Meson-baryon scattering

in manifest Lorentz invariant chiral perturbation theory

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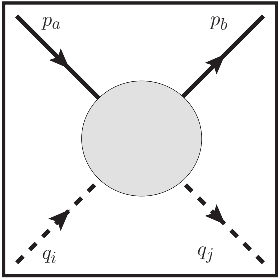
Why and how...

- fundamental part of various processes
- large amount of data up to quite high energies
  $\hookrightarrow$ GWU: 30K data points for $\pi N \to \pi N$
- simplicity of the process



low energy $\longrightarrow$ *effective field theory:*

- $\chi PT_{2(3)}$
  ...expanding the QCD Greens functions in {small meson momenta} and {up, down and (strange)} - quark masses

Weinberg (1979), Gasser and Leutwyler (1984)

How...

Power counting:

$$\mathcal{L}_\phi = \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} + \ldots$$

$$\mathcal{L}_{\phi B} = \mathcal{L}_{\phi B}^{(1)} + \underbrace{\mathcal{L}_{\phi B}^{(2)}}_{\sum_{i=1}^{16} b_i \mathcal{O}_i^2} + \underbrace{\mathcal{L}_{\phi B}^{(3)}}_{\sum_{i=1}^{78} d_i \mathcal{O}_i^3} + \ldots$$

M. Frink, U.-G. Meißner (2006)

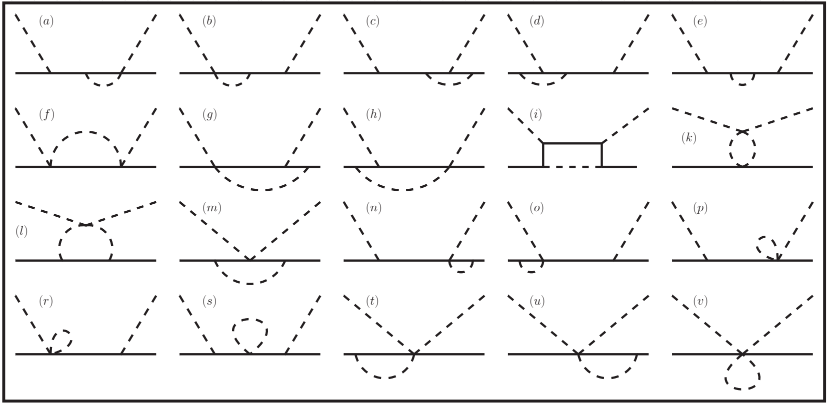
- 1st order: $\mathcal{L}_{\phi B}^{(1)} \longrightarrow$ WT and Born type: D, F, m_0
- 2nd order: $\mathcal{L}_{\phi B}^{(2)} \longrightarrow$ contact terms (11 LECs $\longleftarrow$ FIT)
- 3rd order:

  $\mathcal{L}_{\phi B}^{(3)} \longrightarrow$ contact terms (13 LECs $\longleftarrow$ neglected)

  $\mathcal{L}_{\phi B}^{(1)}, \mathcal{L}_{\phi}^{(2)}, \mathcal{L}_{\phi}^{(4)} \longrightarrow$ wave function renormalization

How...

... $\mathcal{L}^{(1)}_{\phi B} \longrightarrow$ one loop diagrams (+crossed):



$\rightsquigarrow$ regularization:

How...(regularization)

- dim-Reg of the UV-divergencies
- baryons carry intrinsic scale $m_0 \sim 1$ GeV (even if $m_{u,d,s} = 0$)

$$\mathbf{H}(p^2, M^2, m_0^2) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2 - M^2)((k-p)^2 - m_0^2)} = \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}}\int_0^1 \Delta_z^{\frac{d}{2}-2} dz$$

$$\Delta_z = m_0^2 z^2 - 2m_0 M \frac{p^2 - M^2 - m_0^2}{2m_0 M} z(1-z) + M^2(1-z)^2$$

$\rightsquigarrow$ *Infrared Regularization of baryon loops:*

(respects low energy PC) + (manifest Lorentz invariance)

Becher, Leutwyler (1999)

$$\underbrace{\int_0^1 (\ldots) dz}_{\mathbf{H}} = \underbrace{\int_0^\infty (\ldots) dz}_{\mathbf{I}} - \underbrace{\int_1^\infty (\ldots) dz}_{\mathbf{R}}$$

$$\mathbf{I}:\ M^{d-3}\left(c_0 + c_1 M + c_2 M^2 + \ldots\right) \qquad \mathbf{R}:\ \left(d_0 + d_1 M + d_2 M^2 + \ldots\right)$$

Result

- scattering length: $a_{\phi B} = \frac{m_B}{4\pi(m_B+M_\phi)} T_{\phi B}(s_{thr})$
- F_ϕ, M_ϕ, D, F: fixed to the physical values, $m_0 = 1.15$ GeV, 0.938 GeV $< \mu <$ 1.314 GeV
- the HB result is obtained by expanding and truncating $T_{\phi B}$ at finite chiral order
- $\{b_0, b_D, b_F, b_1, ..., b_{11}\} \longleftrightarrow \{\sigma_{\pi N}, \{m_B\}, a^+_{\pi N}, a^{(1)}_{KN}, a^{(0)}_{KN}\}/d_0$

Ellis, Torikoshi (2000), Bernard, Kaiser, Meißner (1993)
Schroeder(πN) (2001), Martin(KN) (1980)

Channel	=	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)_{IR[HB]}$	$+\mathcal{O}(q^3)_{IR[HB]}$	$\sum_{IR[HB]}$	
$a^{(3/2)}_{\pi N}$	=	-0.12	$+0.05[+0.05]$	$+0.04[-0.06]$	$-0.04^{+0.07}_{-0.07}[-0.13^{+0.03}_{-0.03}]$	-0.13 ± 0.01
$a^{(1/2)}_{\pi N}$	=	$+0.21$	$+0.05[+0.05]$	$-0.19[+0.00]$	$+0.07^{+0.07}_{-0.07}[+0.26^{+0.03}_{-0.03}]$	-0.25 ± 0.03
$a^{(3/2)}_{\pi\Xi}$	=	-0.12	$+0.04[+0.04]$	$+0.10[-0.09]$	$+0.02^{+0.06}_{-0.07}[-0.17^{+0.03}_{-0.03}]$	
$a^{(1/2)}_{\pi\Xi}$	=	$+0.23$	$+0.04[+0.04]$	$-0.24[-0.03]$	$+0.02^{+0.08}_{-0.10}[+0.23^{+0.03}_{-0.03}]$	
$a^{(2)}_{\pi\Sigma}$	=	-0.24	$+0.10[+0.07]$	$+0.15[-0.07]$	$+0.01^{+0.04}_{-0.04}[-0.24^{+0.01}_{-0.01}]$	
$a^{(1)}_{\pi\Sigma}$	=	$+0.22$	$+0.09[+0.11]$	$-0.21[+0.00]$	$+0.10^{+0.16}_{-0.17}[+0.33^{+0.06}_{-0.06}]$	
$a^{(0)}_{\pi\Sigma}$	=	$+0.46$	$+0.11[-0.01]$	$-0.47[+0.04]$	$+0.10^{+0.17}_{-0.19}[+0.49^{+0.07}_{-0.08}]$	
$a^{(1/2)}_{\pi\Lambda}$	=	-0.01	$+0.03[+0.03]$	$-0.03[-0.11]$	$-0.01^{+0.04}_{-0.04}[-0.09^{+0.01}_{-0.01}]$	

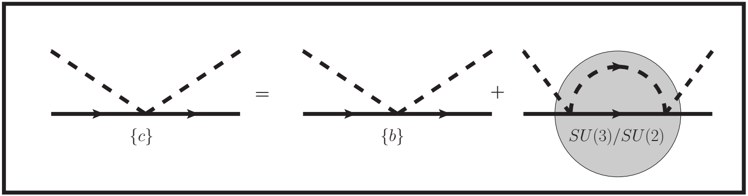
Result

Channel	=	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)_{IR}$	$+\mathcal{O}(q^3)_{IR}$	$\sum_{IR}$	
$a_{KN}^{(1)}$	=	-0.45	$+0.60$	-0.48	$-0.33^{+0.32}_{-0.32}$	-0.33
$a_{KN}^{(0)}$	=	$+0.04$	-0.15	$+0.13$	$+0.02^{+0.64}_{-0.64}$	$+0.02$
$a_{\bar{K}N}^{(1)}$	=	$+0.20$	$+0.22$	$-0.26+0.18i$	$+0.16^{+0.39}_{-0.44}+0.18i$	$+0.37+0.60i$
$a_{\bar{K}N}^{(0)}$	=	$+0.53$	$+0.97$	$-0.40+0.22i$	$+1.11^{+0.47}_{-0.59}+0.22i$	$-1.70+0.68i$
$a_{K\Sigma}^{(3/2)}$	=	-0.31	$+0.33$	$-0.30+0.12i$	$-0.28^{+0.52}_{-0.49}+0.12i$	
$a_{K\Sigma}^{(1/2)}$	=	$+0.47$	$+0.19$	$+0.20+0.01i$	$+0.87^{+0.55}_{-0.64}+0.01i$	
$a_{\bar{K}\Sigma}^{(3/2)}$	=	-0.22	$+0.24$	$-0.35+0.08i$	$-0.33^{+0.44}_{-0.47}+0.08i$	
$a_{\bar{K}\Sigma}^{(1/2)}$	=	$+0.34$	$+0.38$	$+0.27+0.01i$	$+0.98^{+0.59}_{-0.59}+0.01i$	
$a_{K\Xi}^{(1)}$	=	$+0.15$	$+0.34$	$-0.02+0.17i$	$+0.48^{+0.43}_{-0.43}+0.17i$	
$a_{K\Xi}^{(0)}$	=	$+0.66$	$+0.98$	$-0.62+0.14i$	$+1.02^{+0.51}_{-0.68}+0.14i$	
$a_{\bar{K}\Xi}^{(1)}$	=	-0.50	$+0.66$	-0.42	$-0.26^{+0.34}_{-0.34}$	
$a_{\bar{K}\Xi}^{(0)}$	=	-0.15	$+0.02$	$+0.13$	$+0.00^{+0.78}_{-0.68}$	
$a_{K\Lambda}^{(1/2)}$	=	-0.04	$+0.50$	$-0.27+0.14i$	$+0.19^{+0.55}_{-0.56}+0.14i$	
$a_{\bar{K}\Lambda}^{(1/2)}$	=	-0.05	$+0.50$	$-0.40+0.18i$	$+0.04^{+0.55}_{-0.56}+0.18i$	
$a_{\eta N}^{(1/2)}$	=	-0.01	$+0.26$	$-0.13+0.19i$	$+0.13^{+0.60}_{-0.65}+0.19\,i$	$+0.62+0.30i$
$a_{\eta\Xi}^{(1/2)}$	=	-0.09	$+0.84$	$-0.49+0.17i$	$+0.25^{+0.74}_{-0.73}+0.17i$	
$a_{\eta\Sigma}^{(1)}$	=	-0.04	$+0.22$	$-0.15+0.13i$	$+0.03^{+0.24}_{-0.24}+0.13i$	
$a_{\eta\Lambda}^{(0)}$	=	-0.04	$+0.70$	$-0.51+0.38i$	$+0.15^{+0.51}_{-0.55}+0.38i$	$+0.64+0.80i$

Recall: $\mathcal{L}_{\phi B}^{(3)}$ contact terms are neglected

■ *Low energy constants (LECs)*

- integrating out strange quark $SU(3) \longrightarrow SU(2)$



- double scale expansion: $m_0 \gg M_K \gg M_\pi$
  1. *IR-regularized loop integrals in three-flavor formulation*
  2. *expand in* $\{(t - 2M_\pi^2), M_\pi^2, (s - m_0)^2\}$
  3. *expand in* $\{M_K\}$ *to first order*

Low energy constants (LECs)

$$c_1 = b_0 + \frac{b_D}{2} + \frac{b_F}{2} + \frac{M_K}{256\pi F_\pi^2}\left[5D^2 - 6DF + 9F^2 + \frac{2}{3\sqrt{3}}(D-3F)^2\right] + \mathcal{O}(M_K^2),$$

$$c_2 = b_8 + b_9 + b_{10} + 2b_{11} - \frac{M_K}{128\pi F_\pi^2}\left[6 + \frac{19}{3}D^4 + 4D^3F + \frac{58}{3}D^2F^2 - 12DF^3 + 25F^4 - \frac{8(D-3F)^2(D+F)^2}{3\sqrt{3}}\right] + \mathcal{O}(M_K^2),$$

$c_3 = \ldots,\ c_4 = \ldots$

- shifts:

  $\Delta c_1 = +0.2\,\mathrm{GeV}^{-1}$, $\Delta c_2 = -2.1\,\mathrm{GeV}^{-1}$
  $\Delta c_3 = +1.6\,\mathrm{GeV}^{-1}$, $\Delta c_4 = +2.0\,\mathrm{GeV}^{-1}$
  $\Delta(c_2 + c_3 - 2c_1) = -0.1\,\mathrm{GeV}^{-1}$

- the same is performed for the $\pi\Xi$, $\pi\Sigma$ and $\pi\Lambda$ sector

Low energy theorems (LETs)

- πB LETs are useful for chiral extrapolations of LQCD results:

$$T^{+}_{\pi N} = \frac{M_\pi^2}{F_\pi^2}\left\{-\frac{g^2}{4m_N} + 2(c_2 + c_3 - 2c_1) + \frac{3g^2 M_\pi}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^2)\right\}$$

$$T^{-}_{\pi N} = \frac{M_\pi}{2F_\pi^2}\left\{1 + \frac{g^2 M_\pi^2}{4m_N^2} + \frac{M_\pi^2}{8\pi^2 F_\pi^2}\left(1 - 2\log\frac{M_\pi}{\mu}\right) + M_\pi^2 d^r_{\pi N}(\mu) + \mathcal{O}(M_\pi^4)\right\}$$

πN isovector combination is stable with respect to the kaon mass effects (known)

Bernard et al. 1995

- also can be done for the $\pi\Xi$, $\pi\Sigma$ and $\pi\Lambda$ sector

$$\bar{T}^{-}_{\pi\Sigma} = \frac{2M_\pi}{F_\pi^2}\left\{1 + \frac{g_\Sigma^2 M_\pi^2}{16m_\Sigma^2} + \frac{g_{\Sigma\Lambda}^2 M_\pi^2}{4(m_\Lambda + m_\Sigma)^2} + M_\pi^2 d^r_{\pi\Sigma}(\mu) + \frac{M_\pi^2}{8\pi^2 F_\pi^2}\left(1 - 2\log\frac{M_\pi}{\mu}\right) + \mathcal{O}(M_\pi^3)\right\}$$

(new)

Low energy theorems (LETs)

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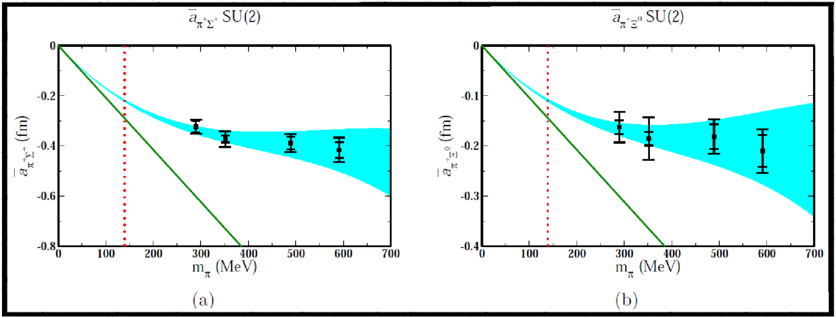

$$a_{\pi^+\Sigma^+} = -0.197 \pm 0.011 fm \qquad a_{\pi^+\Xi^0} = -0.098 \pm 0.017 fm$$

Torok et al. 2009

Summary

- scattering lengths calculated to one loop in $SU(3)-\chi PT$: very slow convergence of the chiral series.(*without third order contact terms*)
- within the πN,$\pi\Xi$, $\pi\Sigma$ and $\pi\Lambda$ sector constraints on the LECs to NLO are calculated.
- novel low-energy theorems in the pion-hyperon sector are derived.