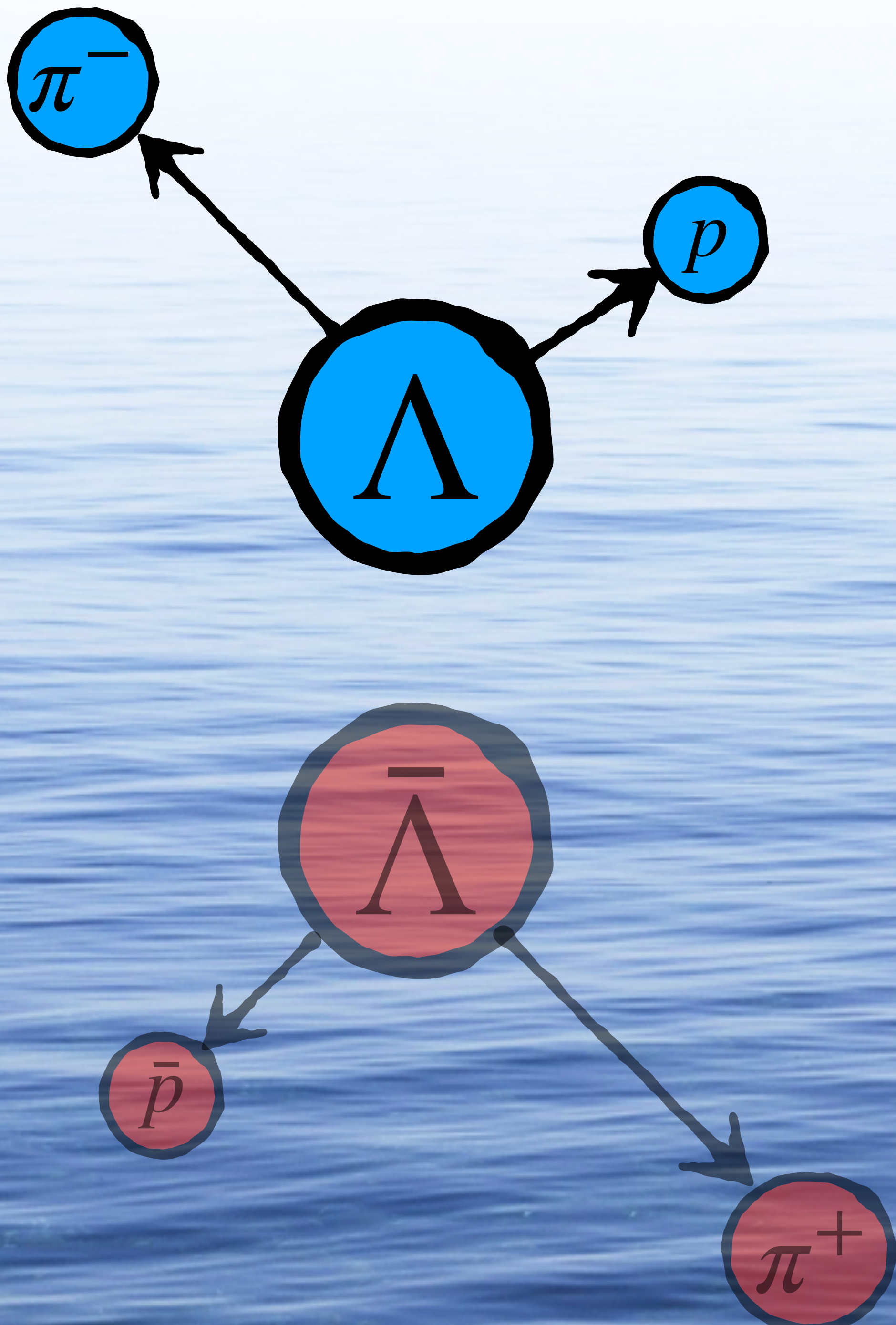
Independent determination of the Lambda decay parameter

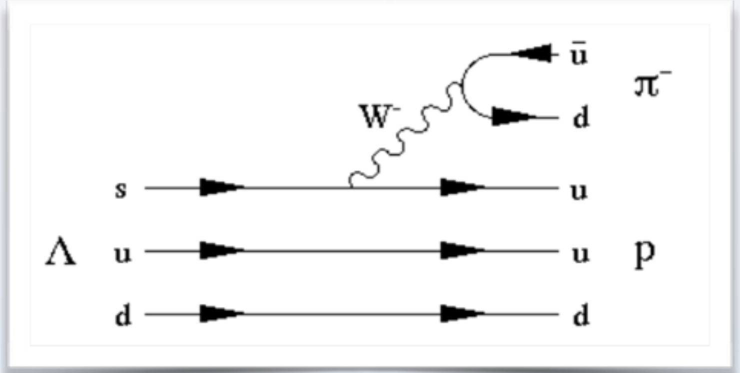
arXiv:1904.07616



Maxim Mai

with
D. G. Ireland
M. Döring
D. I. Glazier
J. Haidenbauer
R. Murray-Smith
D. Rönchen

- Λ **decays weakly to** $p\pi^-$



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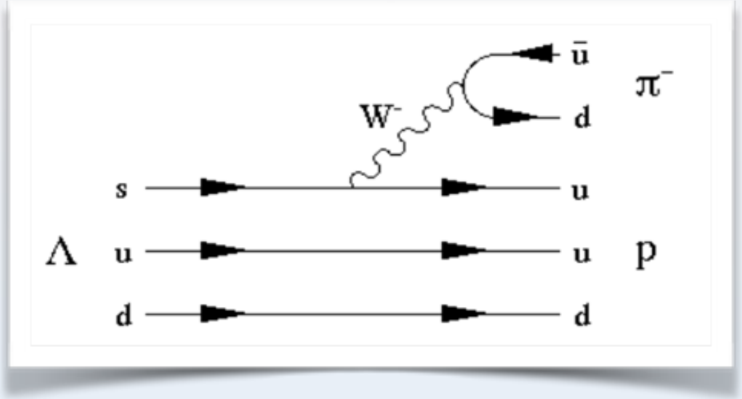

- The decay parameter: α_-

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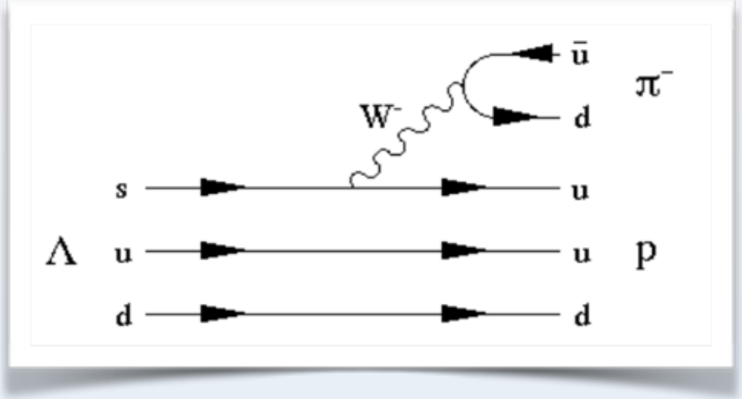
- **The decay parameter: α_-**

    - essential for many modern experiments



e.g. LEAR@CERN, STAR@BNL, ATLAS@CERN

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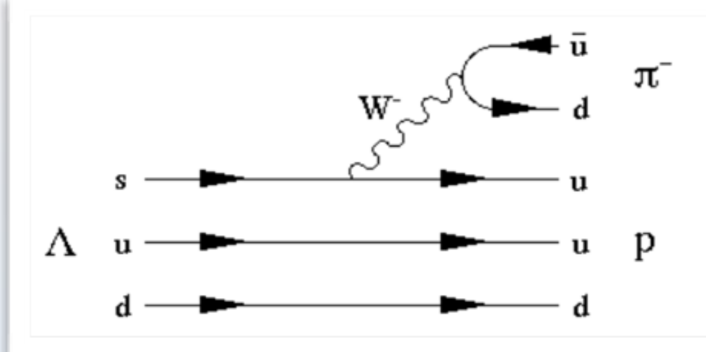

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Ω^- DECAY PARAMETERS

$\alpha(\Omega^-)\ \alpha_-(\Lambda)$ FOR $\Omega^- \to \Lambda K^-$

Some early results have been omitted.

VALUE	*EVTS*
0.0115 ± 0.0015	OUR AVERAGE

Ξ^0 DECAY PARAMETERS

See the ``Note on Baryon Decay Parameters'' i

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This is a product of the $\Xi^0 \to \Lambda\pi^0$ and $\Lambda \to p\pi^-$ asy

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PDG live (2019)

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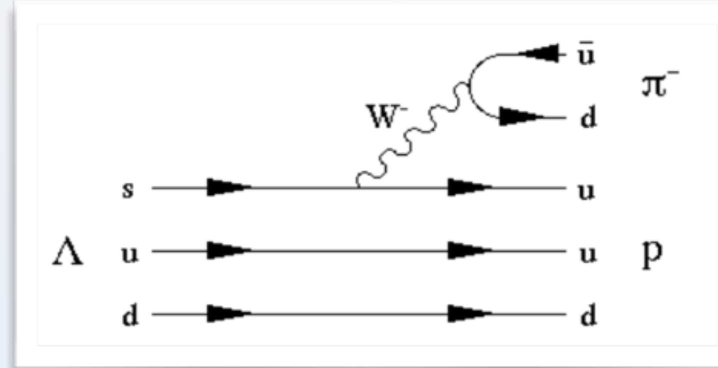

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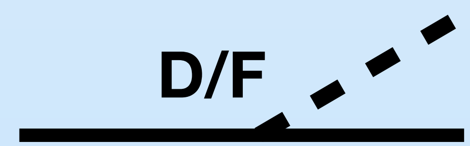

Holstein (2000)
Borasoy/Marco (2003)

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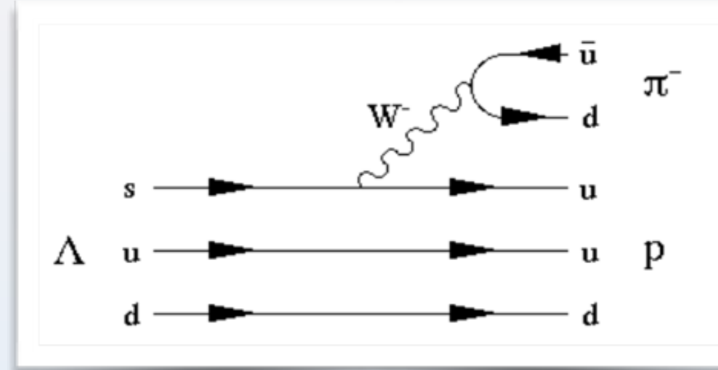

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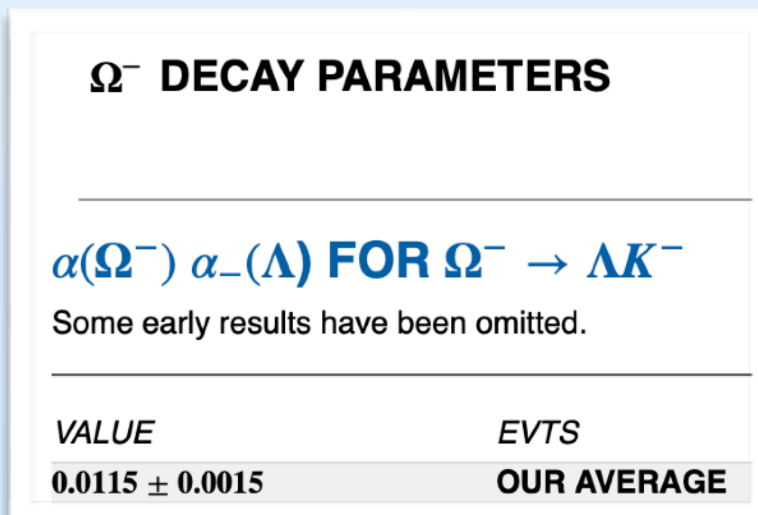

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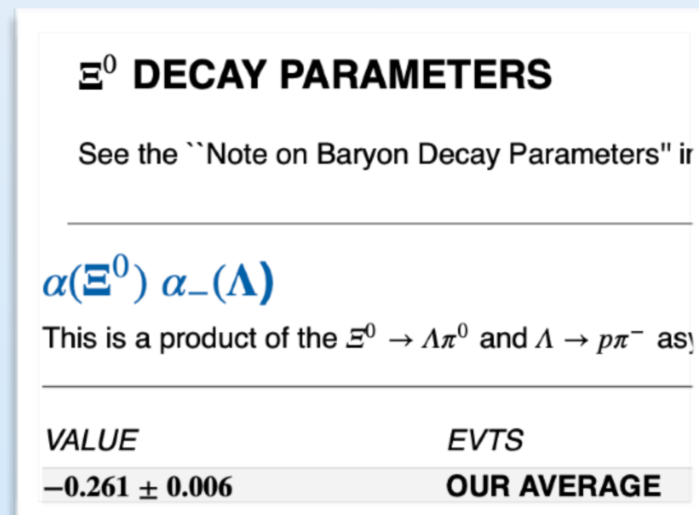

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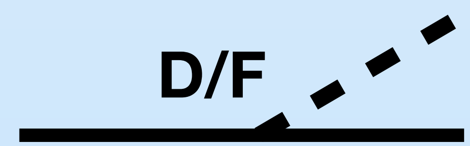

Holstein (2000)
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  - *essential for ($\gamma p \to K^+\Lambda$)*

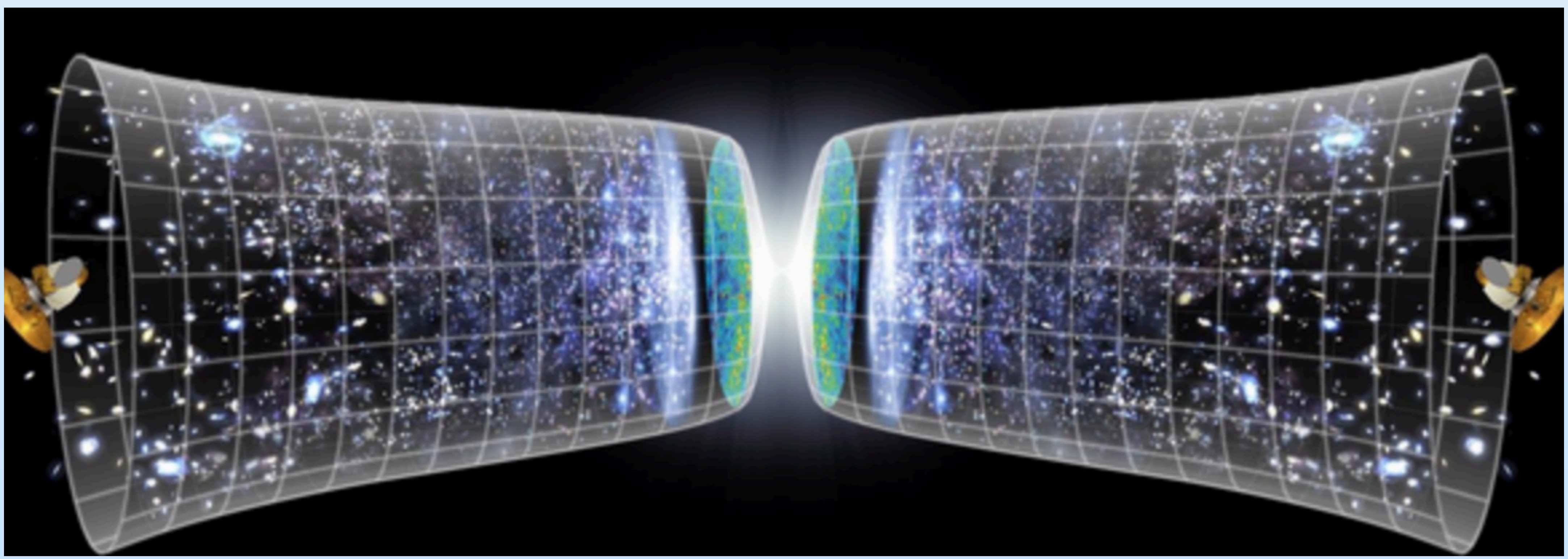
    new measurement by CLAS → **THIS TALK: ESTIMATE α_-**

- **CP-symmetry**

◉ **CP-symmetry**

◉ **Different parameters ➤ matter-antimatter asymmetry**

Sakharov (1967)



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α_+		α_-
-0.71 ±0.08	*PDG average*	0.642±0.013
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RECENT PRESS COVERAGE

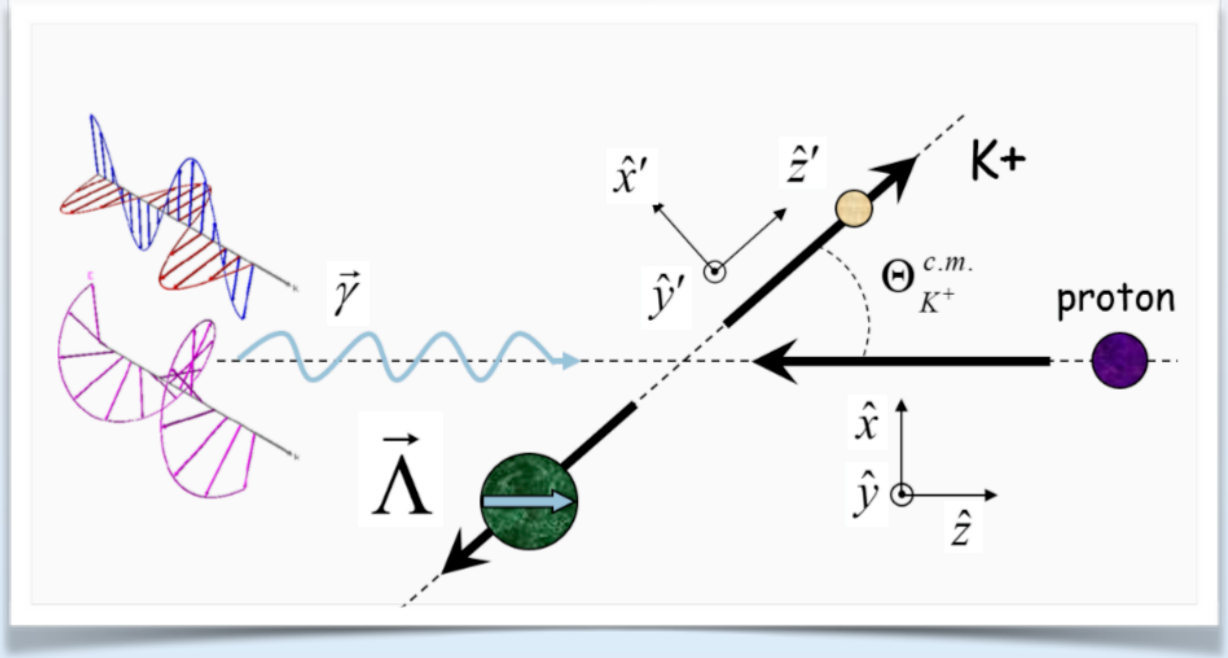
BESIII (2018) & this work

BESIII (2018)

DETERMINATION OF α_- FROM KAON PHOTOPRODUCTION

KAON PHOTOPRODUCTION

Experimental setup



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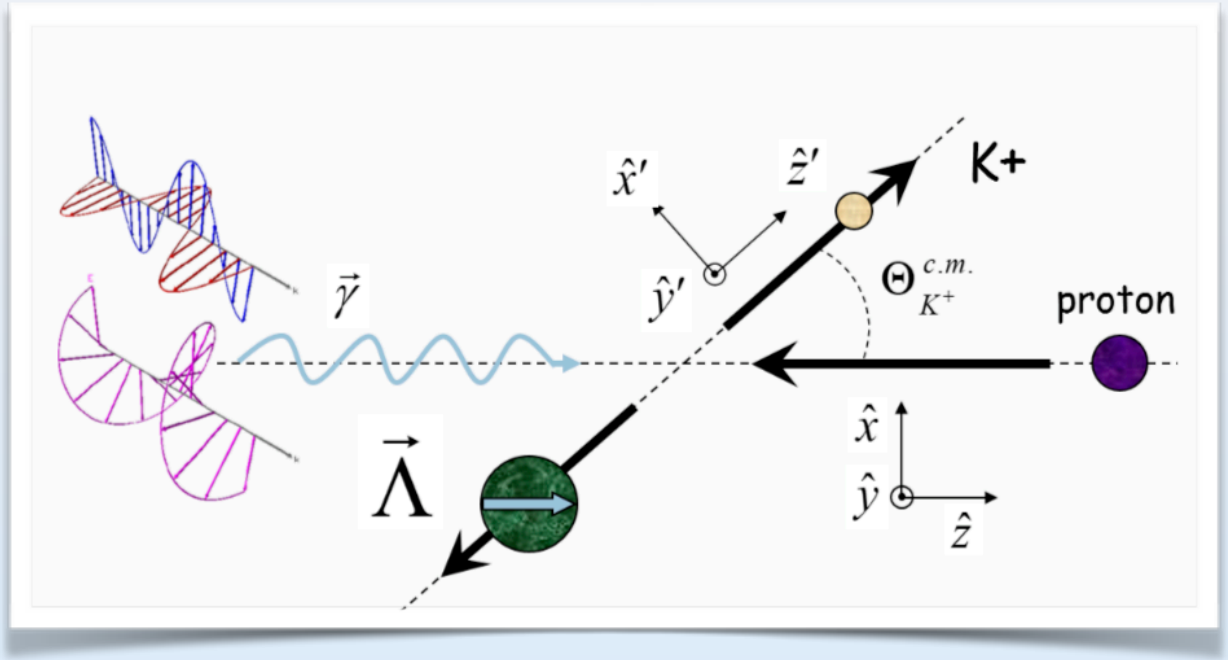

Intensity

$$\begin{aligned}(LP):\ 1&+\alpha_- \cos\theta_y \mathbf{P}\\ &-p_L^\gamma \cos 2\phi \mathbf{\Sigma}\\ &-\alpha_- p_L^\gamma \cos 2\phi \cos\theta_y \mathbf{T}\\ &-\alpha_- p_L^\gamma \sin 2\phi \cos\theta_x \mathbf{O_x}\\ &-\alpha_- p_L^\gamma \sin 2\phi \cos\theta_z \mathbf{O_z}\end{aligned}$$

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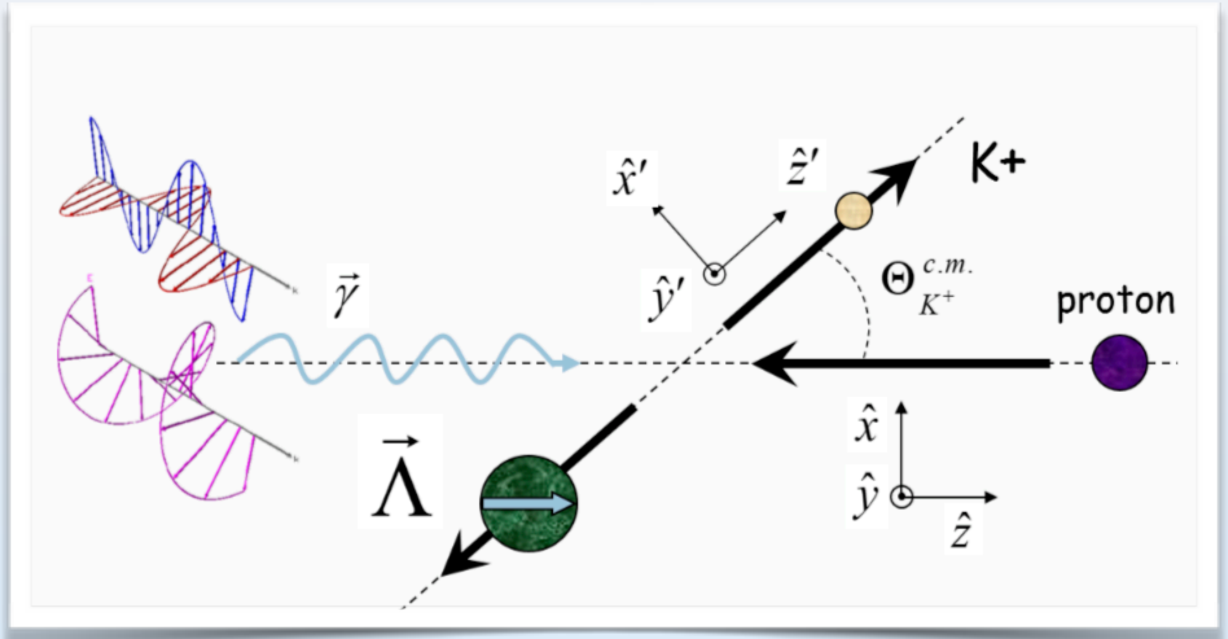

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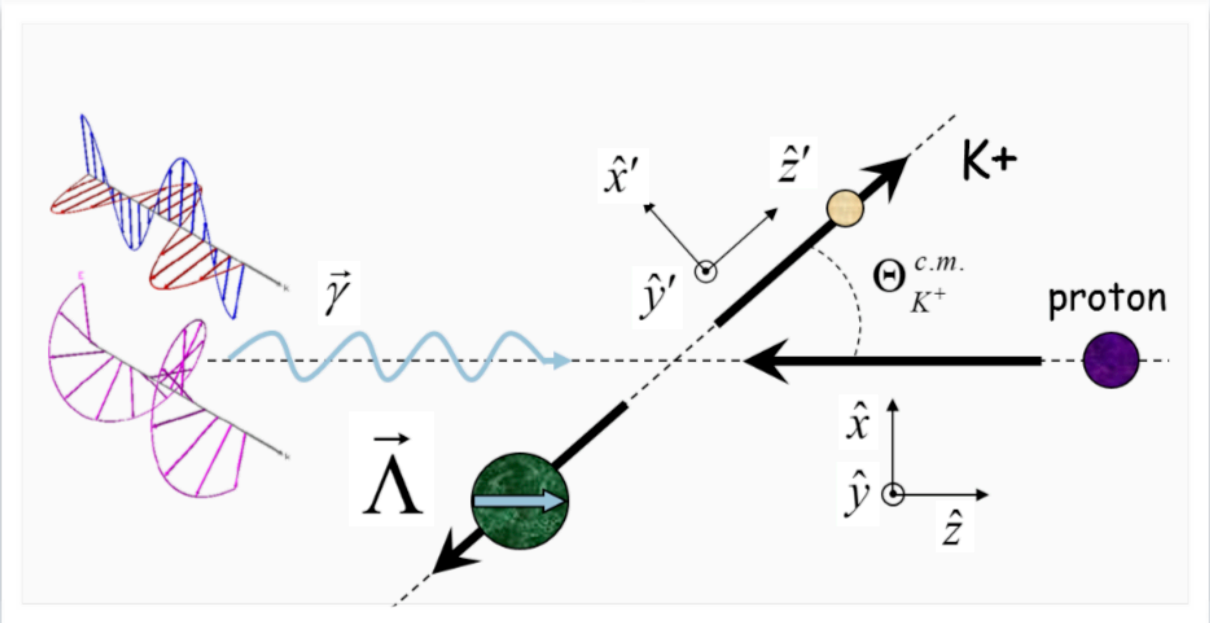

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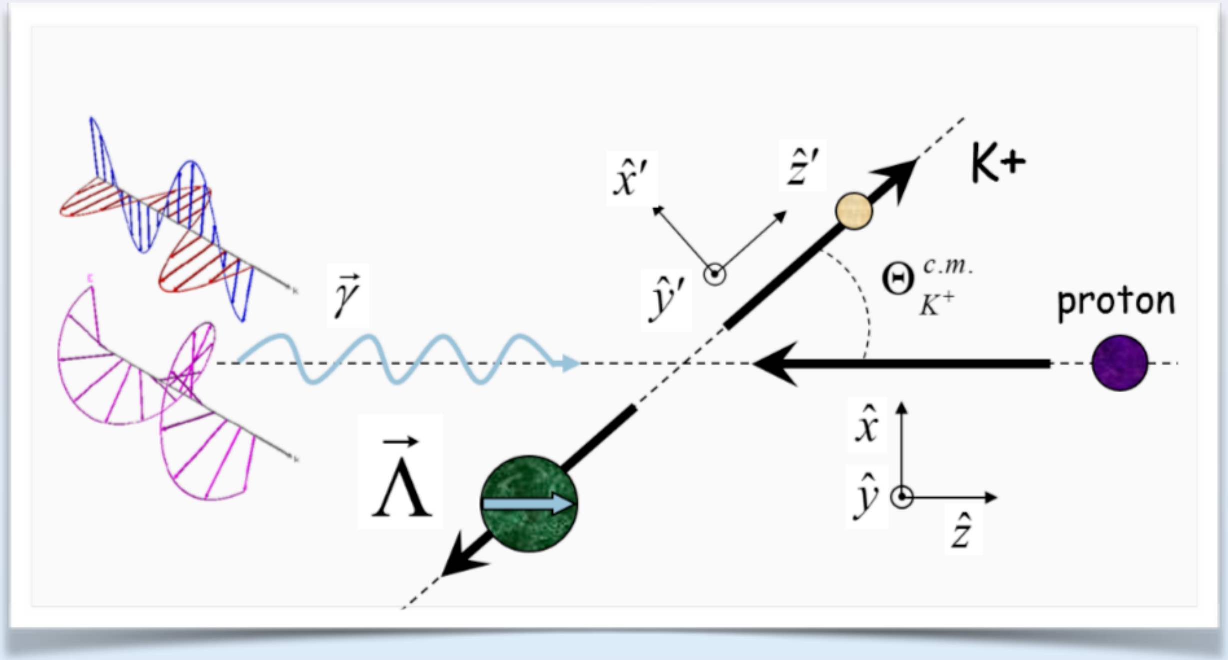

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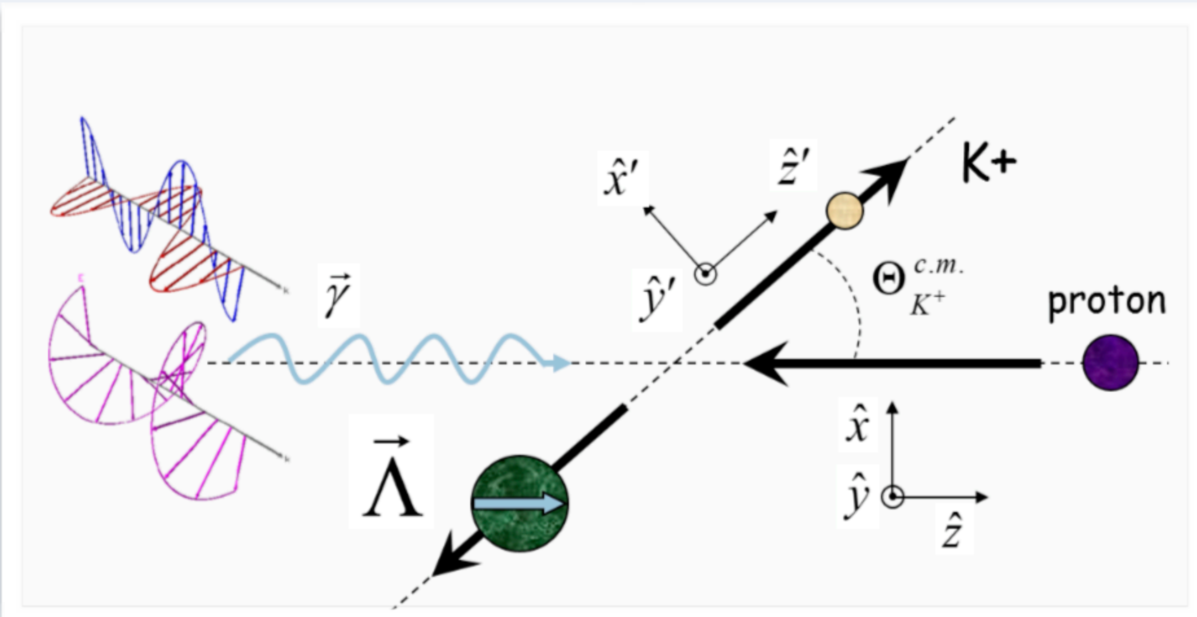

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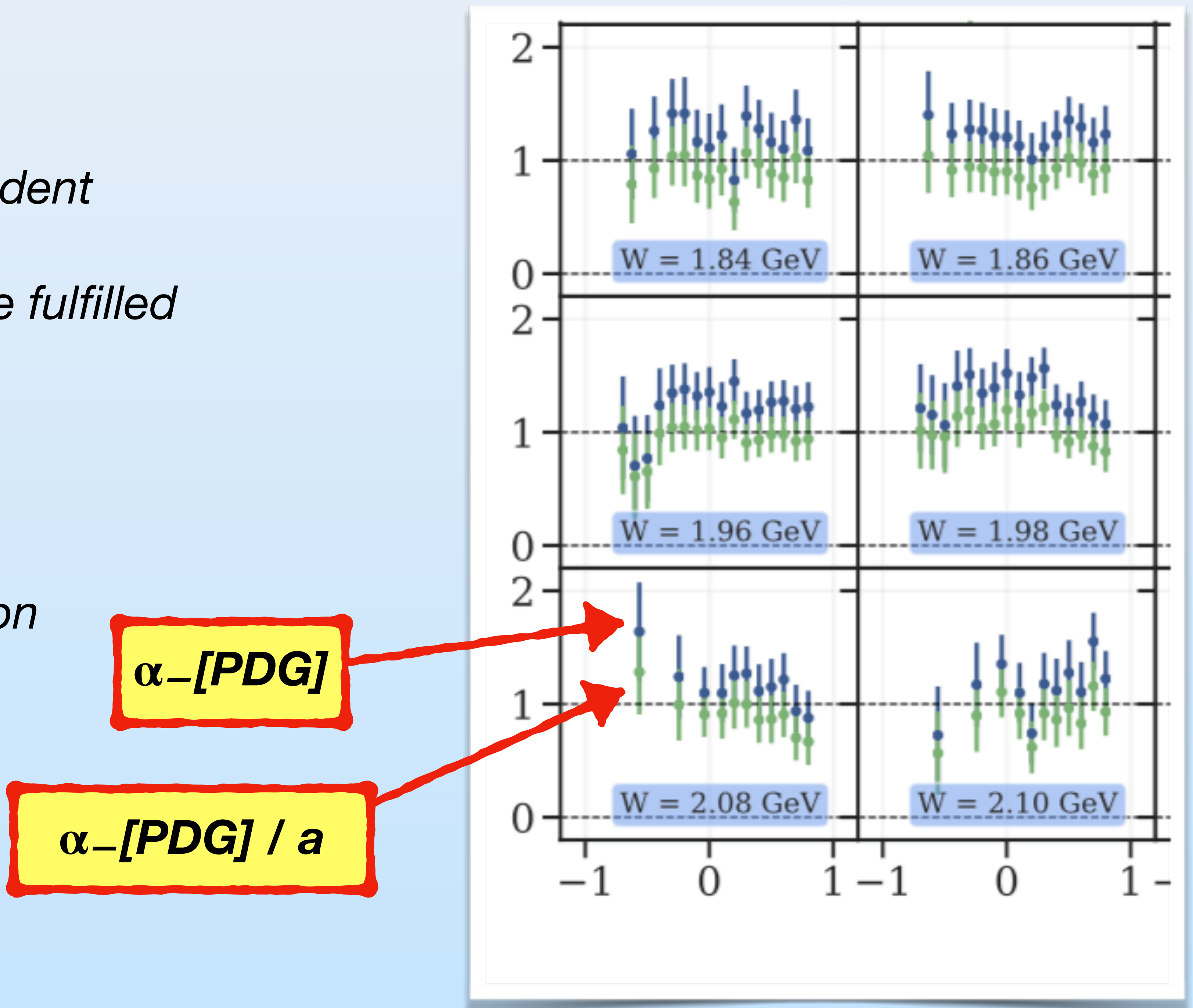
  - ⇒ *Observables are not independent*
  - ⇒ *determine α_- such that FI are fulfilled*
  - ⇒ *statistically non-trivial question*



STATISTICAL ANALYSIS

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- **Define random variables:**

$\mathcal{N}[\mu, \sigma^2]$ **from CLAS measurements**

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- ***FV, a, l, c* become random variables, but:**

A. Scaling:
- *Data and errors are scaled with a, l, c*
- *Normalization of* $PDF[\, a^2 \mathcal{O}^2 \,]$

d'Agostini (1994)

B. Most "observables" and scale parameters enter quadratically

& Is there a closed form of $PDF[\, \mathcal{F}_i \,]$*?*

Roe (2015)

STATISTICAL ANALYSIS

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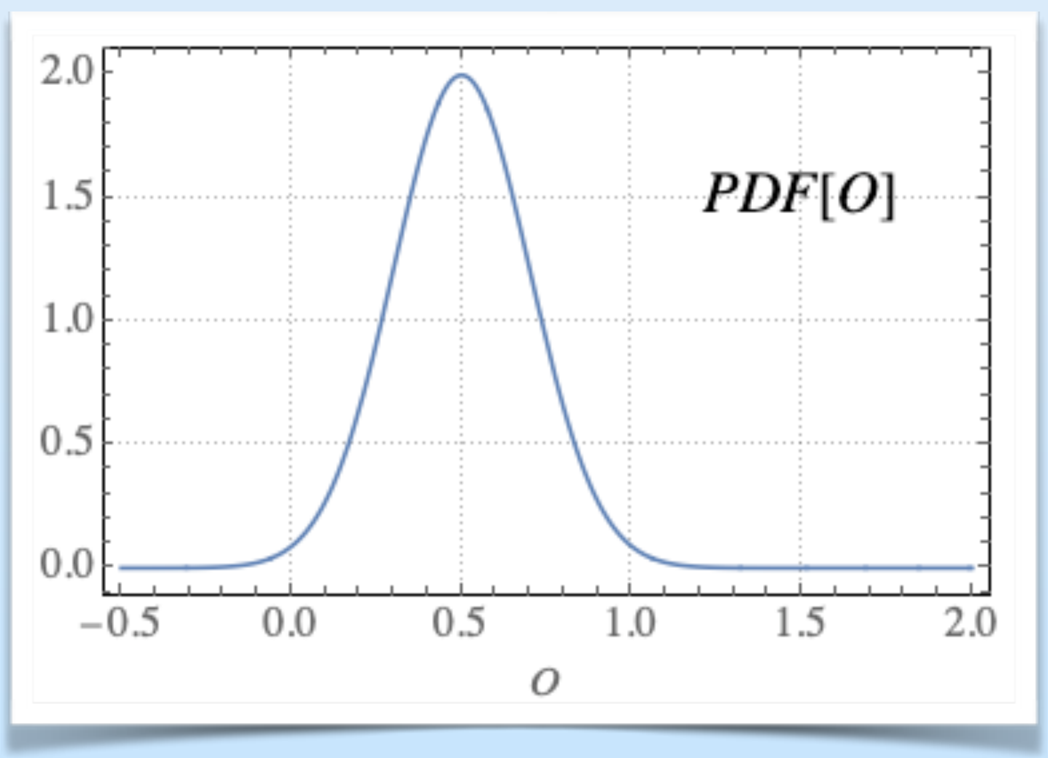

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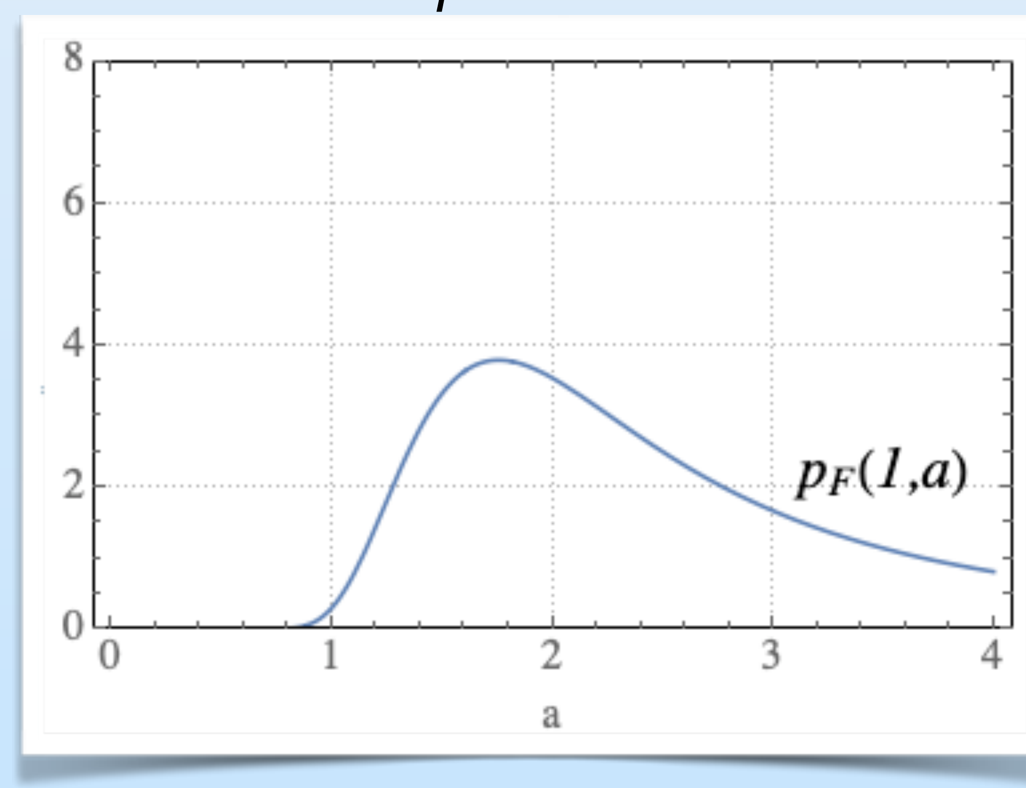

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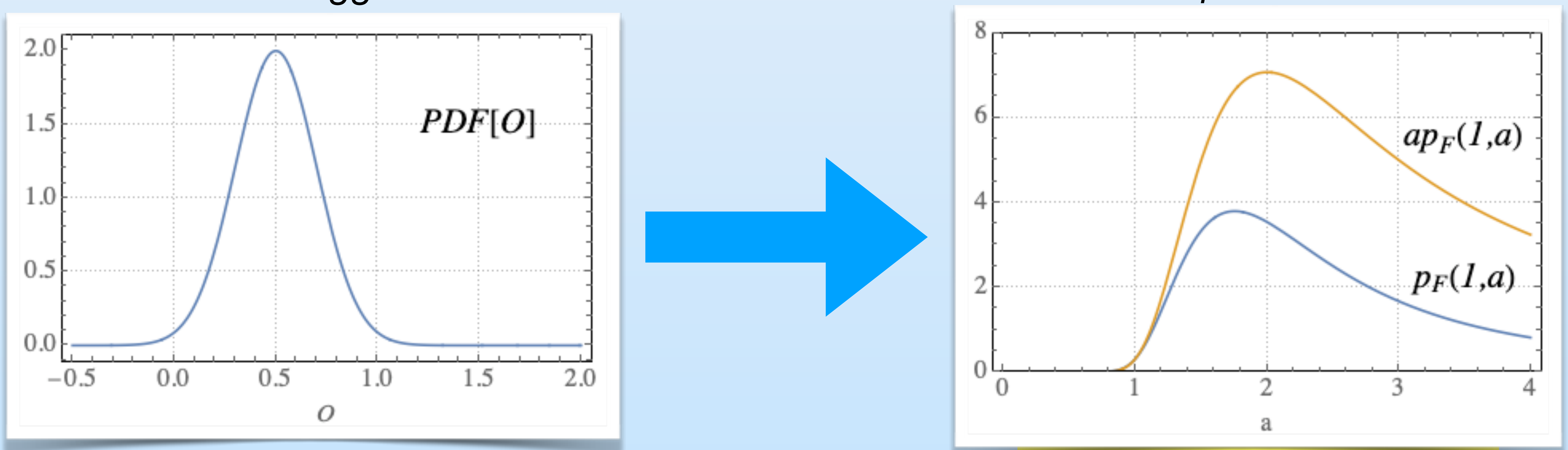

⟹ remove a-dependence from the normalization

STATISTICAL ANALYSIS

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non-central chi squared distribution

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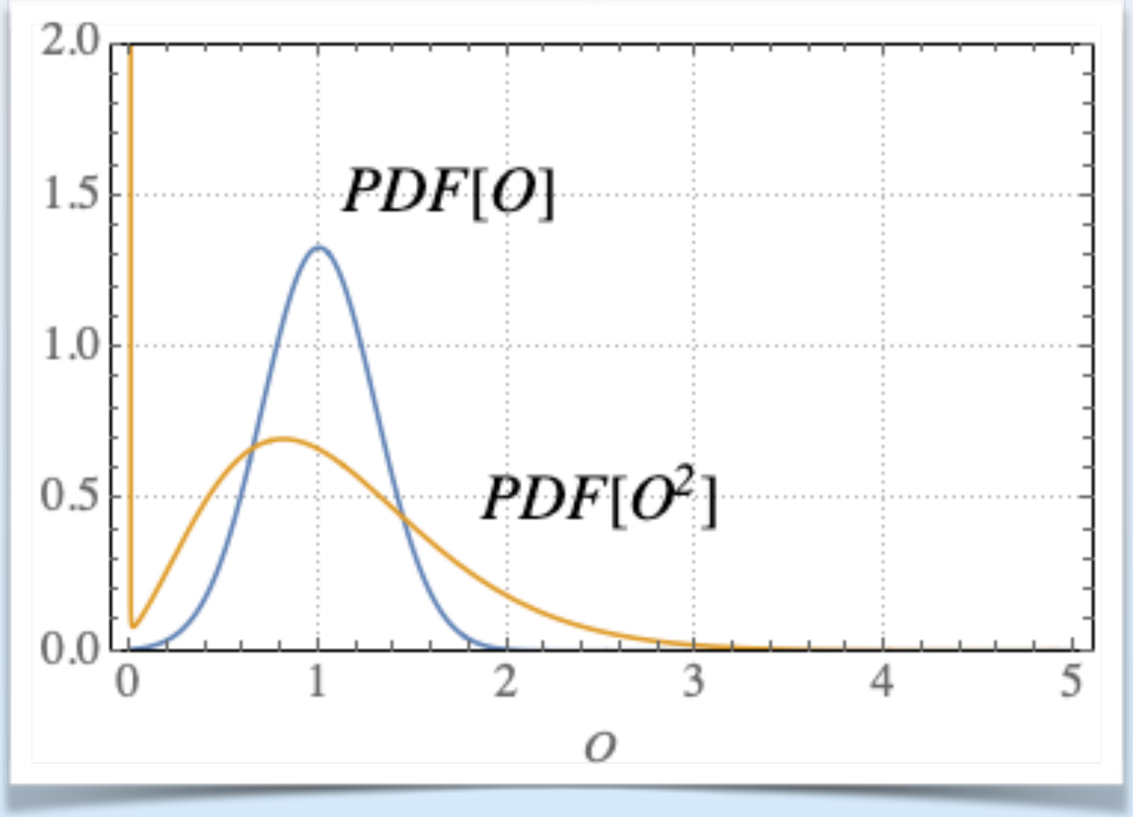

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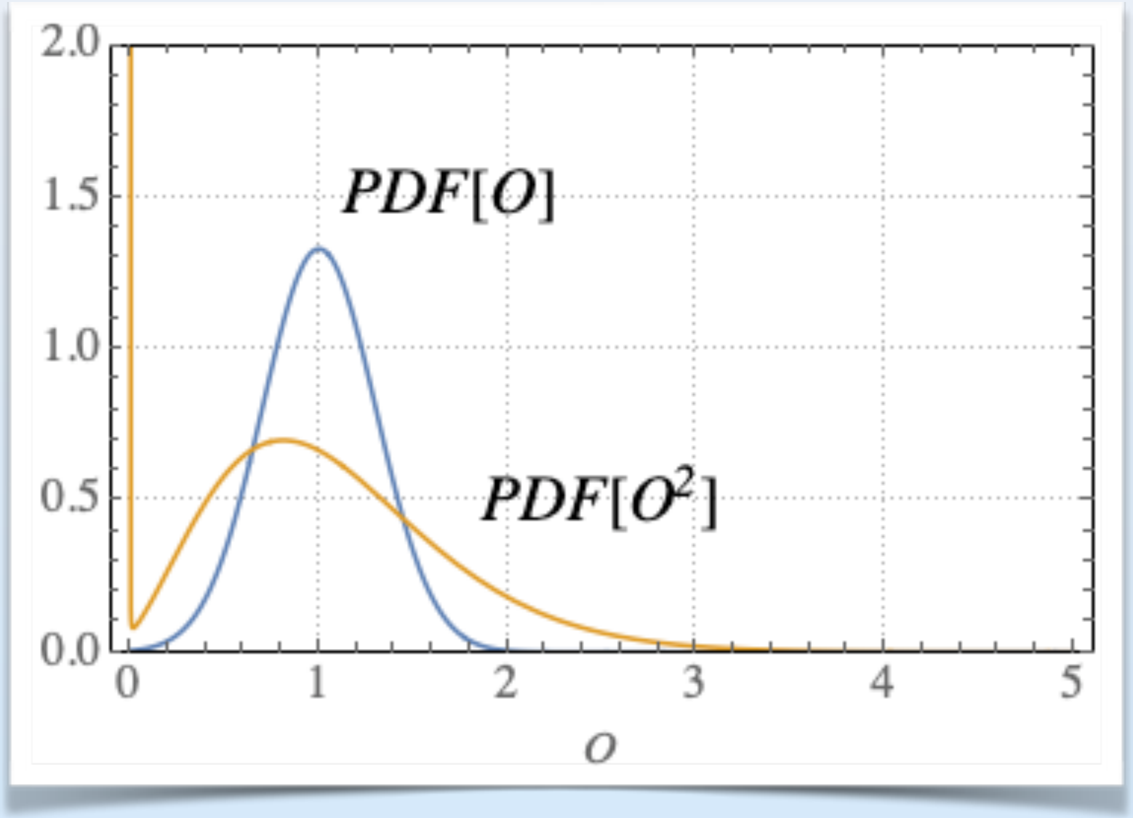


$\Longrightarrow$ Expectation value of Fierz identity ≠ 1

$$\Delta f := 1 + a^2\,\sigma_P^2 - a^2\,l^2\,\sigma_T^2 + \ldots$$

STATISTICAL ANALYSIS

- **Combined likelihood function:**

$$\prod_{kin.\ points} p^{(1)}(f_i^{(1)} = \Delta f_i \,|\, a, l, c) \cdot p^{(2)}(f_i^{(2)} = 0 \,|\, a, l, c)$$

- *likelihood for a, l, c to fulfill F.I.*
- ***attention: Δf ≠ 1***

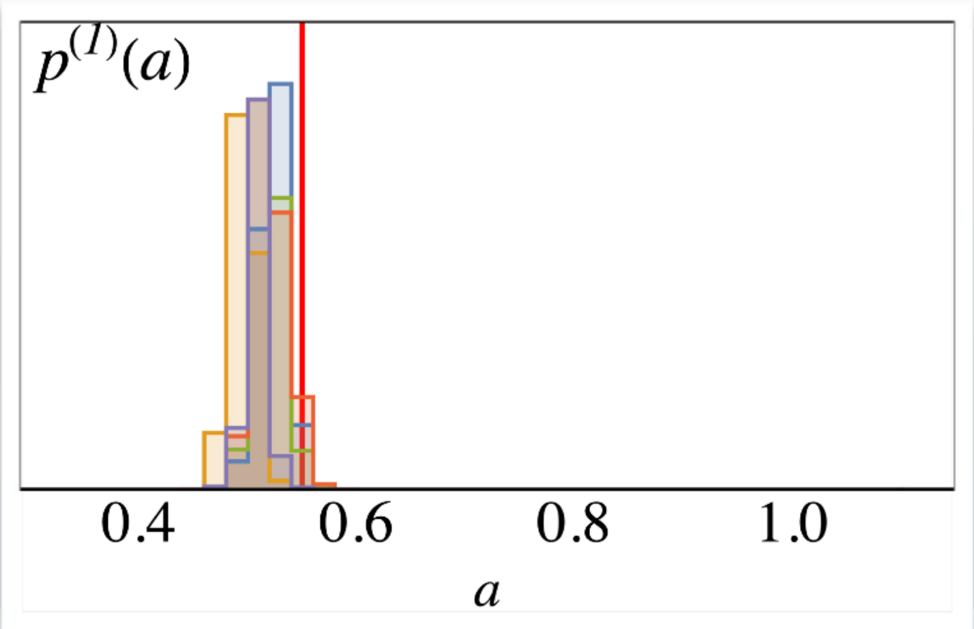
$$\mathscr{P}(a, l, c \,|\, \{\mathscr{O}\}) \quad \propto \quad \mathscr{P}(\{\mathscr{O}\} \,|\, a, l, c) \cdot \mathscr{P}(l, c)$$

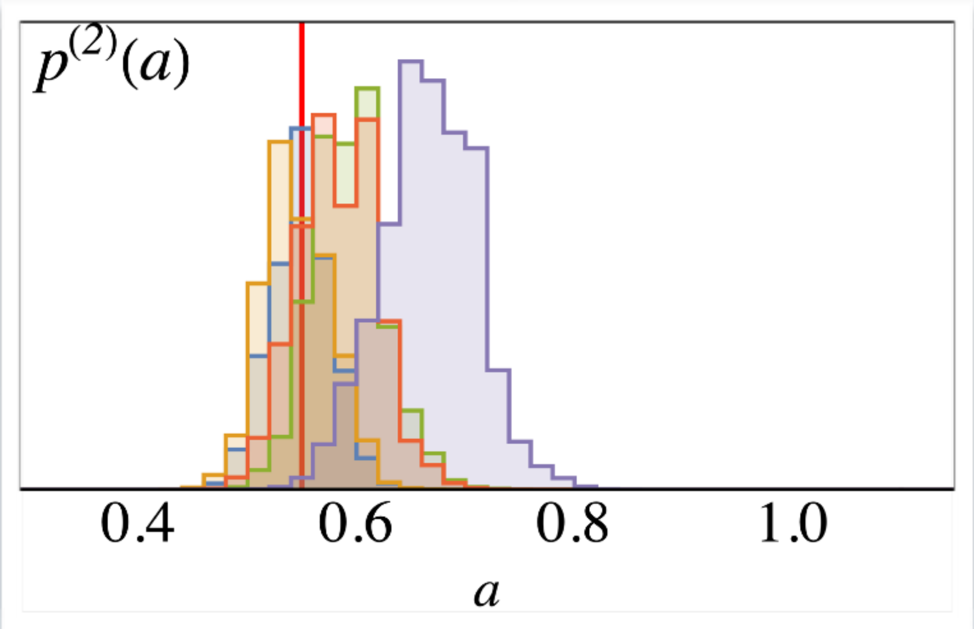
- *prior knowledge of calibration parameters*

  $\delta_l = 0.05$, $\delta_c = 0.02$
- *test various forms ➤ systematics*

STATISTICAL ANALYSIS

- **Ultimately— blind test on synthetic data**



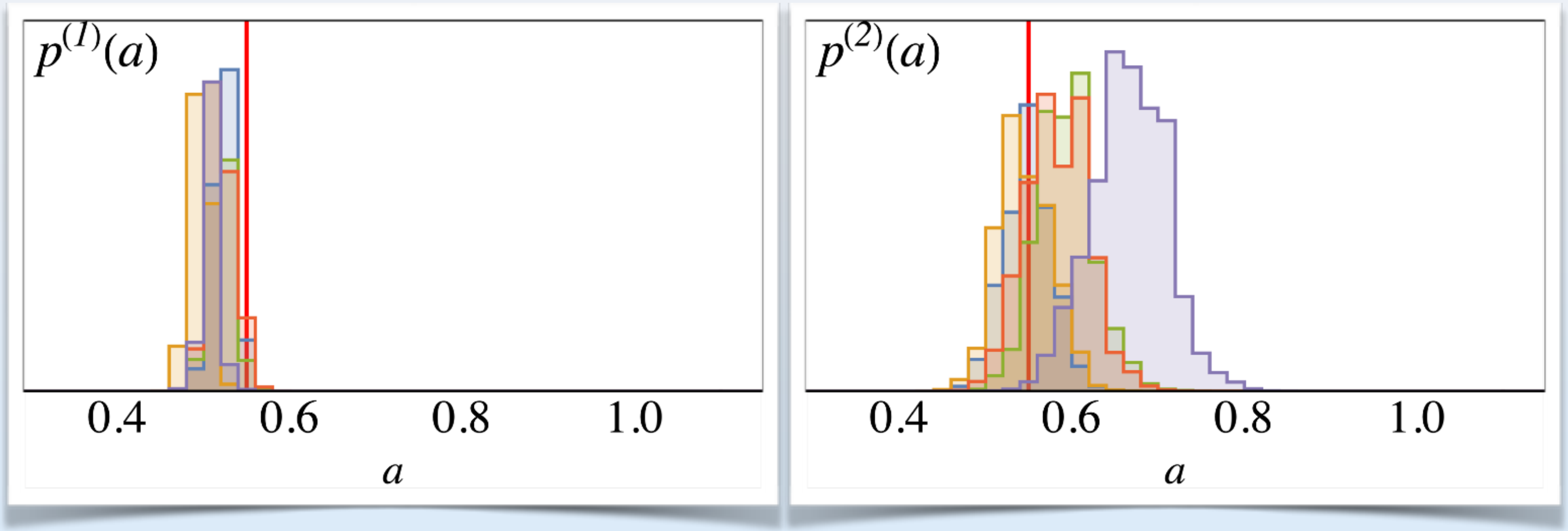


re-sampling test of both Fierz identities:

- *300 kin. points*
- *200 000 samples*
- ***a_{test} = 0.55***

STATISTICAL ANALYSIS

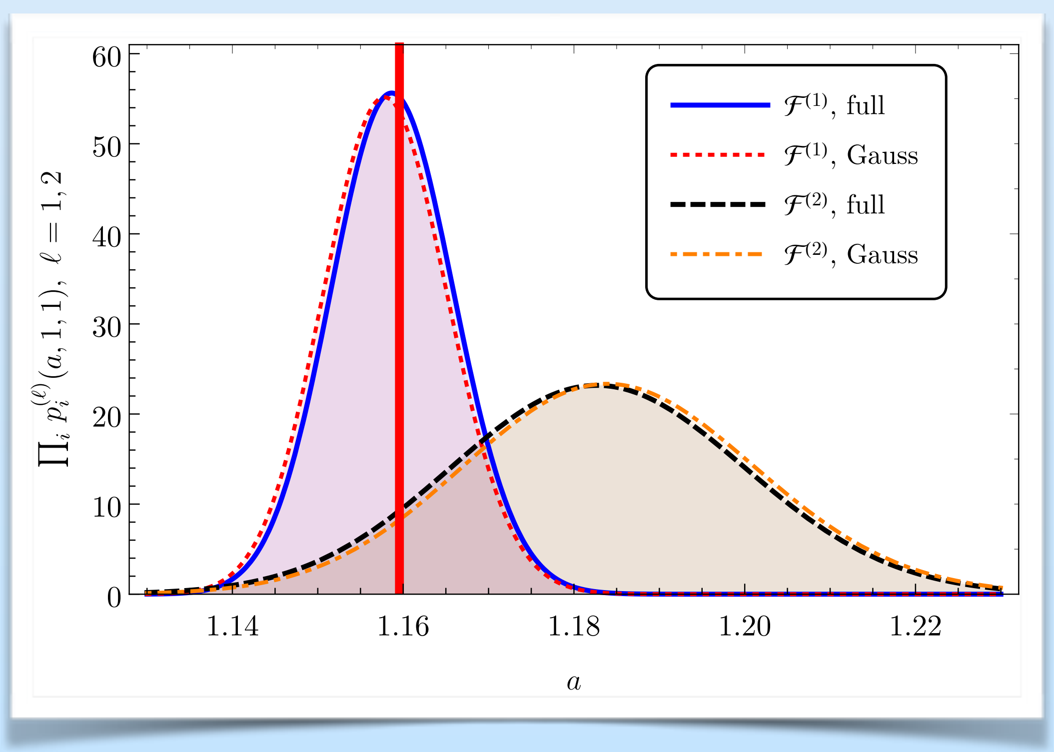
◉ **Ultimately— blind test on synthetic data**



re-sampling test of both Fierz identities:

- *300 kin. points*
- *200 000 samples*
- ***a_{test} = 0.55***

◉ **Or… blind test on model data (JuBo 2019)**



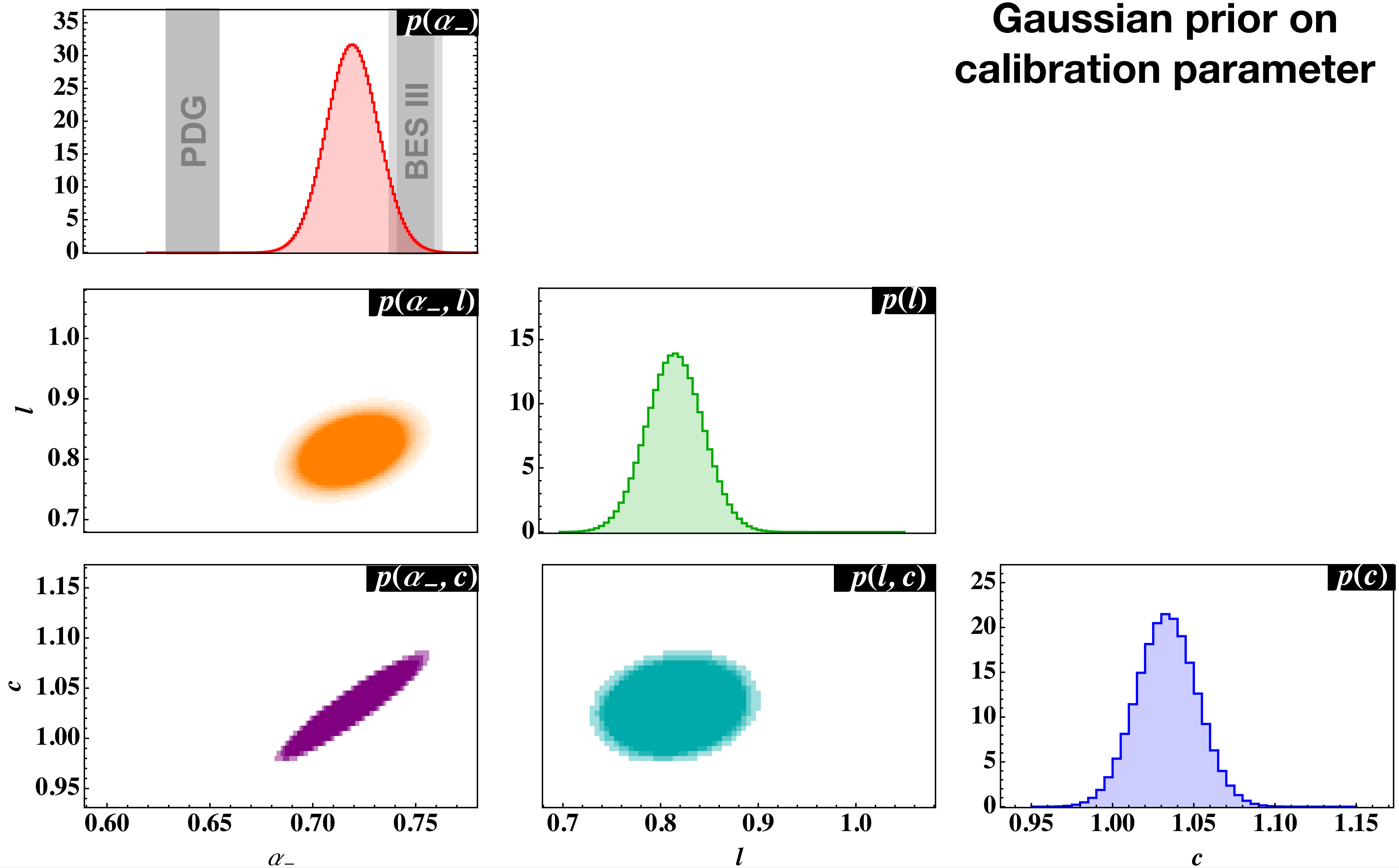
- *500 sets of observables from JuBonn model*

Rönchen et al 2014

- *Wrong α_- is dialed in*

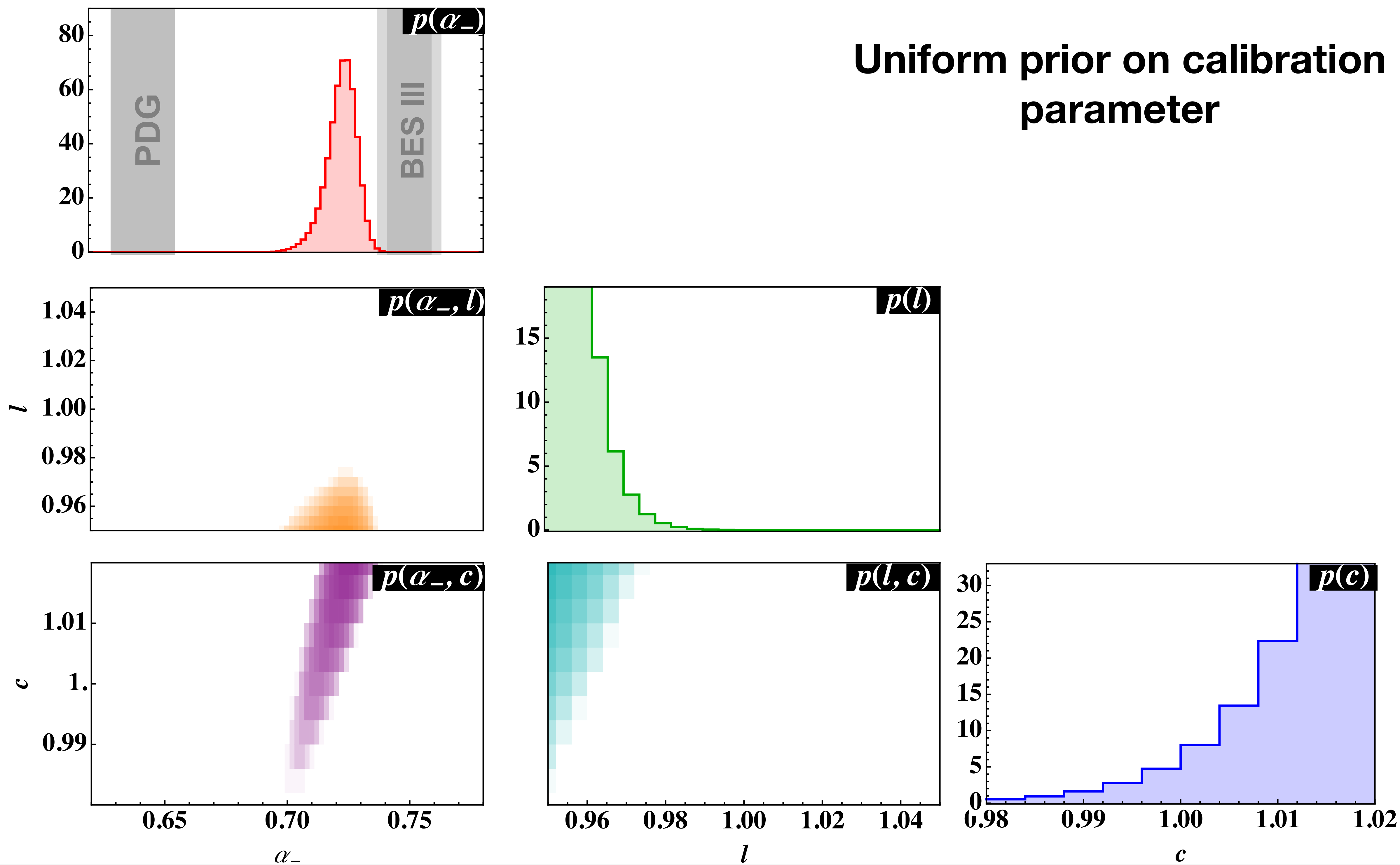
RESULTS

RESULTS



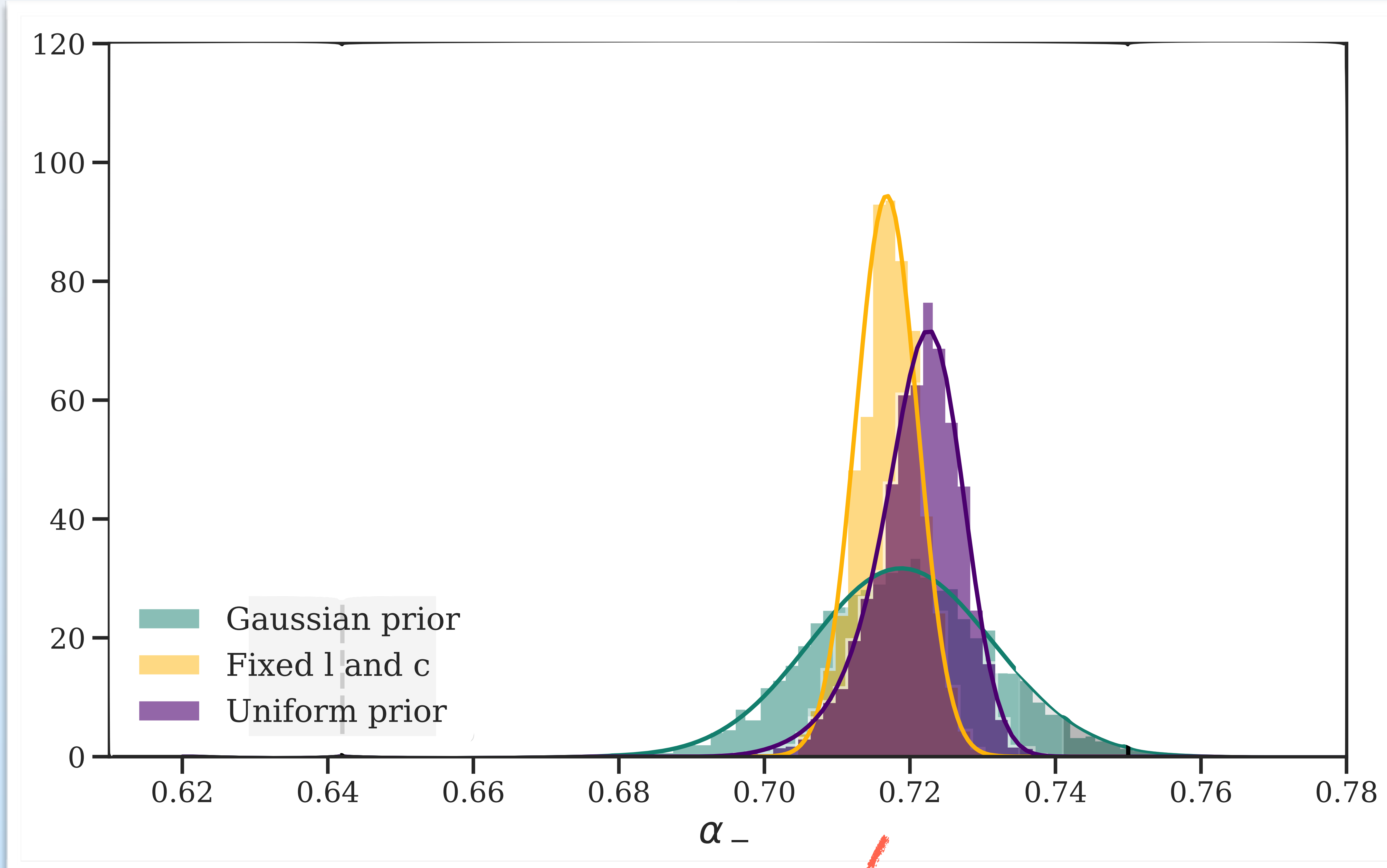
Gaussian prior on calibration parameter

RESULTS



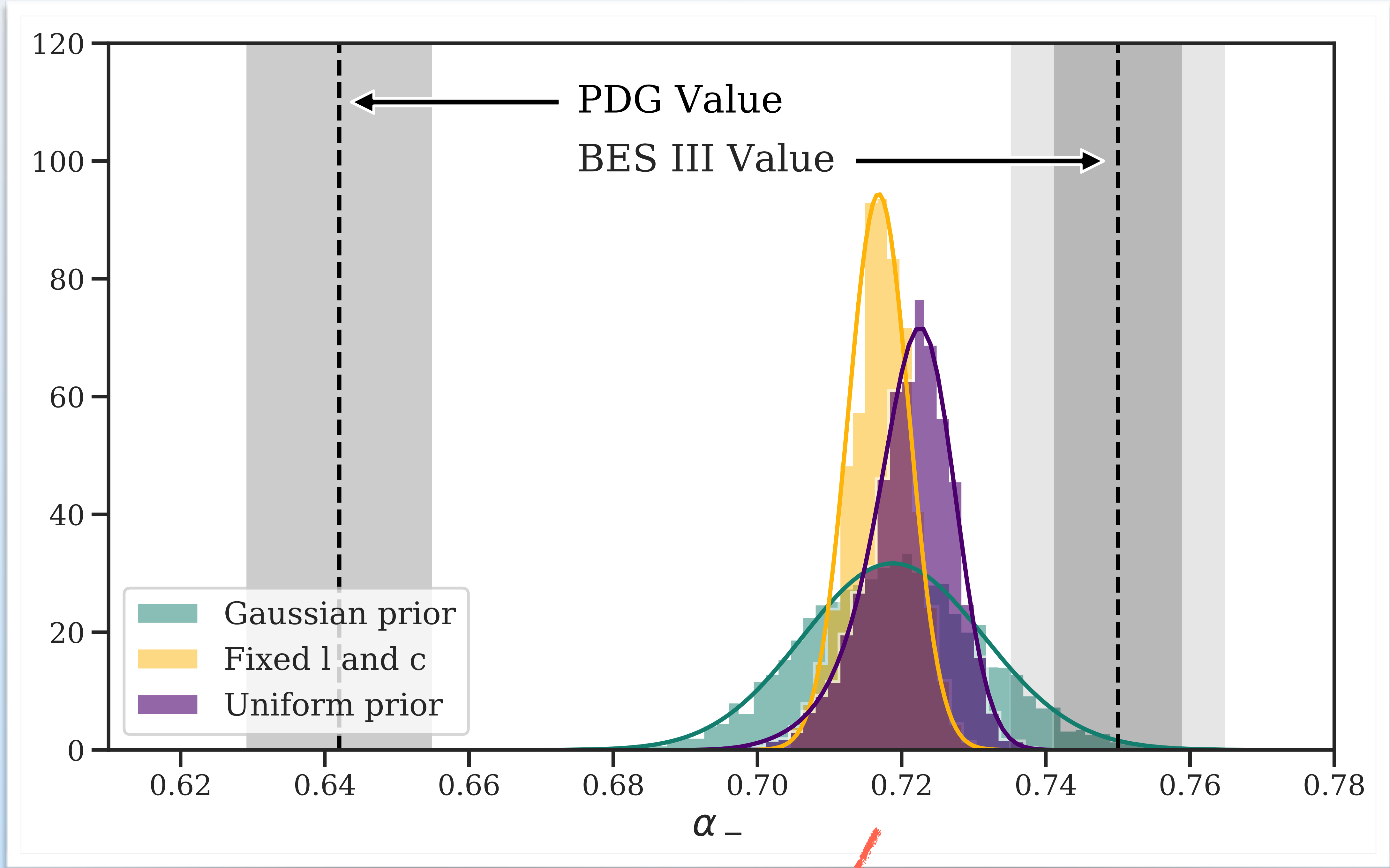
Uniform prior on calibration parameter

RESULTS



$$\alpha_- = 0.721 \pm 0.006 \pm 0.005$$

RESULTS



$\alpha_{-} = 0.721 \pm 0.006 \pm 0.005$

SUMMARY

- **Kaon photoproduction data contains information on Lambda decay parameter: α_-**

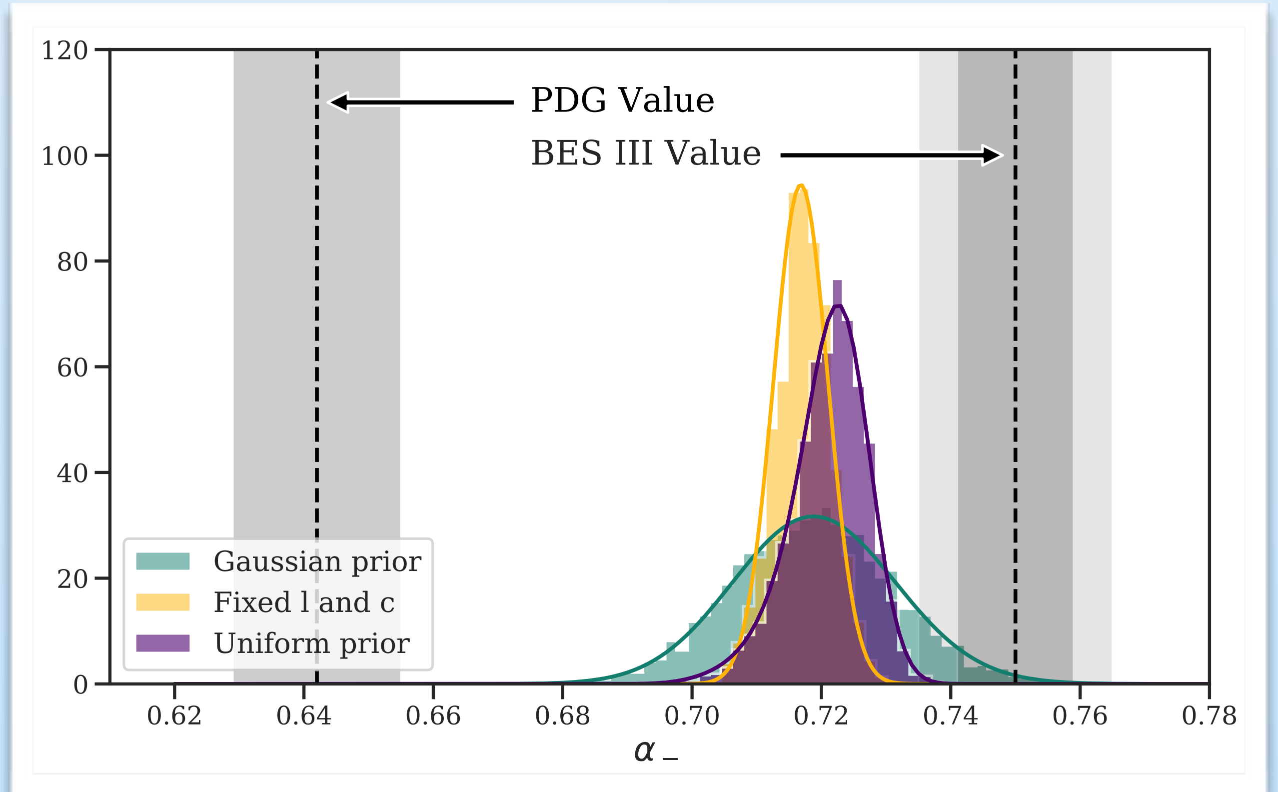
- **Fierz identities allow for an extraction of α_-**

- **Careful statistical interpretation is in order:**

  - Scaling bias (similar to d'Agostini bias) *- Non-linearities* *- ...*

  multiple simulations and tests ➤ "bias-free" procedure

- **Final result is close to the recent BES III value**



SPARES

- Ultimately— blind test on model data (JuBo 2019)

SPARES

Observable (# data points)	χ^2/n (Refits) $\alpha_- = 0.642$	0.75	0.721
$d\sigma/d\Omega$ (421) [15]	1.11	1.03	0.95
Σ (314) [17]	2.55	2.61	2.56
T (314) [17]	1.75	1.74	1.69
P (410) [15]	1.84	1.66	1.62
C_x (82) [16]	2.15	1.72	1.34
C_z (85) [16]	1.58	1.83	1.62
O_x (314) [17]	1.44	1.53	1.51
O_z (314) [17]	1.34	1.58	1.49
all (2254)	1.67	1.66	1.59

TABLE II. χ^2/data point of the Jülich-Bonn refits for different values of α_-. The value of $\alpha_- = \alpha_-^{\mathrm{PDG}} = 0.642$ corresponds to the refit to unscaled data, $\alpha_- = 0.75$ correponds to the BES-III result [1] and $\alpha_- = 0.721$ uses the data-driven result of this study as input for the refit.