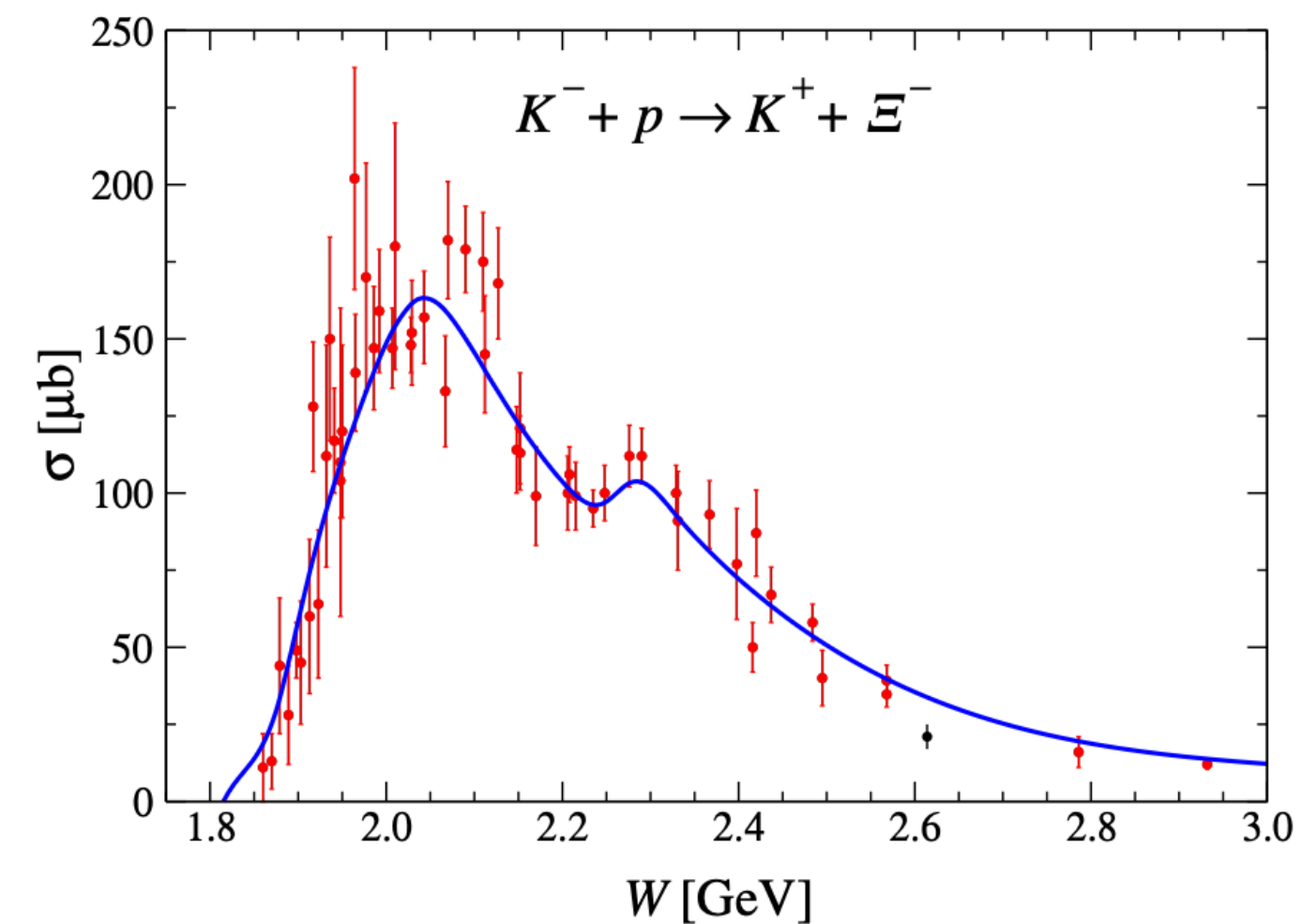
# FAKE RESONANCE PROBLEM

## Big data models (data driven models)

- e.g. Dynamical coupled-channel models, ...  
Review: Döring, Haidenbauer, MM, Sato PPNP2025
- $N_{\text{data}} \sim \mathcal{O}(10^5)$
- Very flexible parameterizations  $N_{\text{par}} \sim \mathcal{O}(10^3)$
- $\chi^2$  minimization criterion

## Possible issue

- adding new resonances improves data description
- Resonances are “seen”



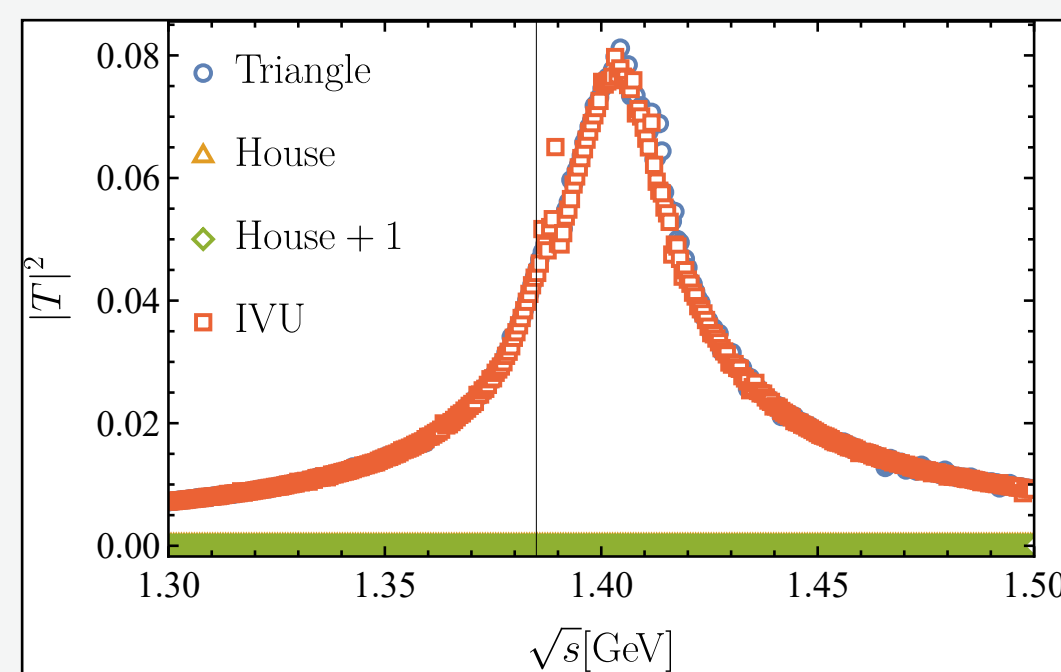
# POSSIBLE SOLUTIONS

## Better physical constraints

- S-matrix constraints: Thresholds, Triangle singularities, ...

### Triangle singularity $a_1(1420)$

GUO+ PPNP 112 (2020) 103757 [arXiv:1912.07030]



Sakthivasan/MM/... JHEP 10 (2024) 246

- Effective Lagrangian (QCD symmetries)
  - Controlled hadron-hadron dynamics
  - $\Lambda(1405)$ ,  $\Lambda(1380)$ , ...

Review: Eur.Phys.J.ST 230 (2021) 6, 1593-1607

## Statistics/MachineLearning/Data-driven tools

- Bayesian criteria
- Least Absolute Shrinkage and Selection Operator (LASSO)
  - minimal spectrum to describe a hadronic reaction

R. Tibshirani, J. R. Stat. Soc. 73, 273 (2011).

T. Hastie, R. Tibshirani, and J. Friedman, The Elements of Statistical Learning  
G. James, D. Witten, T. Hastie, and R. Tibshirani, An Introduction to Statistical Learning

- **Main idea:** penalize the appearance of new resonances

$$\chi_T^2 := \chi^2 + P(\lambda)$$

- How to define penalty function  $P(\lambda)$ ?
- How to determine optimal penalty factor  $\lambda$ ?

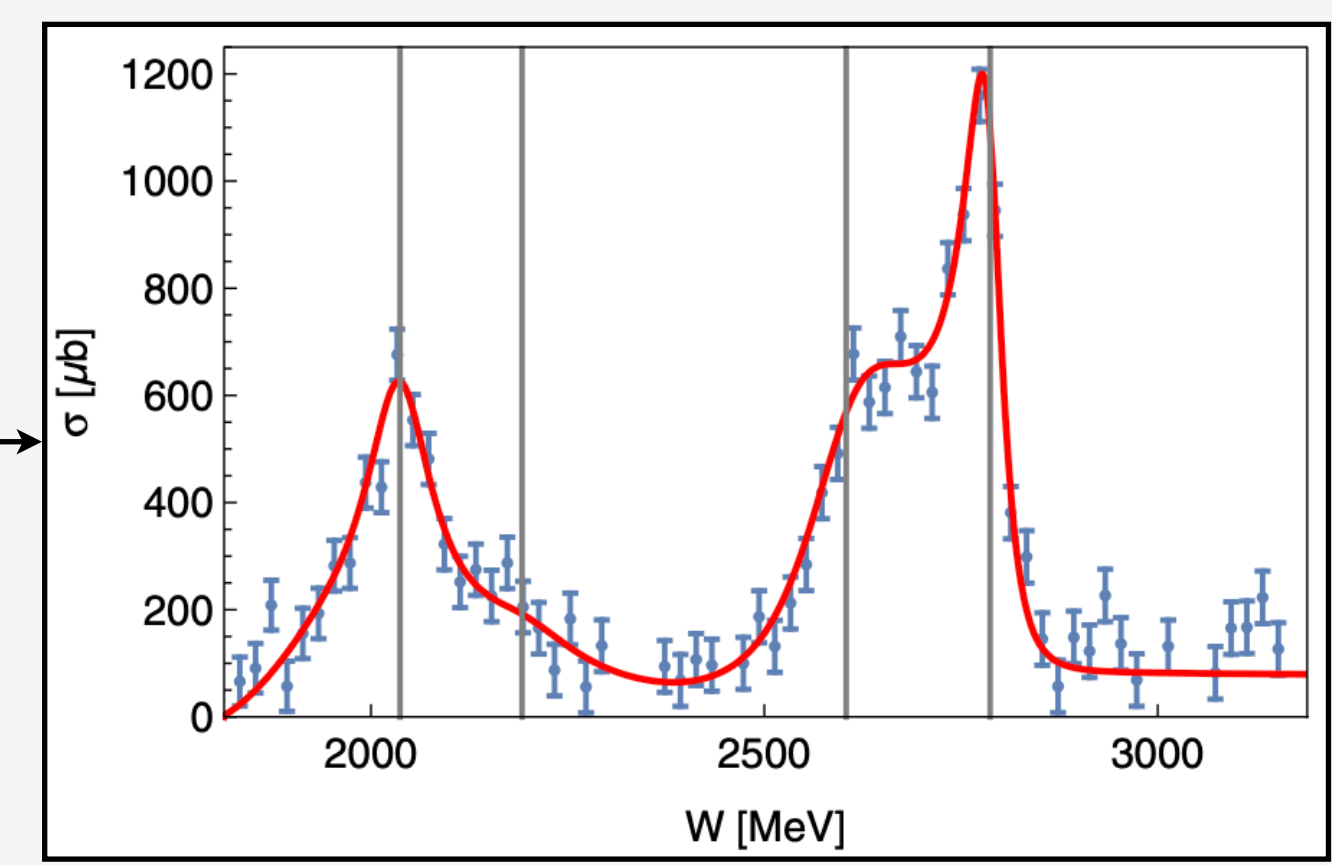
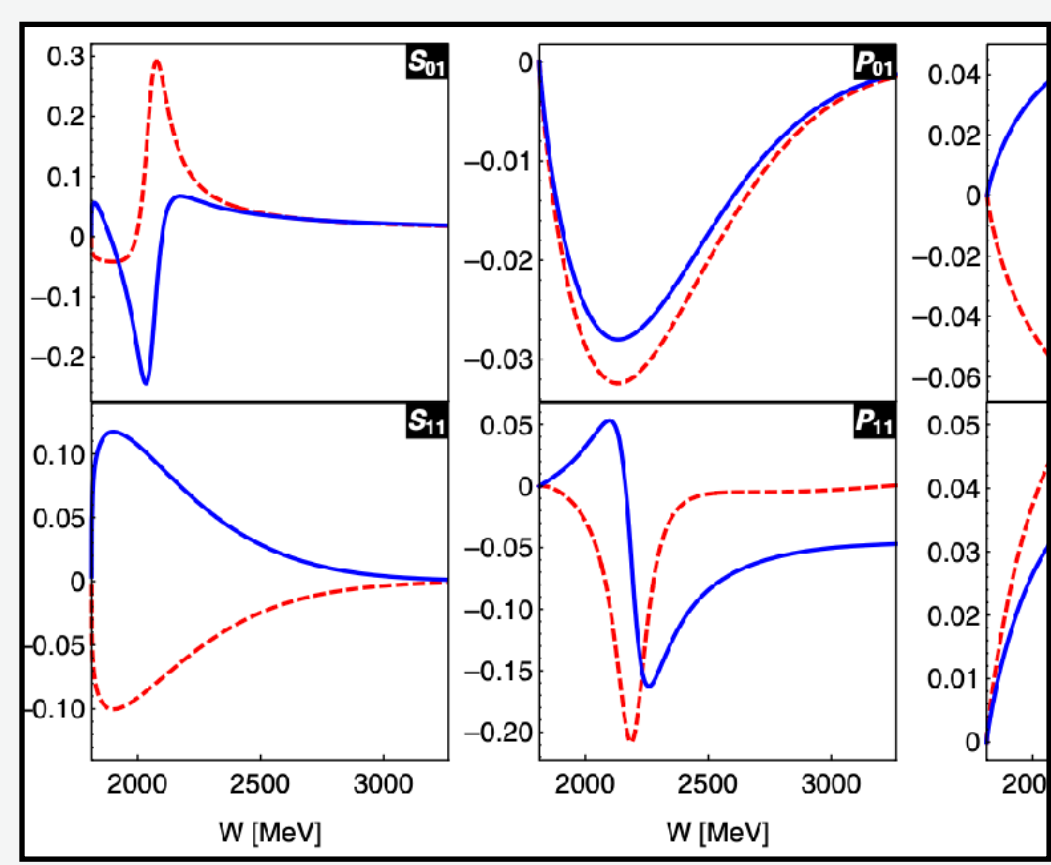
# SYNTHETIC DATA TESTS

## Controlled environment

- 10 partial waves/10 resonance candidates

$$\tau(W) = e^{i\phi} \left( \frac{k_f(W)}{\Lambda} \right)^{L+\frac{1}{2}} \times \left( a e^{-\alpha^2 \left( \frac{k_f(W)}{\Lambda} \right)^2} - x e^{i\Phi} \frac{\Gamma/2}{W - M + i\Gamma/2} \right)$$

- Synthetic data with 4 active resonances



## Penalty function

- Residua

$$P(\lambda) = \lambda^4 \sum_{i=1}^{i_{\max}} |x_i|$$

- Second derivative

$$P(\lambda) = \lambda^5 \sum_{i=1}^{10} \frac{\int_{m_K+m_\pi}^{W_{\max}} \left| \frac{\partial^2 \tau_i(W)}{\partial W^2} \right|^2 dW}{\int_{m_K+m_\pi}^{W_{\max}} |\tau_i(W)|^2 dW}$$

physically informed

## Procedure

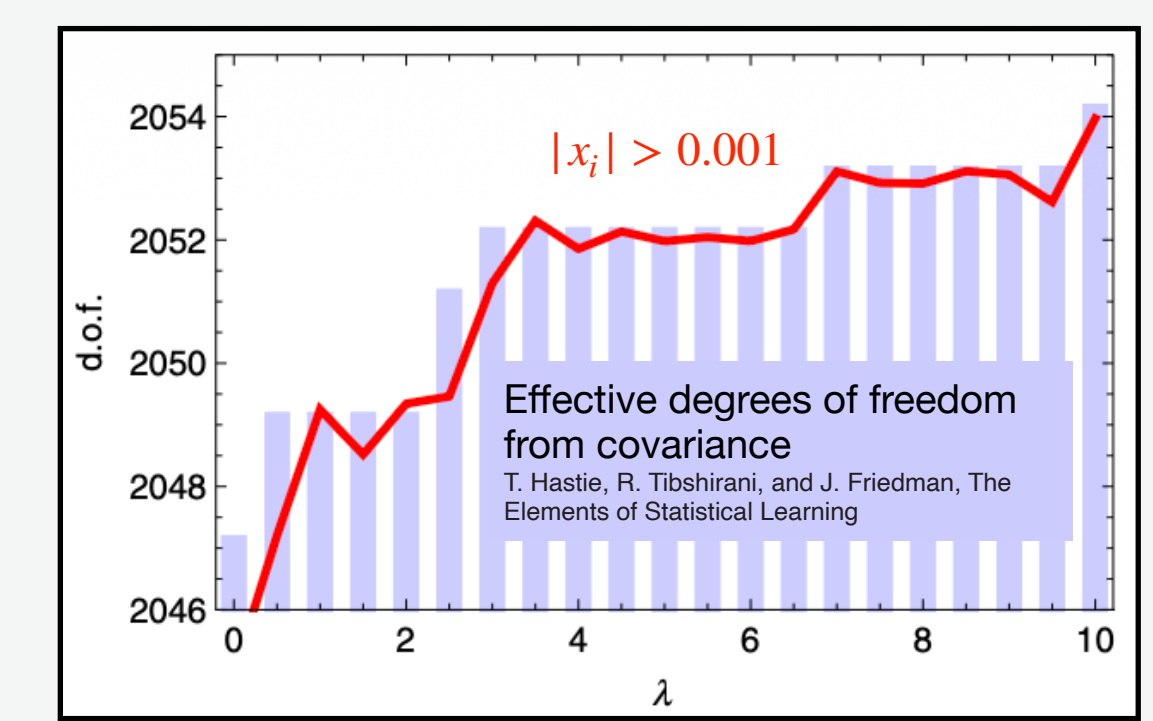
Minimize  $\chi_T^2 := \chi^2 + P(\lambda)$  stepwise as

- $(\lambda = 10) \rightarrow (\lambda = 0)$  forward LASSO
- $(\lambda = 0) \rightarrow (\lambda = 10)$  backward LASSO (automatic shutoff)
- ...

## Optimal penalty factor

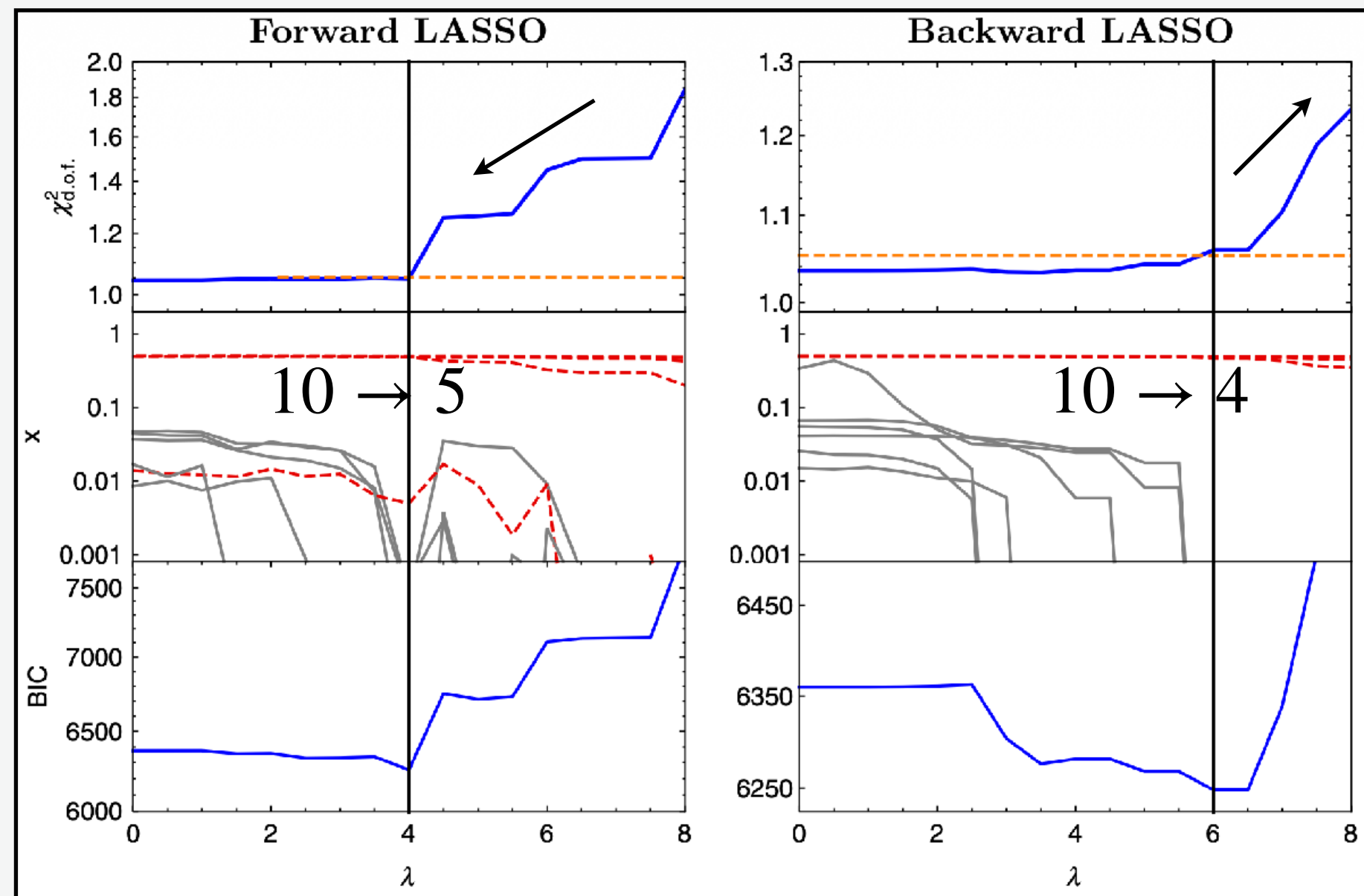
- Bayesian information criterion

$$BIC = k_{\text{eff}} \log(n) + \chi^2 + c$$



# RESULTS

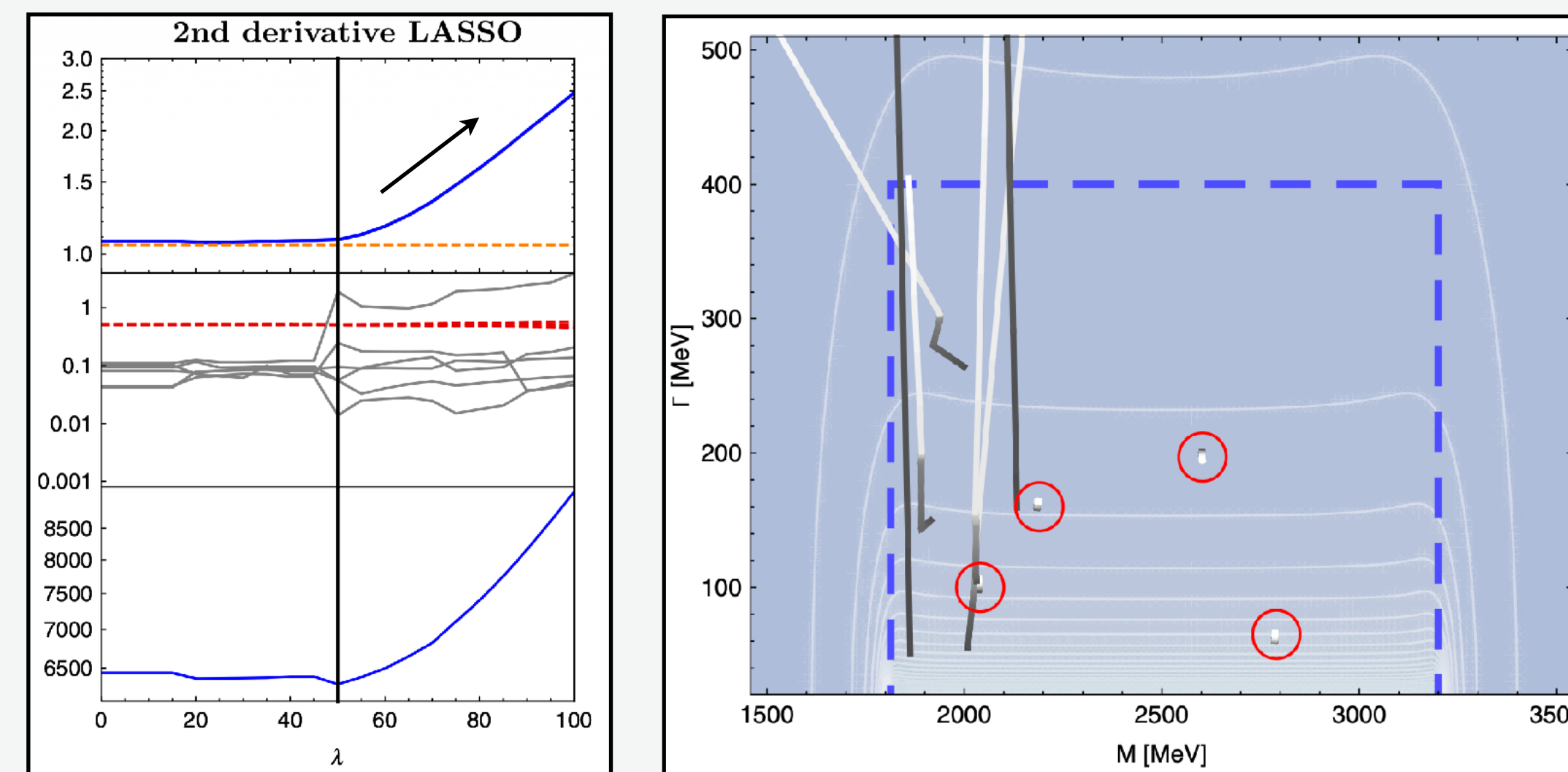
## Parametric penalty



- ➡ Resonance content reduced
- ➡ good local minima
- ➡ background flexibility dependence

M. Williams, J. Instrum. 12, P09034 (2017).

## Second derivative penalty



- backward LASSO (second derivative)
  - ➡ picks the 10 → 4 correct resonances
  - ➡ Inactive resonances driven into the complex plane
  - ➡ BIC minimum is less pronounced
- for real data ( $K^-p \rightarrow K\Xi$ ) + data pruning
  - ➡ 21 → 10 resonances

# APPLICATION 2

---

## LATTICE QCD – PHENOMENOLOGY

Dimitri Agadjanov  
Michael Doring  
Maxim Mai  
Ulf-G. Meißner  
Akaki Rusetsky

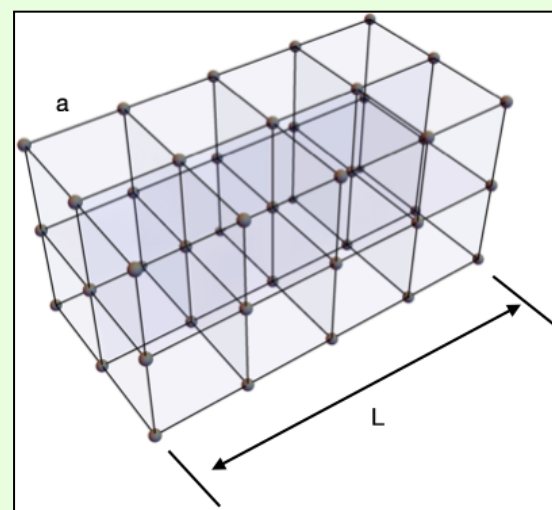
**The Optical Potential on the Lattice**

**JHEP 06 (2016) 043 • e-Print: [1603.07205](https://arxiv.org/abs/1603.07205) [hep-lat]**

# BACK TO QCD

## Lattice Gauge Theory (LQCD)

Wilson, Phys. Rev. D10 (1974) 2445 , ...



$$Z[J] = \int [DU] e^{-S_E} \det[M[U]]$$

[+] QCD degrees of freedom

[+/-] unphysical quark masses

[+] non-perturbative access to QCD Green's functions

[-] discretized space-time

[-] Euclidean metric

[-] finite-volume (no S-matrix!)

## 2-hadron systems

- $\rho(770)$ ,  $f_0(500)$ ,  $K^*(700)$ ,  $K^*(892)$ , ...

ETMC/GWQCD/HadSpec/CLQCD/...

- Meson-baryon, 2-baryon systems are harder (noise etc..)

BaSC/ETMC/HadSpec/NPLQCD/HALQCD/...

## 3-hadron systems

- new formalisms
- finite-volume unitarity

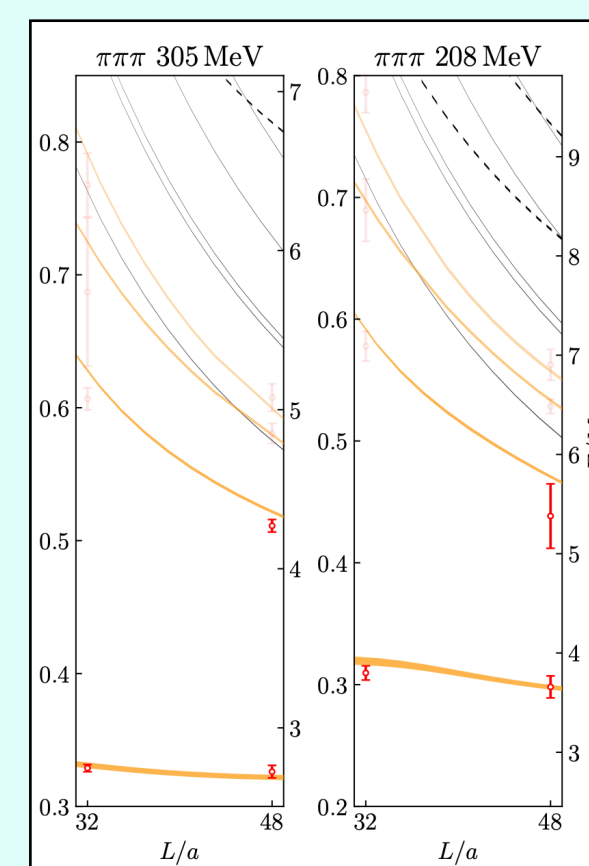
Rusetsky/Döring/Hansen/Davoudi/Sharpe/Guo/MM...

MM/Döring Eur.Phys.J.A 53 (2017) 12, 240

- new results of resonant systems

$\pi(1300)$  Phys.Rev.Lett. 136 (2026) 14

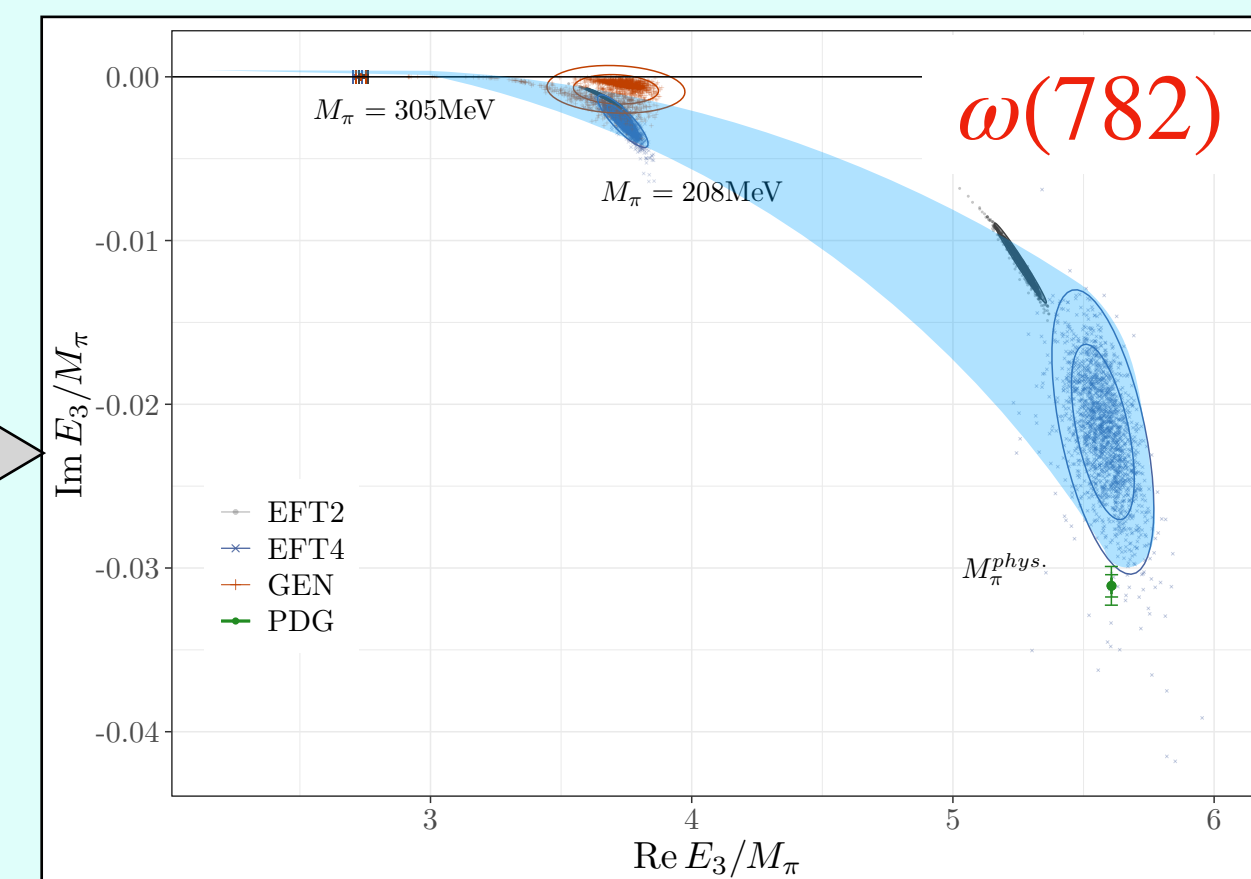
$\omega(782)$  Phys.Rev.Lett. 133 (2024) 21



FVU

$$\det \left[ 2L^3 E_p (\tilde{K}^{-1} - \Sigma^L) - B - C \right]^\Lambda \equiv 0$$

MM/Döring  
Eur.Phys.J.A 53 (2017) 12, 240

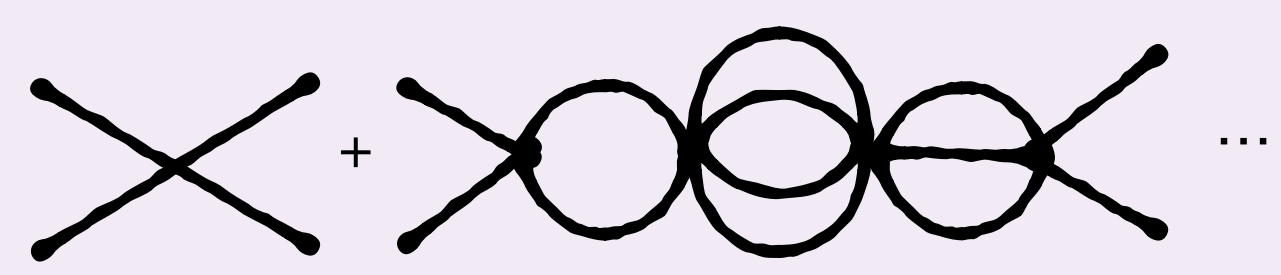
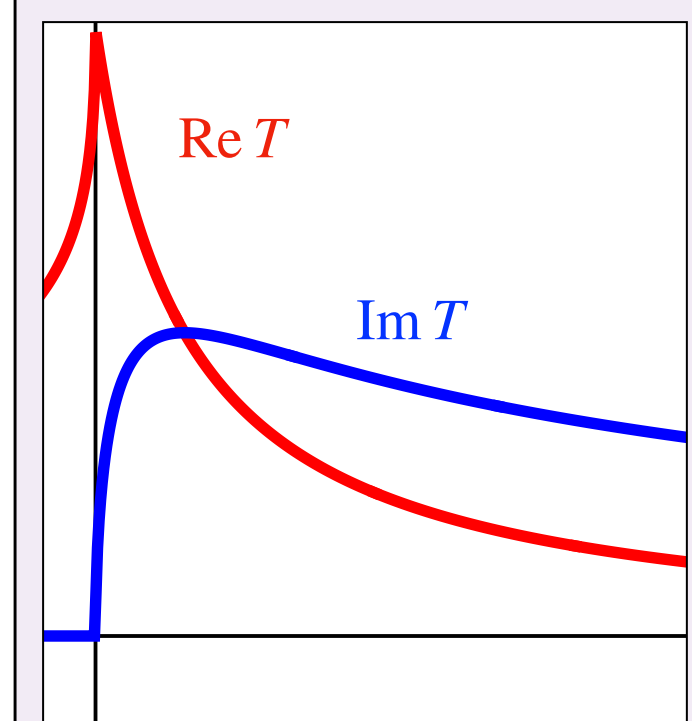


# QUANTIZATION CONDITIONS

## Continuum QFT

- Asymptotic states at  $t \rightarrow \pm \infty$
- underlying quantity:  $(\sigma \sim |S|^2, \dots)$

$$S = 1 + iT \in \mathbb{C}$$



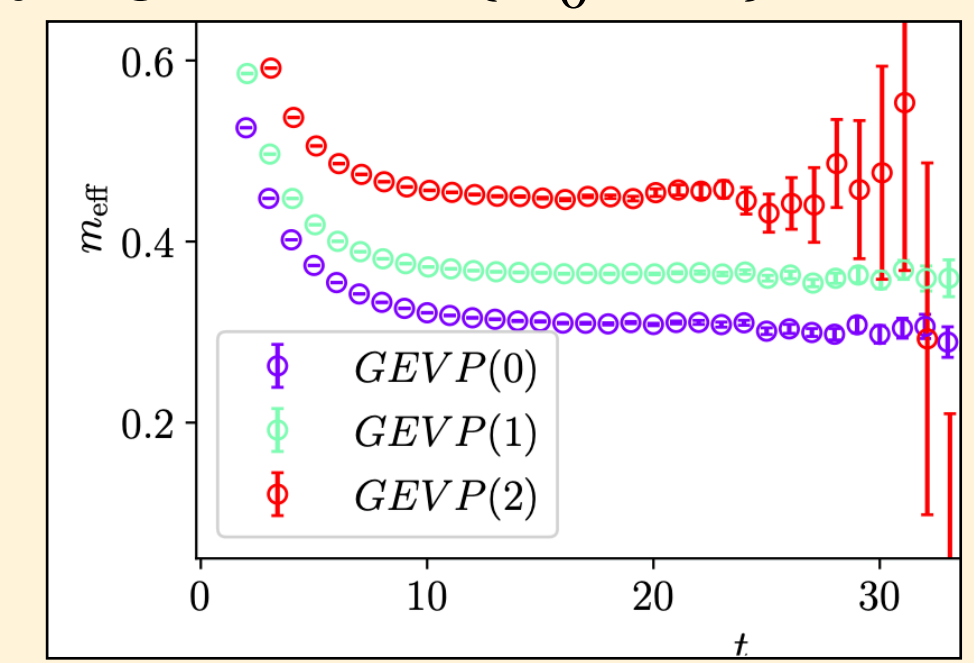
$$\text{disc } T \sim |T|^2 \quad \sqrt{s} < 3m$$

unitarity condition (on-shell-ness)

$$T^{-1} = K^{-1} - \int_l \frac{1}{2E_l} \frac{1}{(s - 4E_l^2 + i\epsilon)} = p \cot \delta - \left( \int \dots - \text{Re} \int \dots \right)$$

## Finite-volume setup

- Input: energy eigenvalues  $\{E_0, \dots\} \in \mathbb{R}$



- 2-body case

$$p \cot \delta(E_i) = Z_{00}(E_i) + \mathcal{O}(e^{-ML})$$

M. Lüscher, Nucl. Phys. B 354, 531 (1991)

- 3-body case

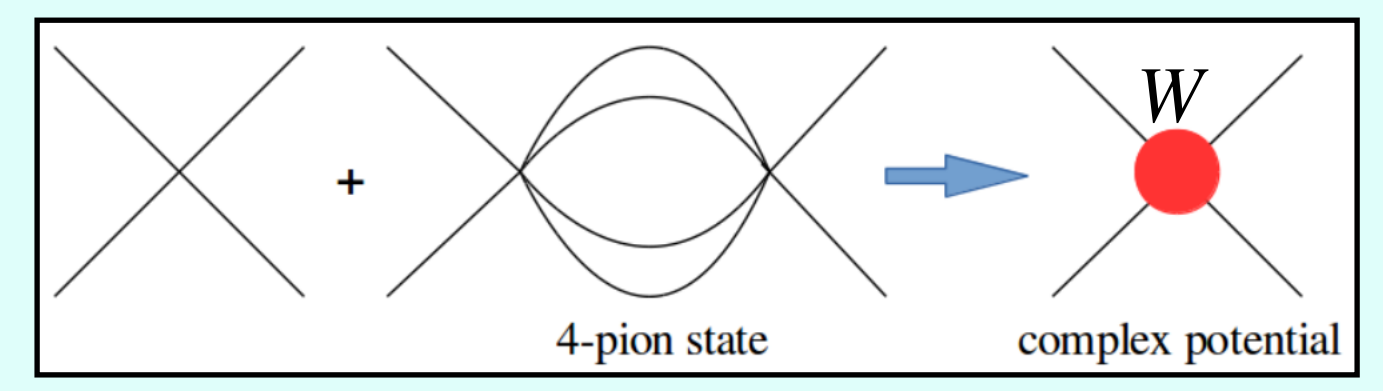
$$\det \left[ 2L^3 E_p \left( \tilde{K}^{-1} - \Sigma^L \right) - B - C \right]^\Lambda \equiv 0$$

MM/Döring Eur.Phys.J.A 53 (2017) 12, 240

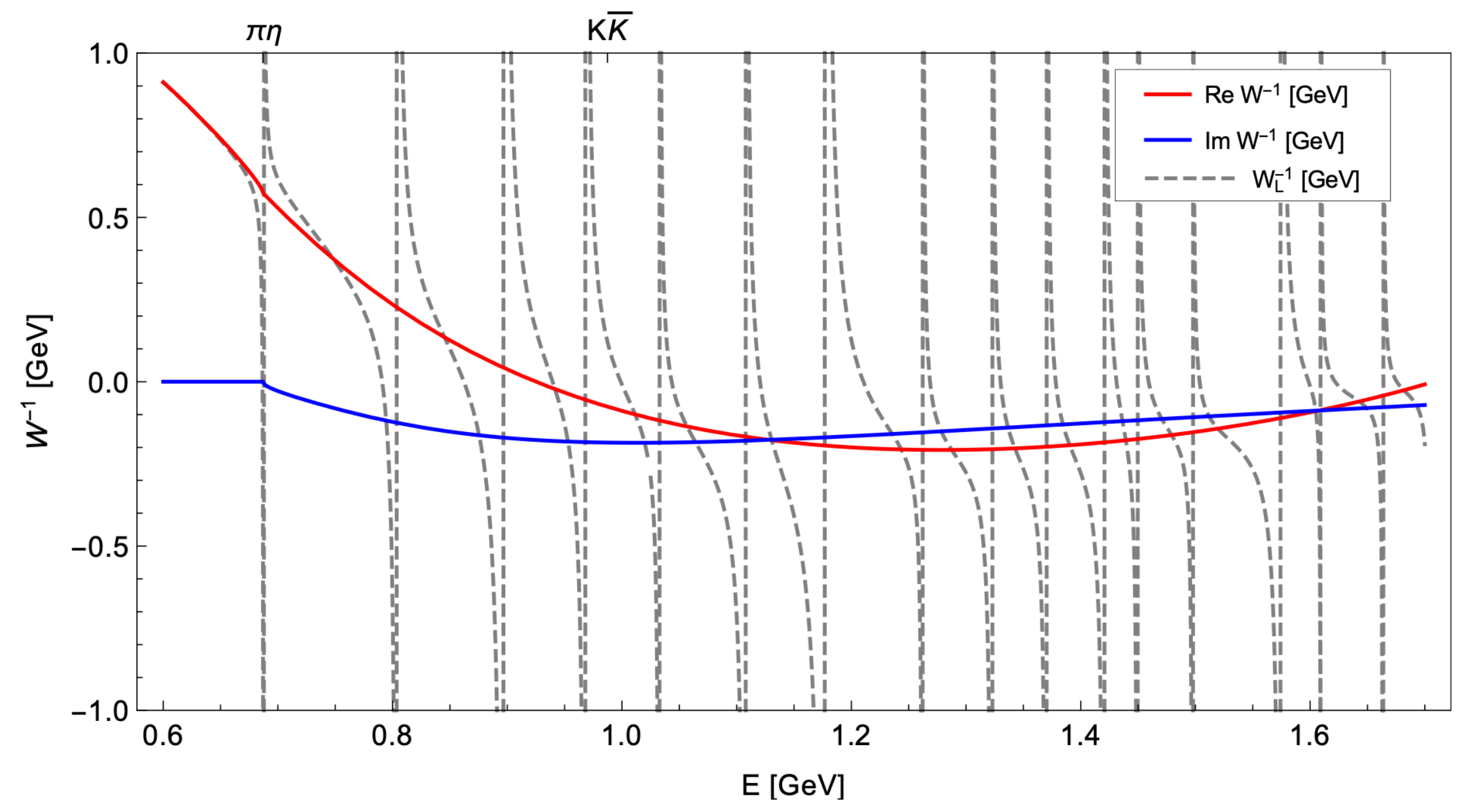
- n-body case?

# BULK PROPERTIES OF FV SPECTRA

- It is not always necessary to explicitly parameterize complicated intermediate states
- “uninteresting” dynamics in a optical potential  $W \in \mathbb{C}$
- How to reconstruct true OP (complex) from finite-volume OP (real)?

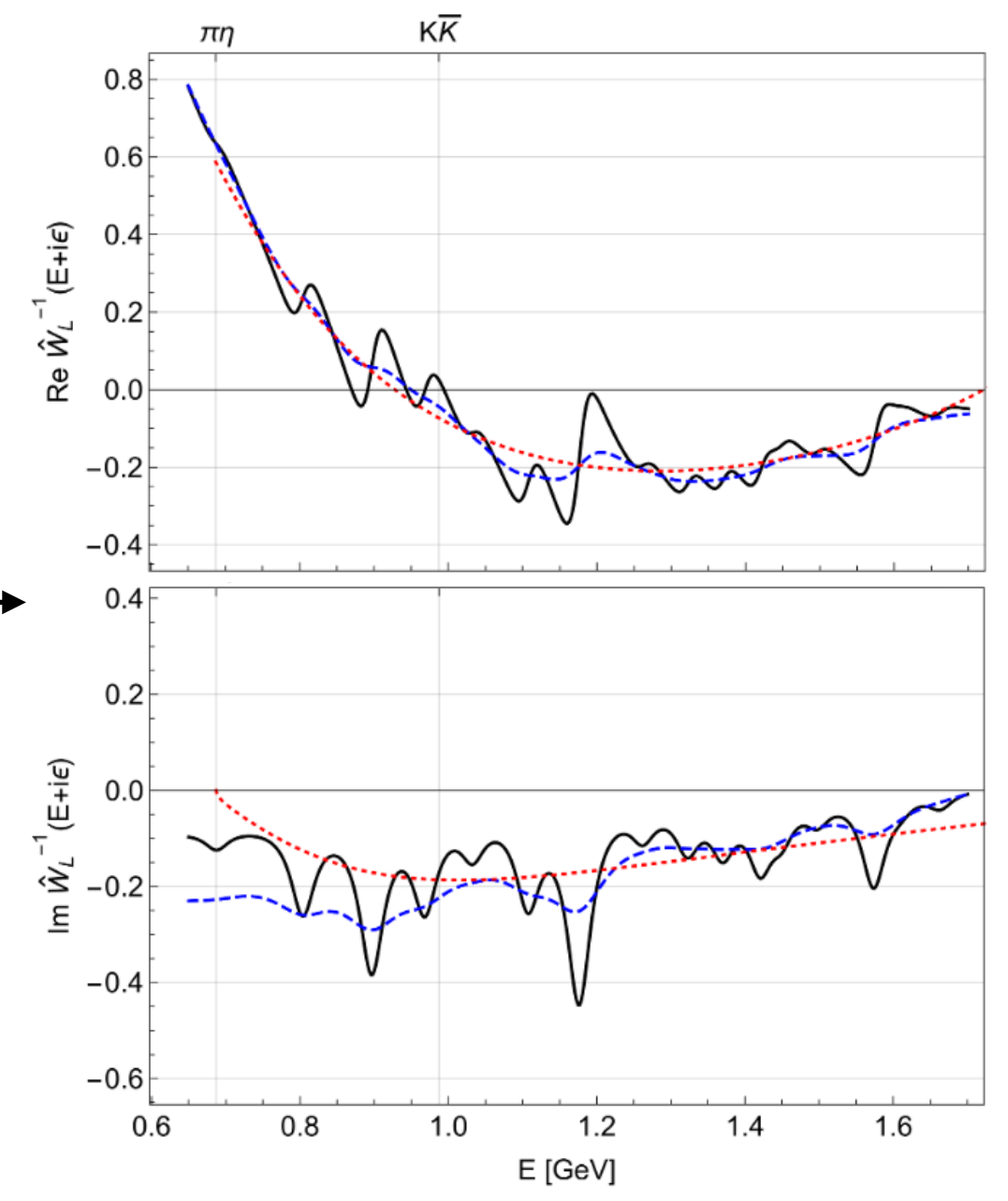


$$\hat{W}_L^{-1}(E) = \sum_i \frac{Z_i}{E - Y_i} + D_0 + D_1 E + \dots$$



$$W^{-1}(E) = \lim_{\epsilon \rightarrow 0} \lim_{L \rightarrow \infty} W_L^{-1}(E + i\epsilon)$$

double limit



# BULK PROPERTIES OF FV SPECTRA

## 1. Fit general form of the OP

$$\hat{W}_L^{-1}(E) = \sum_i \frac{Z_i}{E - Y_i} + D_0 + D_1 E +$$

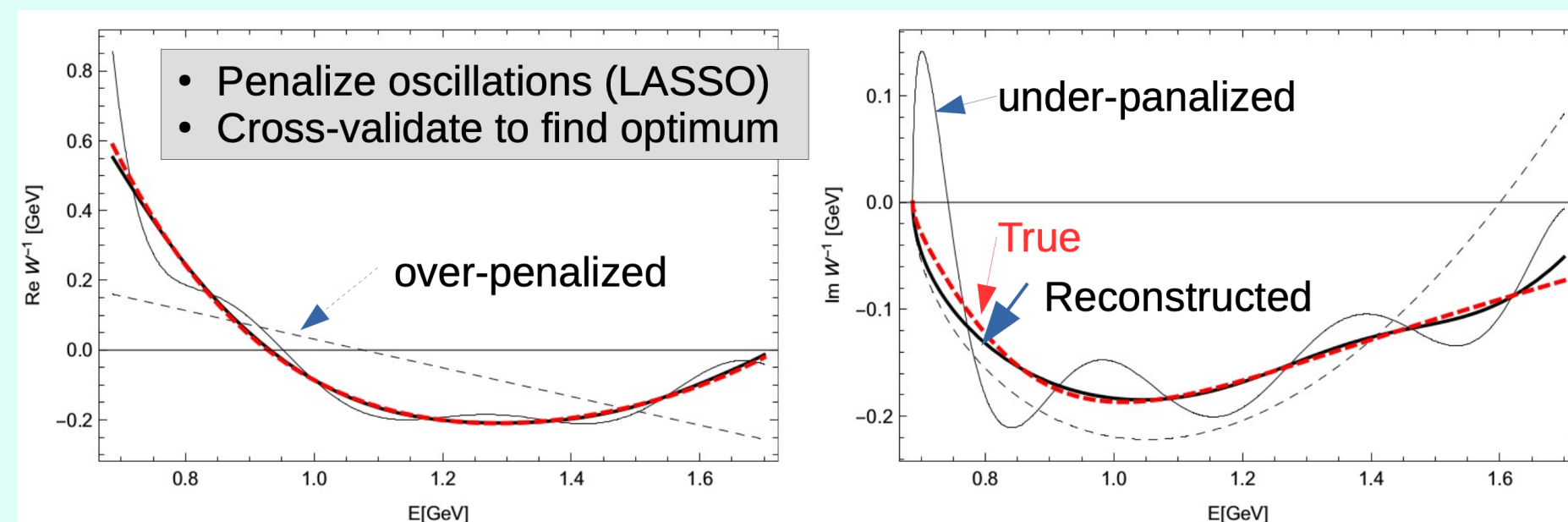
## 2. Extrapolate to complex plane $W_L(E + i\epsilon)$

## 3. Smoothing oscillations

... = infinite-volume limit

### LASSO + cross validation

- ▶ fit at finite a  $\epsilon$
- ▶ validate at a different  $\epsilon'$



# BULK PROPERTIES OF FV SPECTRA

## 1. Fit general form of the OP

$$\hat{W}_L^{-1}(E) = \sum_i \frac{Z_i}{E - Y_i} + D_0 + D_1 E +$$

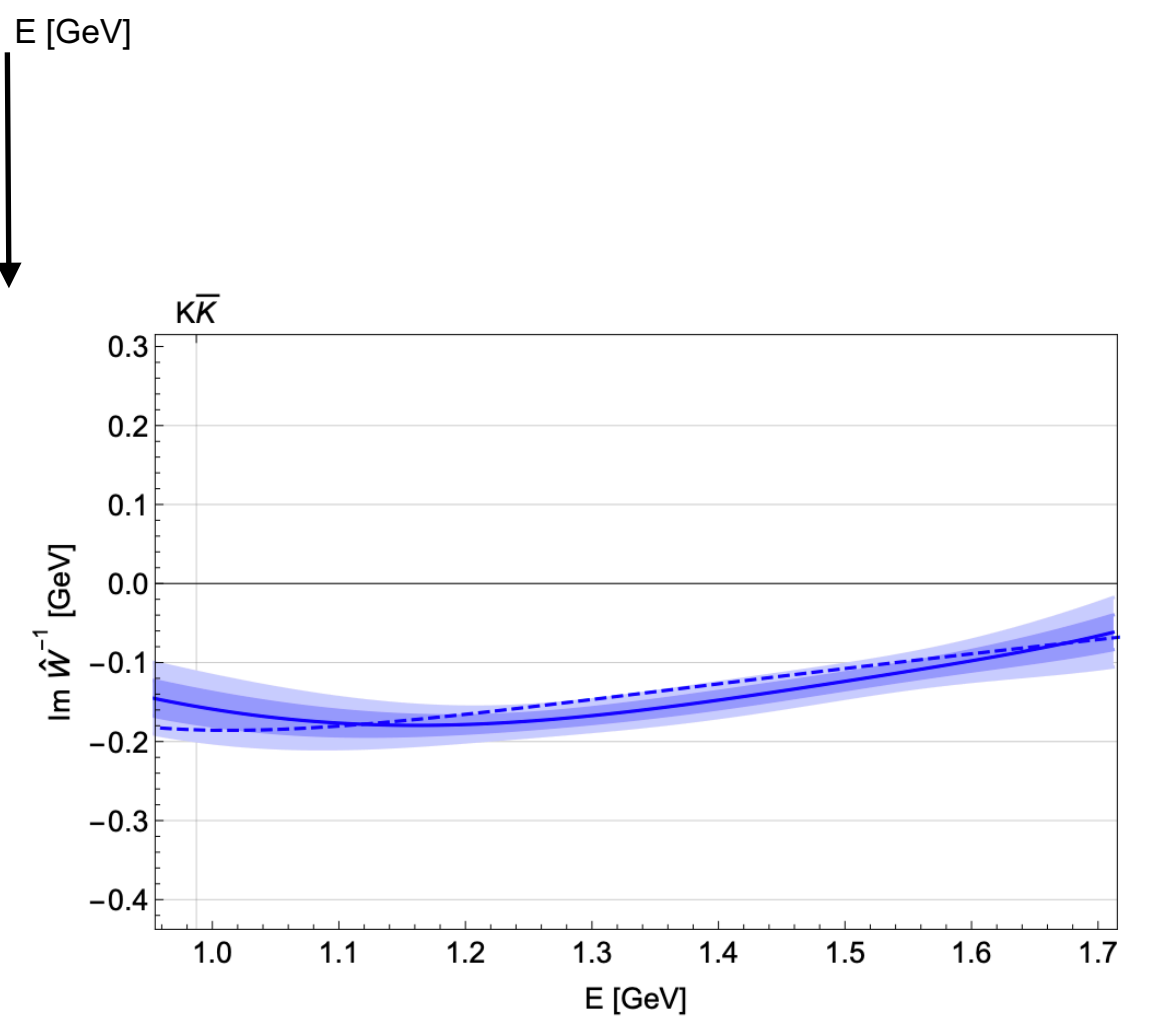
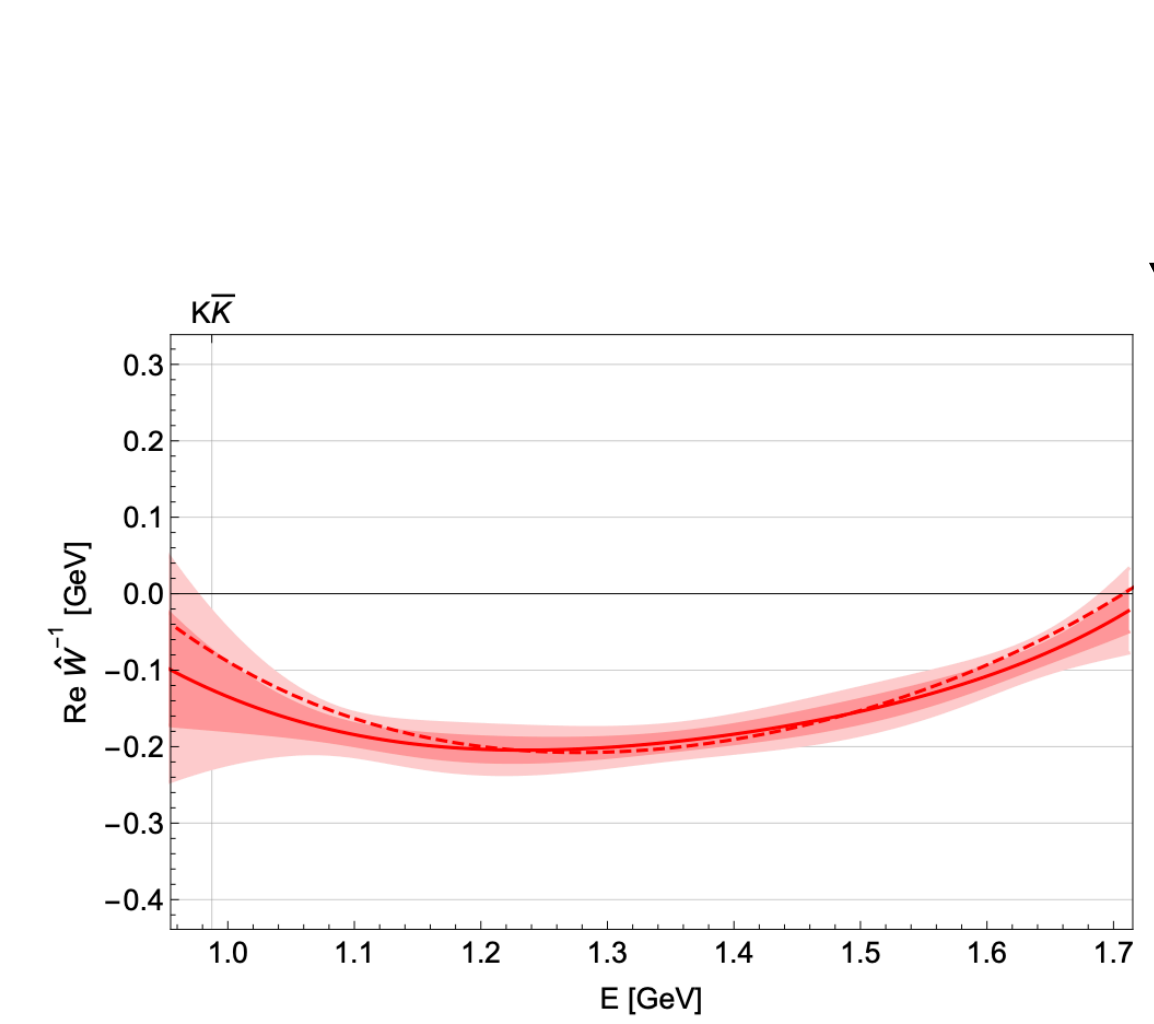
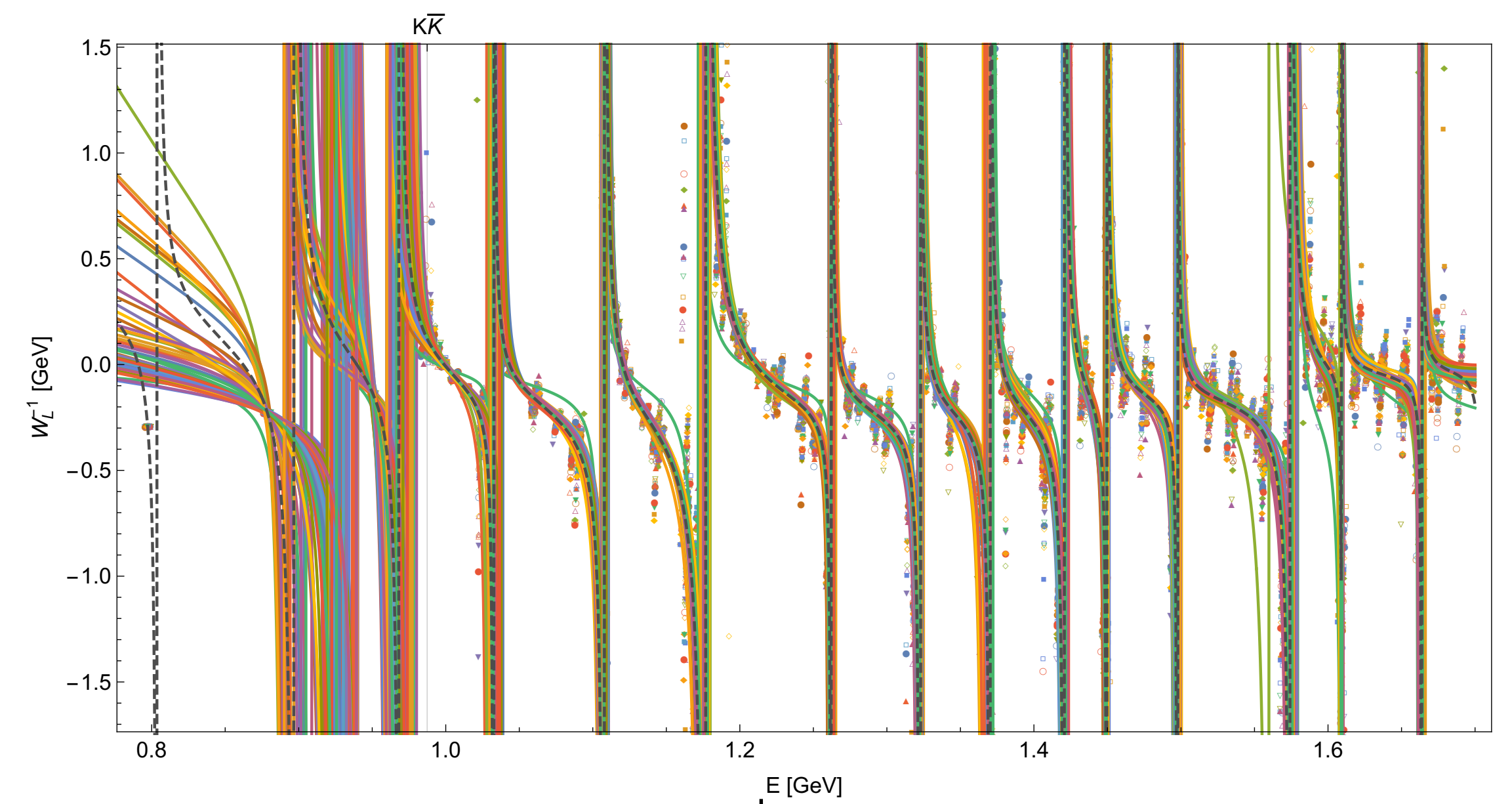
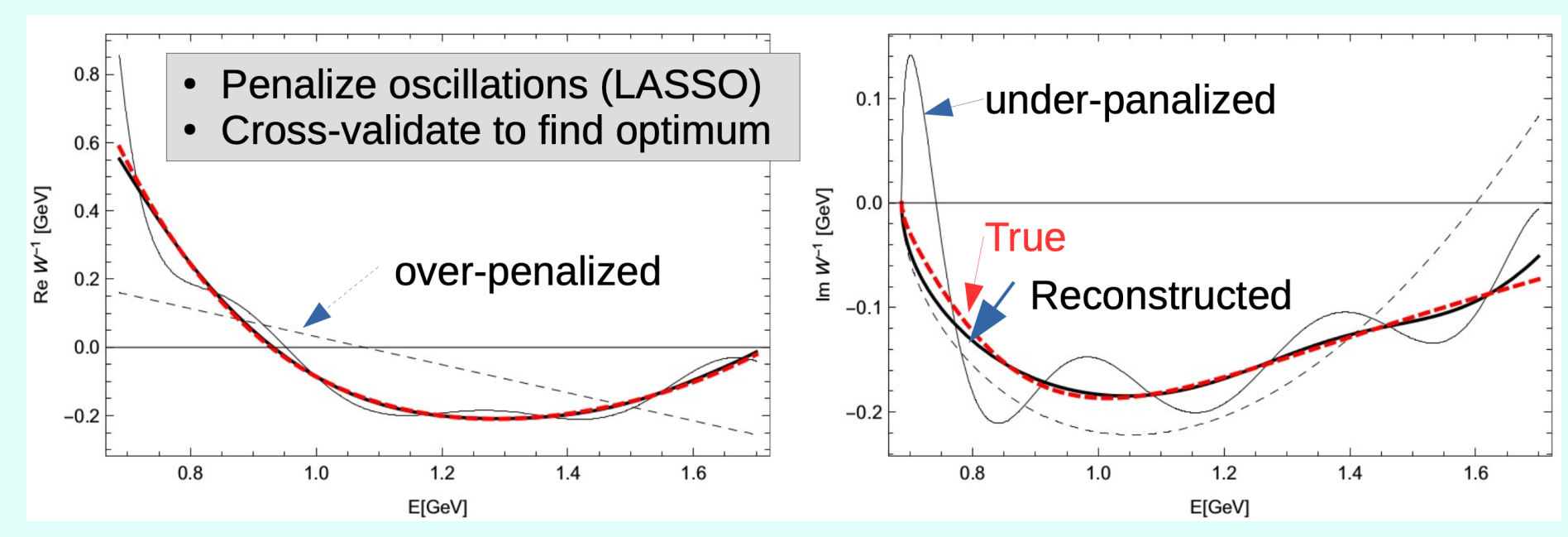
## 2. Extrapolate to complex plane $W_L(E + i\epsilon)$

## 3. Smoothing oscillations

... = infinite-volume limit

### LASSO + cross validation

- ▶ fit at finite a  $\epsilon$
- ▶ validate at a different  $\epsilon'$



# APPLICATION 3

---

# SIMULATION-BASED INFERENCE

Daniel Sadasivan  
Isaac Cordero  
Andrew Graham  
Cecilia Marsh  
Daniel Kupcho  
Melana Mourad  
Maxim Mai

**Deep Neural Network Driven Simulation Based Inference Method for Pole Position Estimation under Model Misspecification**

**e-Print: 2507.18824 [hep-ph]**

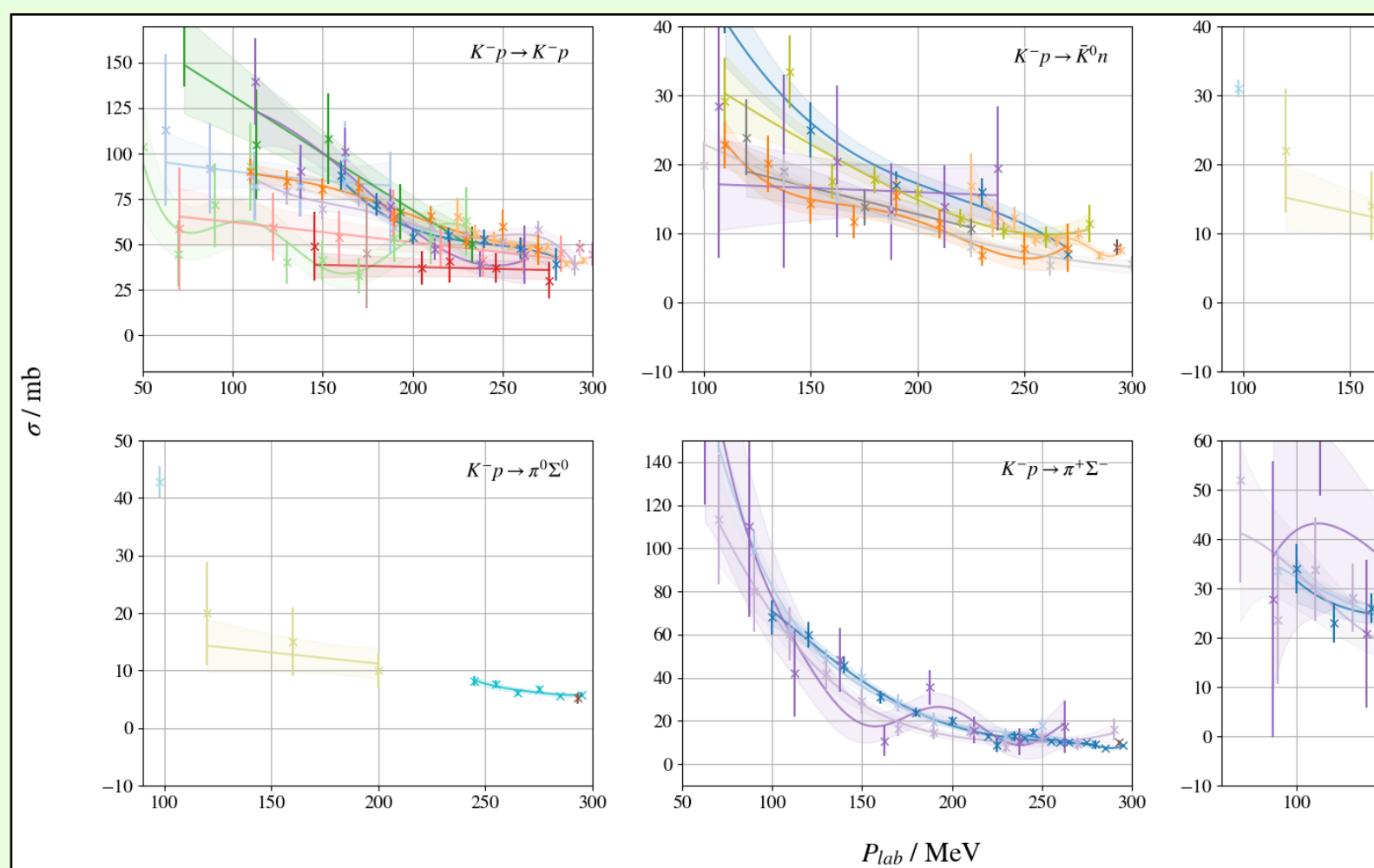
# FROM DATA TO POLES

## Hadron physics relevant experimental data

- often old (1950's+)
- sometimes low/wrong statistics
- barely any correlations
- unknown systematics/bias

G. D'Agostini Nucl.Instrum.Meth.A 346 (1994), 306-311, ...

Ball et al. JHEP05(2010)075



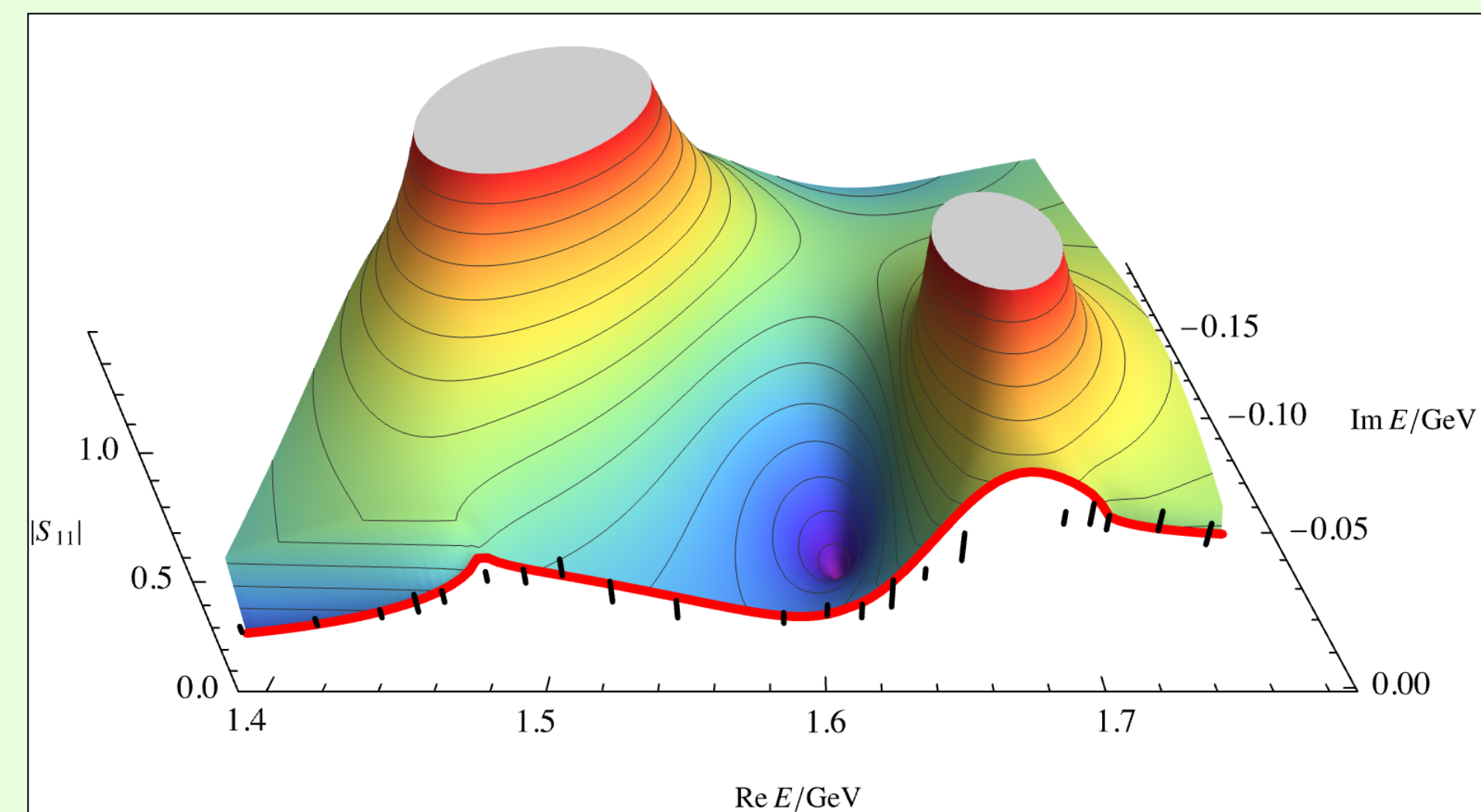
Review: MM Eur.Phys.J.ST 230 (2021) 6, 1593-1607

## Resonance parameter

- full solution from QCD does not exist
- model parameters  $T(x | \vec{a}) \in \mathbb{C}$  from

$$\chi^2 = \sum_i \left( \frac{|T(x_i, \vec{a})| - y_i}{\Delta y_i} \right)^2$$

- Analytic continuation of the amplitude  $x \in \mathbb{C}$



# MODEL MISSPECIFICATION

## $\chi^2$ statistic

most probable parameters a given the measured data  $y_i$  under the assumption that the **data are normally distributed** about the function for some values of  $a$ , and the **prior uniform probability distribution** of fit parameters.

## Model misspecification

no values of  $a$  exist for which the data are normally distributed about the function

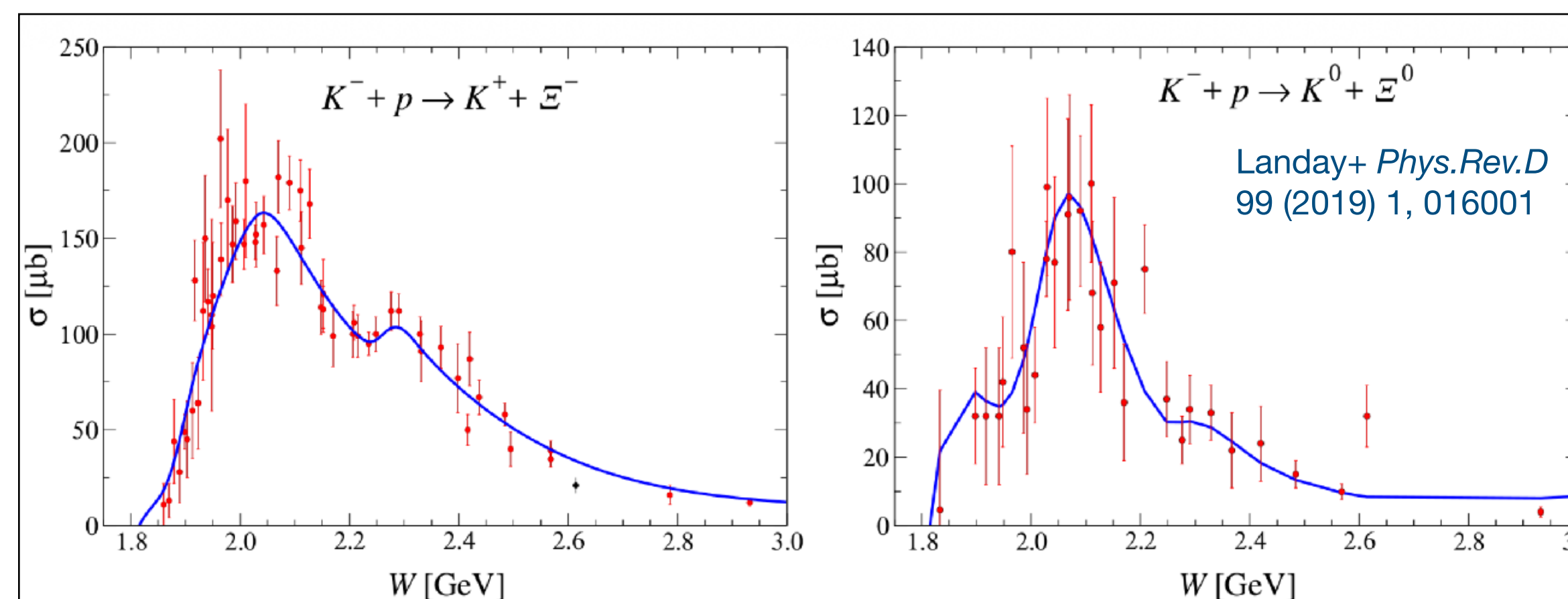
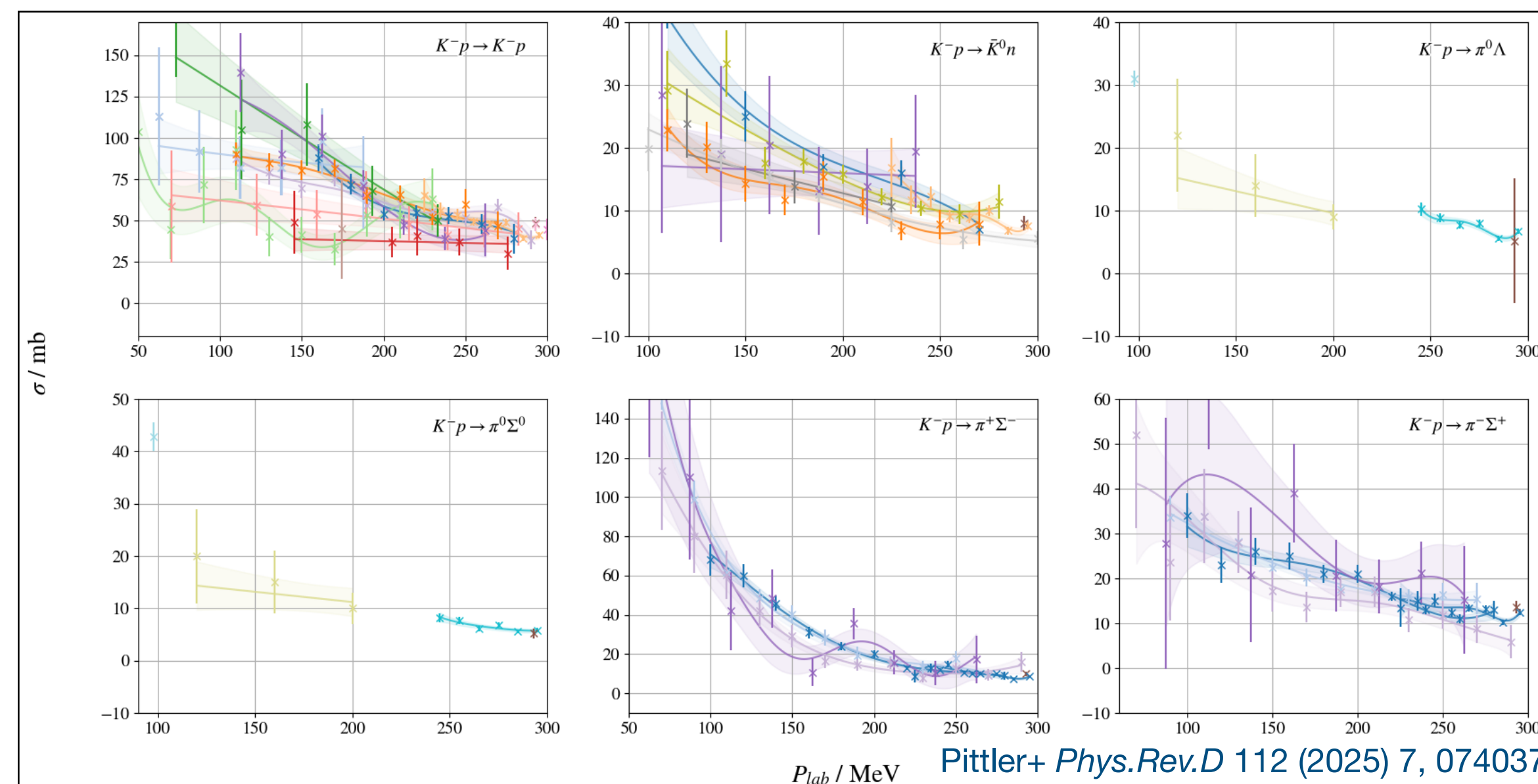
- outliers
- non-gaussian (systematic) uncertainties
- ...

### Solutions:

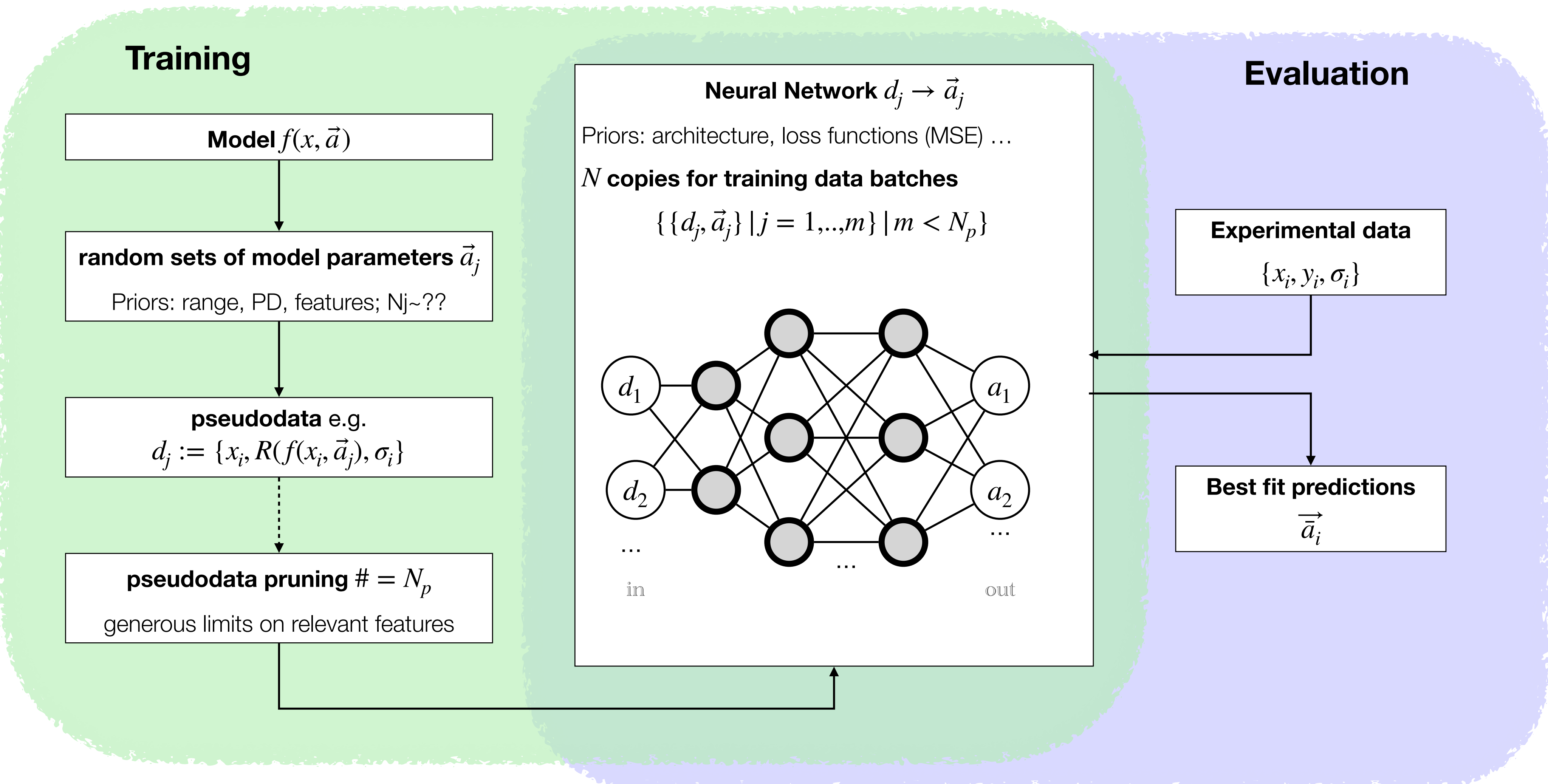
- ➡ Data pruning
- ➡ LASSO
- ➡ ...
- ➡ **Simulation-Based Inference**

Tavare+ Genetics 145, 505–518 (1997)  
Astro Automata, "A tutorial on simulation-based inference," (2020)  
Johann Brehmer, Cranmer 2010.06439 [hep-ph]

...



# SIMULATION-BASED INFERENCE



# TOY EXAMPLES

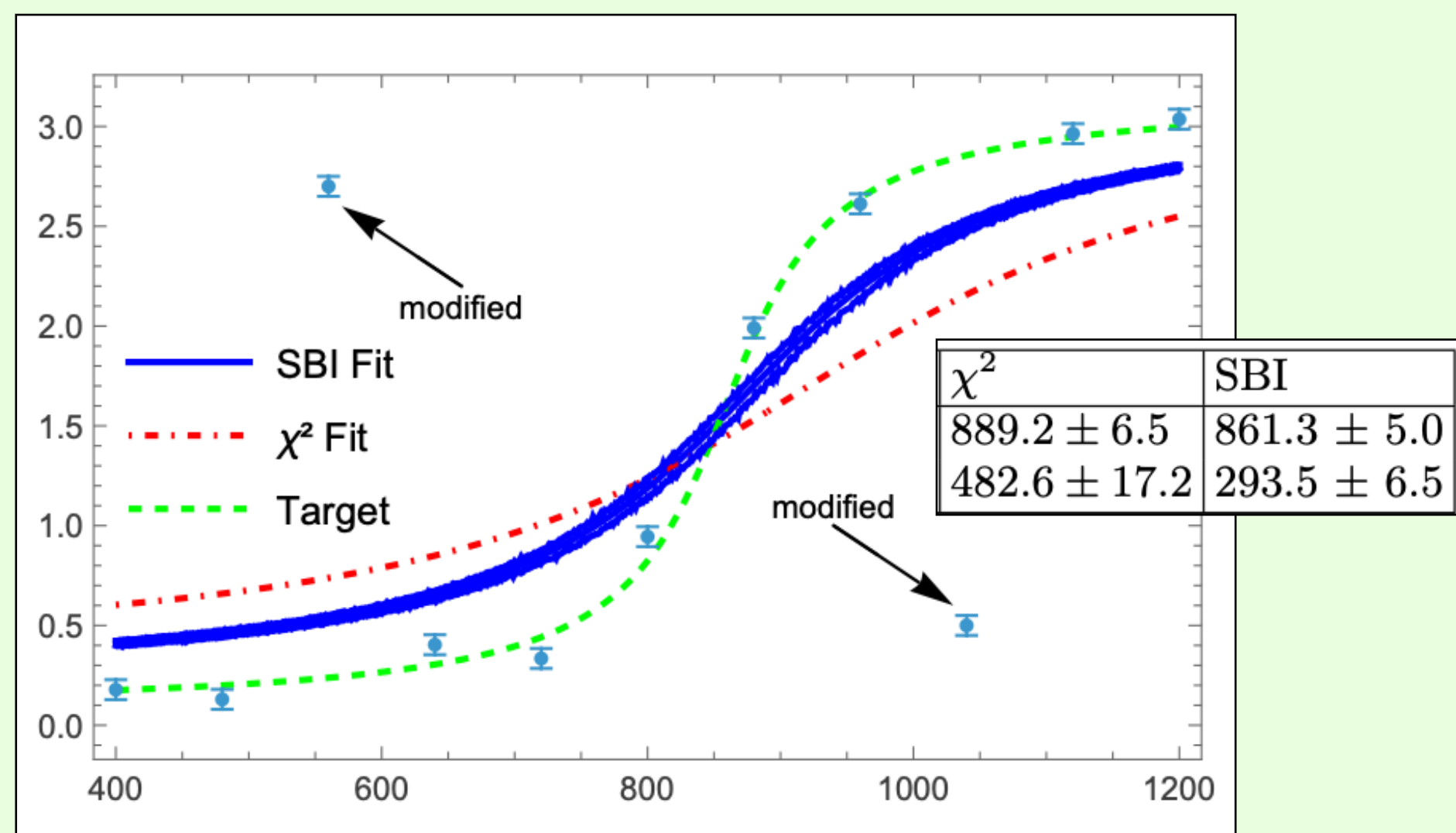
## Toy example 1

- target function

$$T(E) = \frac{\sqrt{k}}{(E^2 - M^2) - iM\Gamma}$$

Modified	Generating
$M$ [MeV]	857.0
$\Gamma$ [MeV]	119.0

- add visible outliers



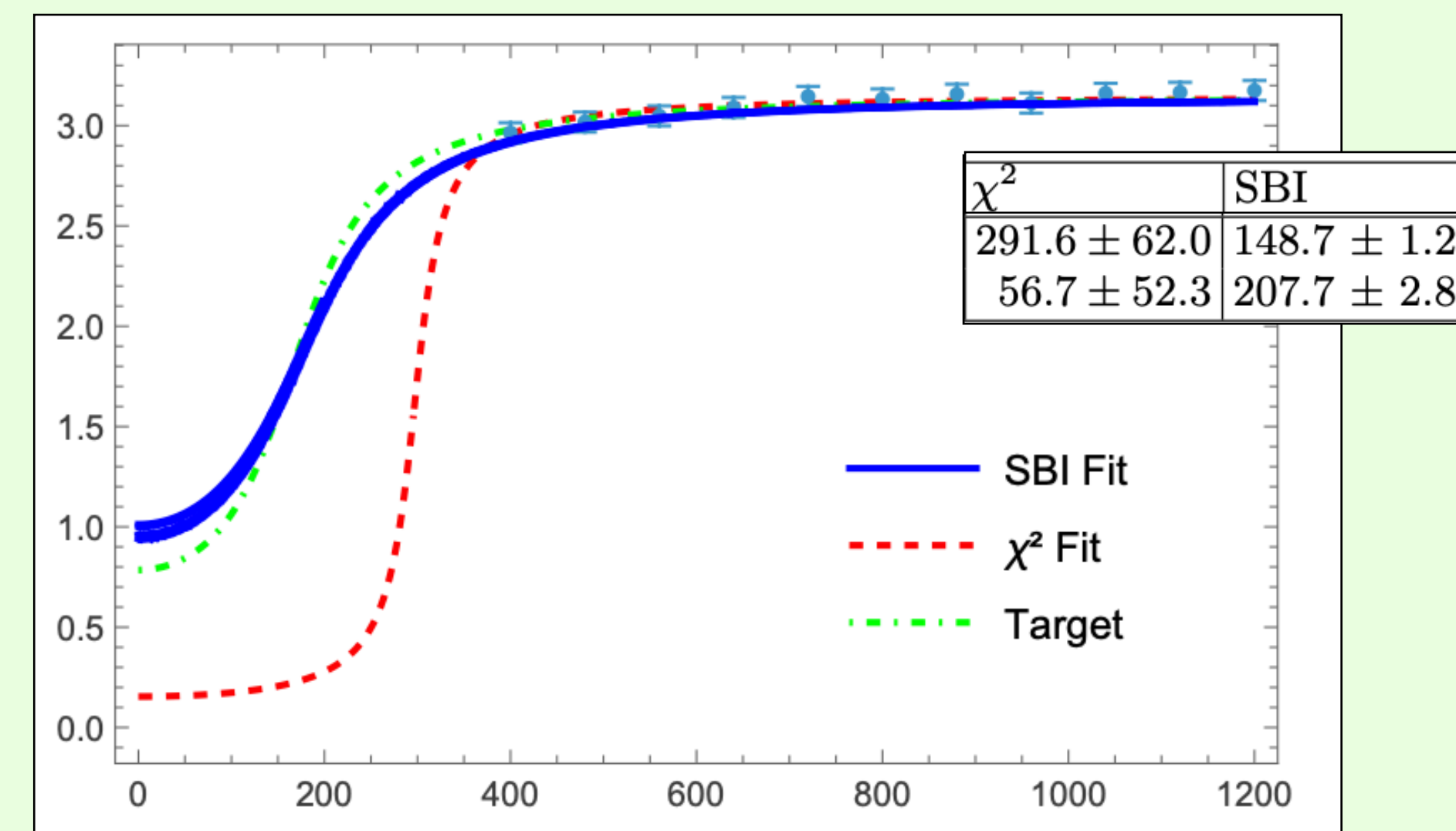
## Toy example 2

- target function

$$T(E) = \frac{\sqrt{k}}{(E^2 - M^2) - iM\Gamma}$$

Non-gaussian	
$M$ [MeV]	150.0
$\Gamma$ [MeV]	150.0

- data outside the resonance region
- non-Gaussian noise



# $\pi\pi$ SCATTERING

## Model

- generic K-matrix form

$$T(E) = \tilde{v}(k_{\text{cm}}) \frac{1}{K_n^{-1}(E) - \Sigma(E)} \tilde{v}(k_{\text{cm}}),$$

$$K_n^{-1}(E) = M_\pi^2 \sum_{i=0}^{n-1} a_i \left(\frac{E^2}{M_\pi^2}\right)^i,$$

$$\Sigma(E) = \int_0^\infty \frac{dk k^2}{(2\pi)^3} \frac{1}{2E_k} \left(\frac{E^2}{4E_k^2}\right)^n \frac{\tilde{v}^2(k)}{\sigma - 4E_k^2 + i\epsilon},$$

- flexibility: 2, 3 subtractions

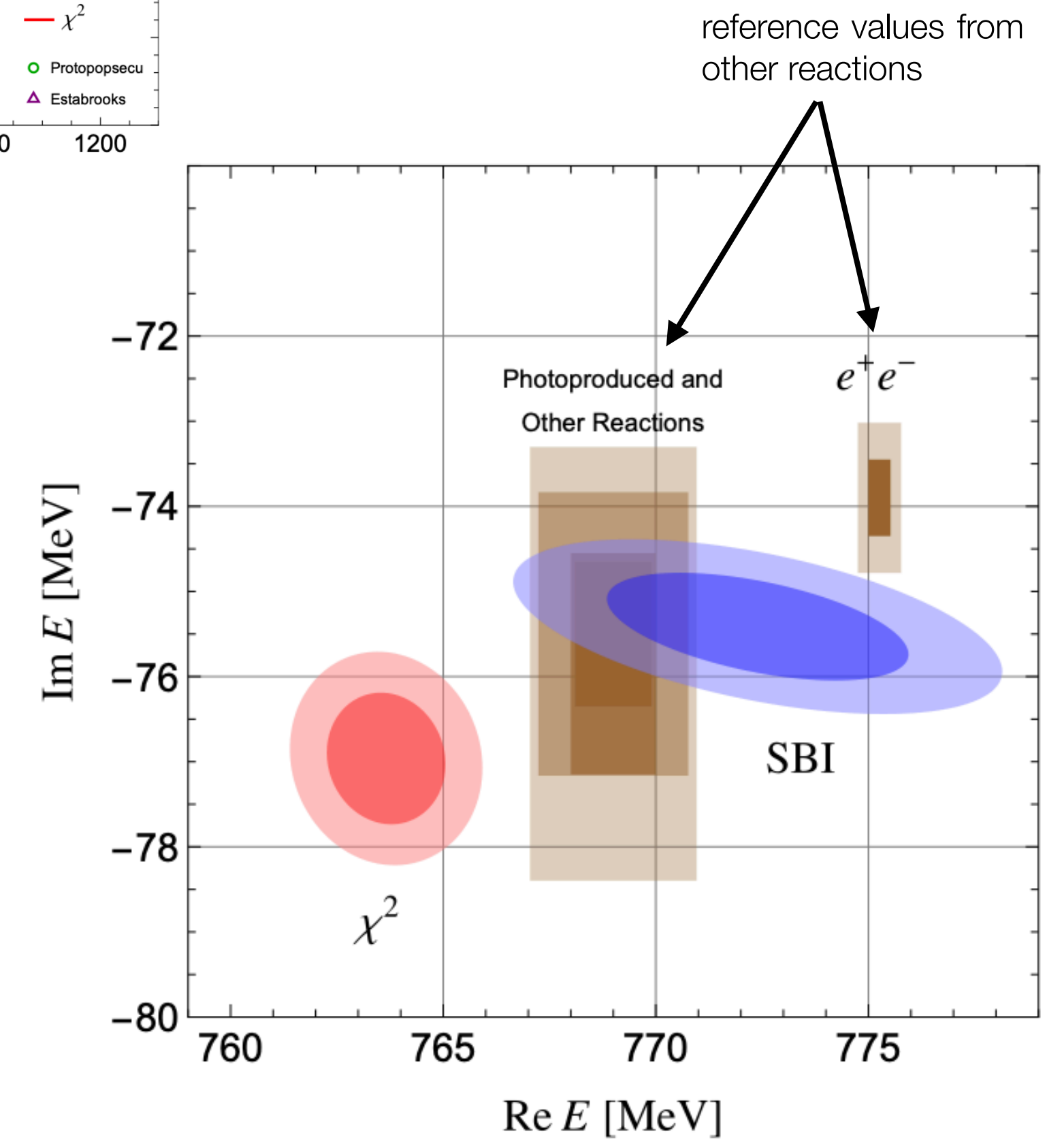
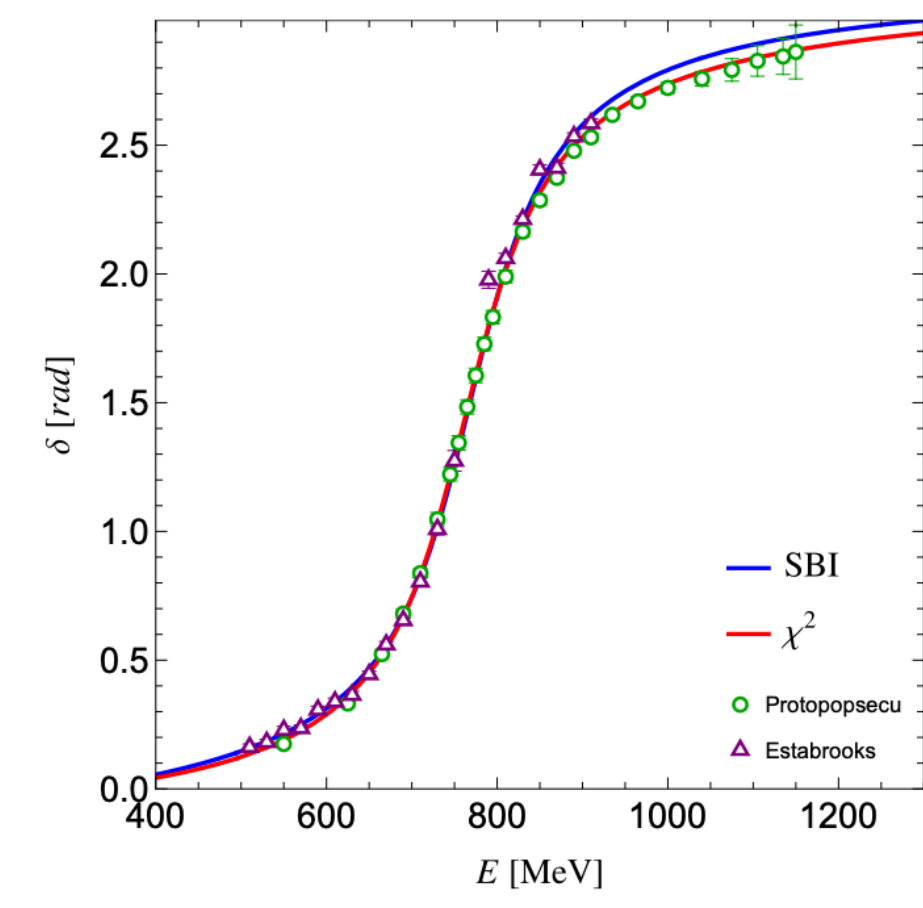
## SBI vs $\chi^2$

SBI					
Data	$n$	$a_0 \cdot 10^3$	$a_1 \cdot 10^3$	$a_2 \cdot 10^3$	$E^* [\text{MeV}]$
All	3	-187	-3.17	+0.42	$772.4(2.3) - i 75.4(0.4)$

$\chi^2$				
$a_0 \cdot 10^3$	$a_1 \cdot 10^3$	$a_2 \cdot 10^3$	$\chi^2_{\text{dof}}$	$E^* [\text{MeV}]$
-290	+4.10	+0.29	3.1	$763.7(0.9) - i 76.9(0.6)$

- bias through outliers in  $\chi^2$  method
- **SBI pole positions overlapping w. higher-data-PDG-result**



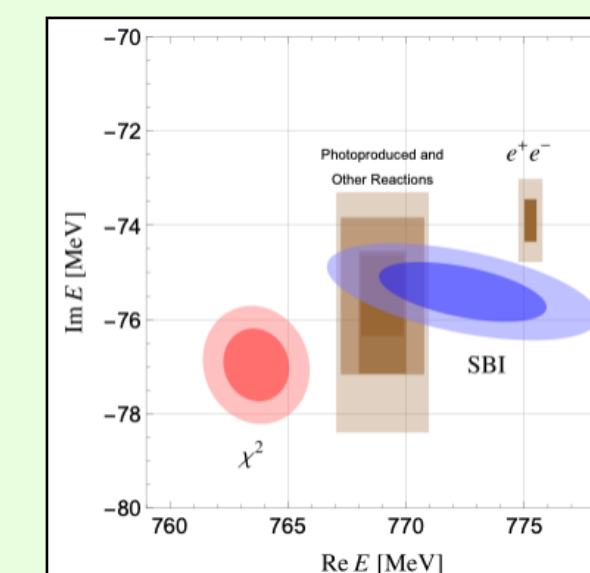
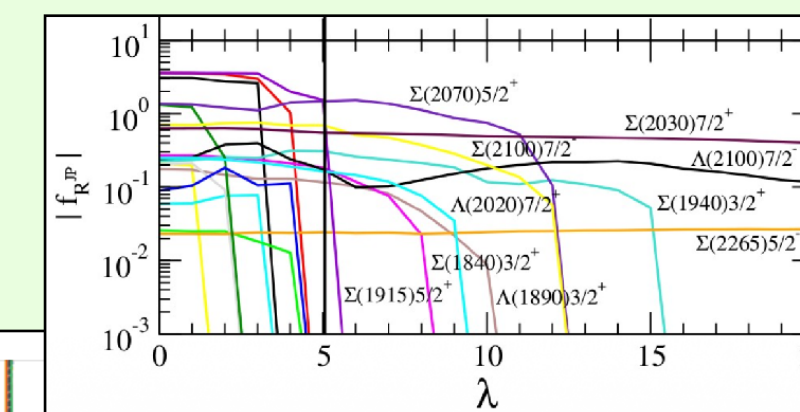
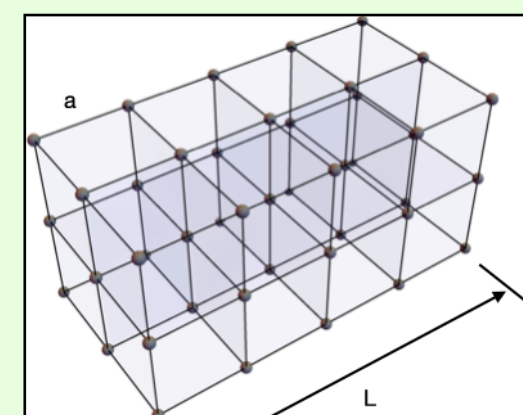
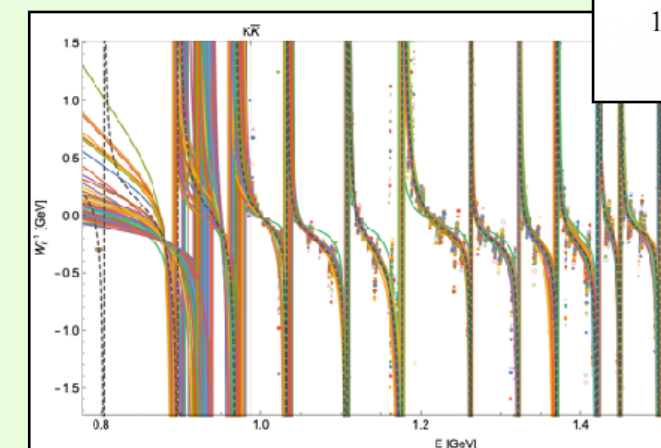
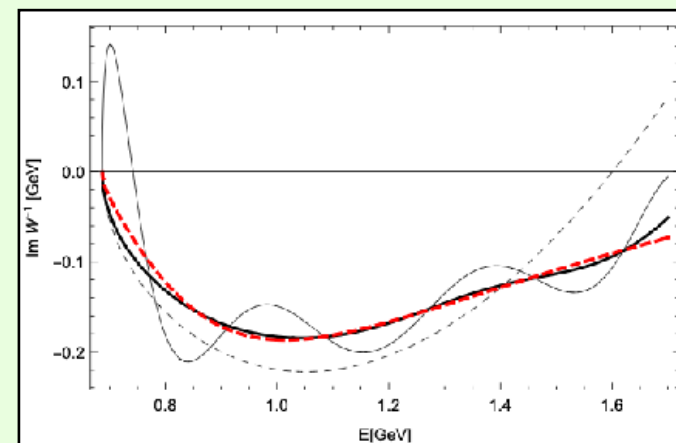
# SUMMARY

## Experiment: Data abundance

- ▶ many new experiments
- ▶ new technology

## Theory: No surprises in the Standard Model

- ▶ consistency/precision tests become more important
- ▶ new techniques (EFTs, Lattice, Machine learning)



## Future directions and open questions

### \* Universality:

convolution networks, spaces, loss functions, ...

### \* Skepticism: How to filter hype from necessity?

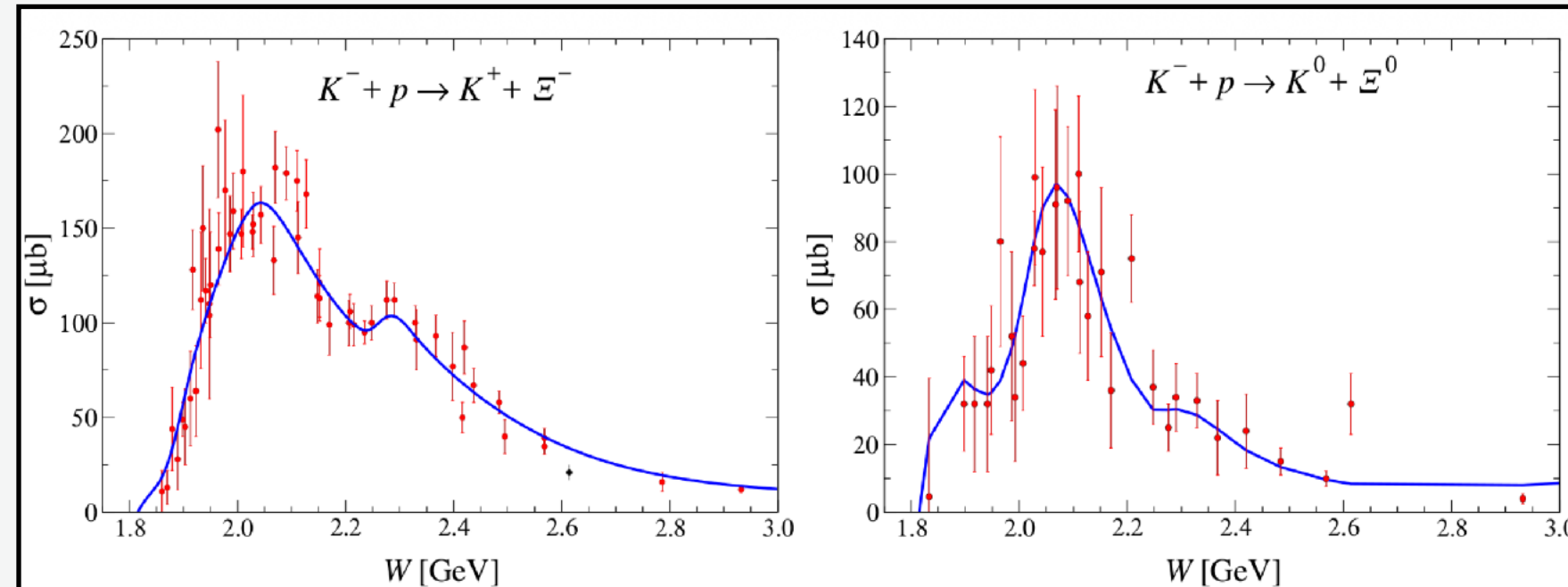
*“never trust a statistics you have not faked yourself” (Germany Saying)*

### \* Good practice:

Toy models, Formal proofs?

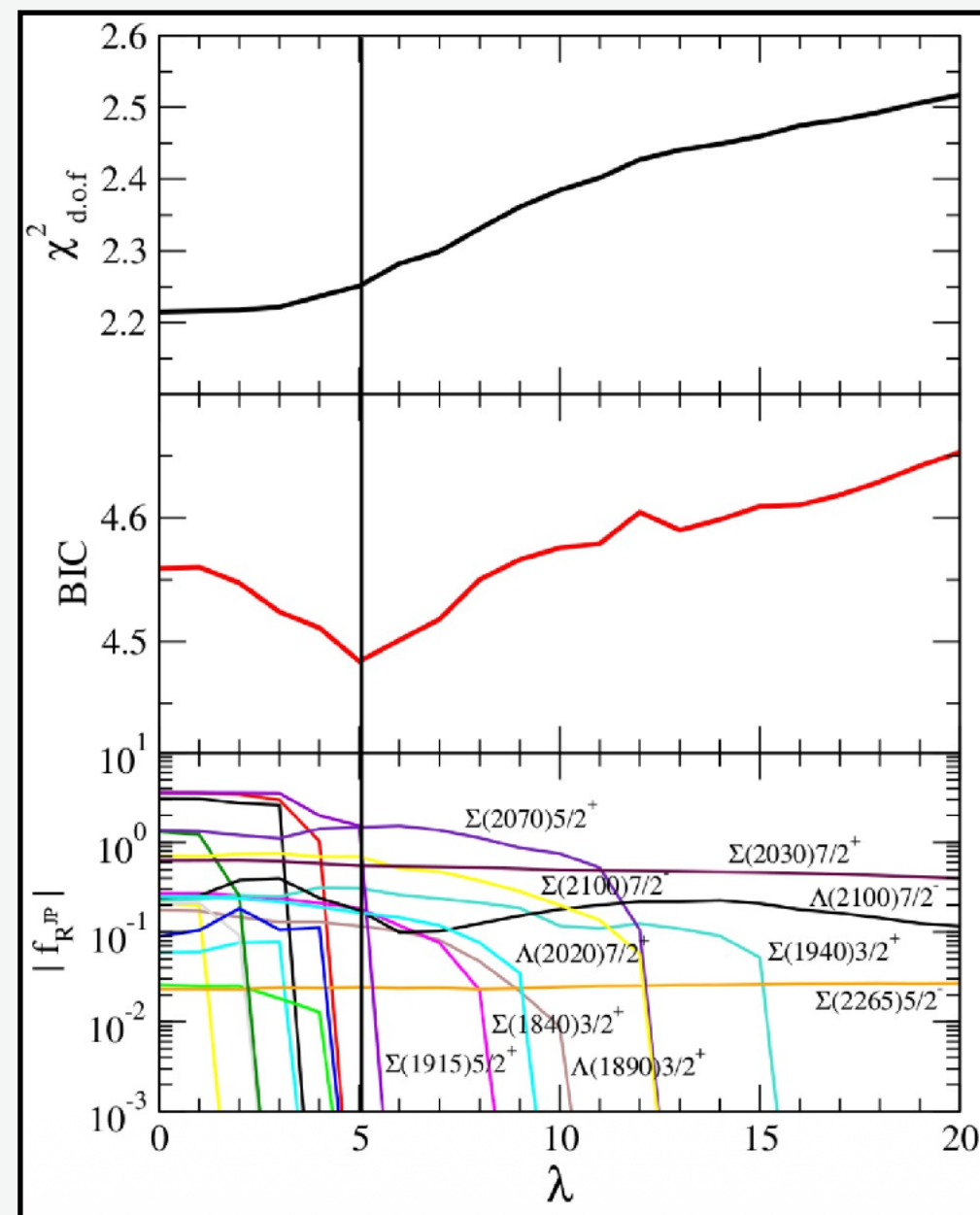


# REAL DATA $K^- p \rightarrow K \Xi$



Pjerrou:1962pc, Carmony:1964zza, Berge:1966zz, Haque:1966mda, London:1966zz, Tripp e:1967wat, Trower:1968zz, Merrill:1968zz, Burgun:1969ee, Dauber:1969hg

- Data pruning for outliers using smoothness as criterion (self-consistent  $3\sigma$  criterion) Navarro Perez,+ Phys. Rev. C 88, 064002 (2013);
- d'Agostini bias potentially an issue if rescaling is included



- LASSO
  - Backward selection (automatic shutoff LASSO)
  - All 21 resonance candidates from PDG
  - Masses and widths fixed to PDG values
- Only x fitted
- 10 out of 21 resonances identified

Solving eq. (2.8), one finds an explicit expression of the on-shell  $T$ -matrix in terms of the optical potential  $W(E)$ :

$$T_{K\bar{K} \rightarrow K\bar{K}}(E) = \frac{1}{W^{-1}(E) - ip_{K\bar{K}}}. \quad (2.10)$$

Note that the form of eq. (2.10) with a complex optical potential  $W$  is a completely general expression for the case of the multi-channel and -particle scattering problem.

It is often useful to introduce the so-called  $M$ -matrix  $M = V^{-1}$ , i.e.,

$$M = \frac{1}{\Delta} \begin{pmatrix} V_{\pi\eta \rightarrow \pi\eta} & -V_{K\bar{K} \rightarrow \pi\eta} \\ -V_{K\bar{K} \rightarrow \pi\eta} & V_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix}, \quad \Delta = V_{K\bar{K} \rightarrow K\bar{K}} V_{\pi\eta \rightarrow \pi\eta} - V_{K\bar{K} \rightarrow \pi\eta}^2. \quad (2.11)$$

In terms of this quantity, the above formula can be rewritten in the following form:

$$W^{-1}(E) = M_{K\bar{K} \rightarrow K\bar{K}} - \frac{M_{K\bar{K} \rightarrow \pi\eta}^2}{M_{\pi\eta \rightarrow \pi\eta} - ip_{\pi\eta}}. \quad (2.12)$$

$$W_L^{-1}(E) := \frac{2}{\sqrt{\pi L}} Z_{00}(1; q_{K\bar{K}}^2) = M_{K\bar{K} \rightarrow K\bar{K}} - \frac{M_{K\bar{K} \rightarrow \pi\eta}^2}{M_{\pi\eta \rightarrow \pi\eta} - \frac{2}{\sqrt{\pi L}} Z_{00}(1; q_{\pi\eta}^2)}. \quad (2.15)$$