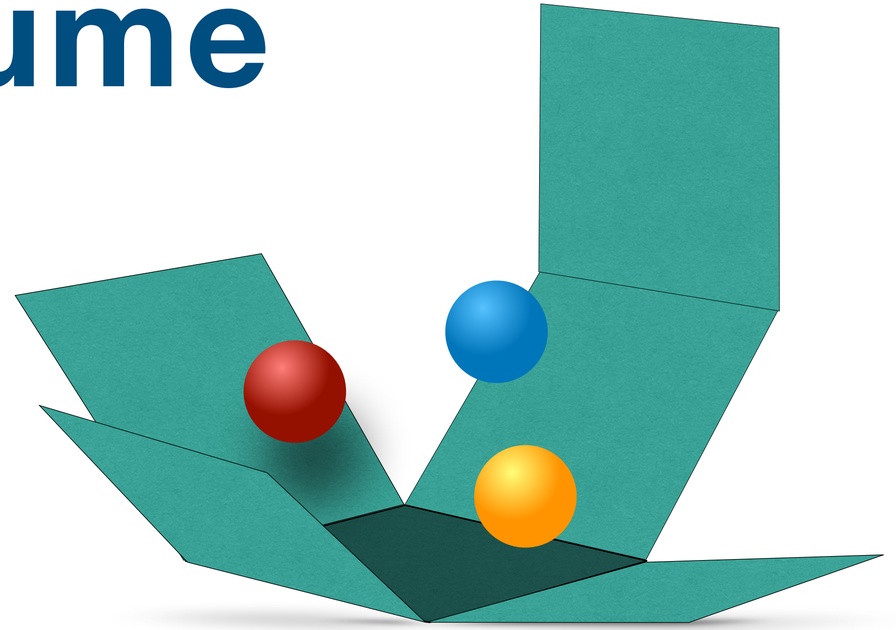
# Multihadron resonances in a finite volume



*Maxim Mai*

University Bonn

The George Washington University (adjunct)

# RESONANCES

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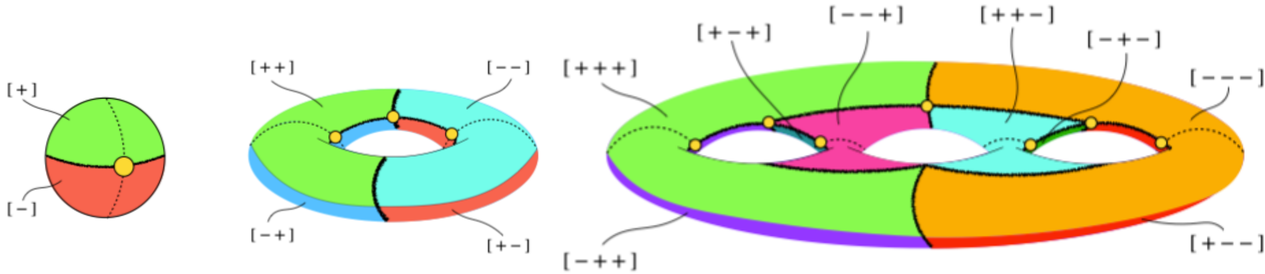



MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.

# RESONANCES

Hadron spectrum:

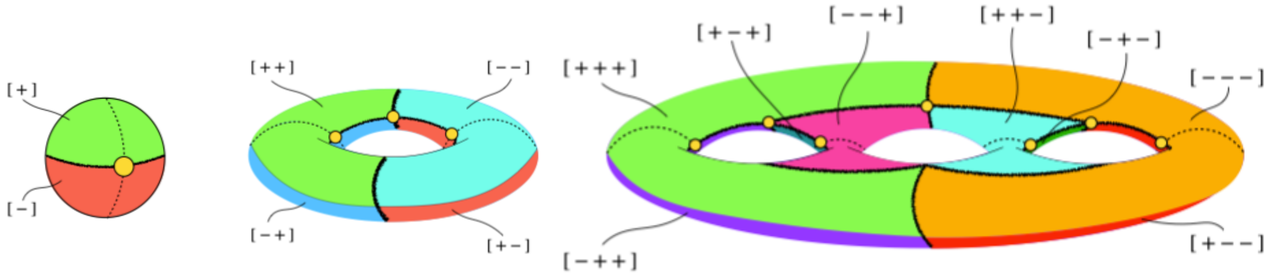
- PDG: ~100(50) excited meson(baryon) states (****)
- reaction-independent (*universal*) parameters:

  > *poles on the Riemann Surface*

- physical information ( **E** $\in \mathbb{R}$ )

  > *experiment*

  > *theory -- Lattice QCD*

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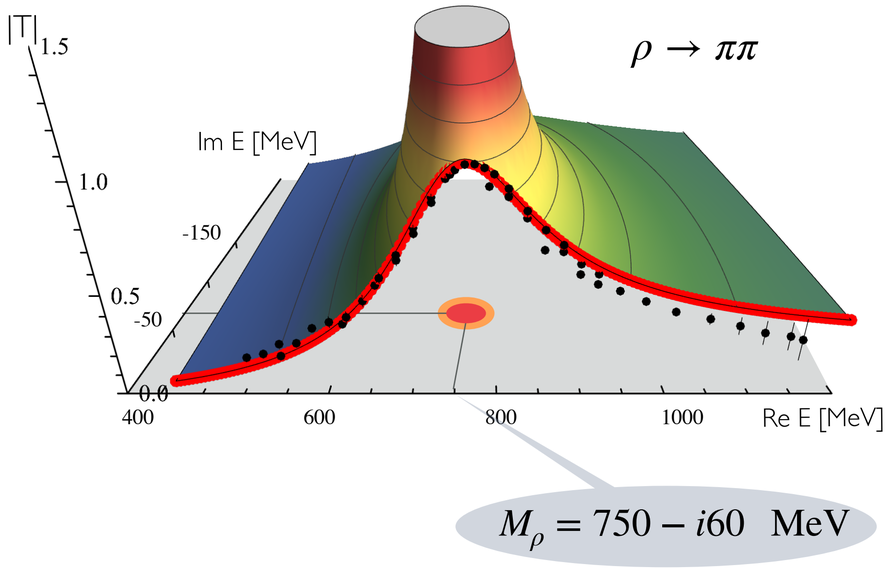


Data: Estabrooks et al. NPB 79 (1974); Protopopescu et al. PRD 7 (1973);

# LATTICE HADRON SPECT.

Finite-volume spectrum

- complementary scenarios:

  > *large pion mass*

  > variating dynamical flavours, ...

- requires mapping to infinite volume quantities (transition amplitudes)

  > *Quantization Condition*

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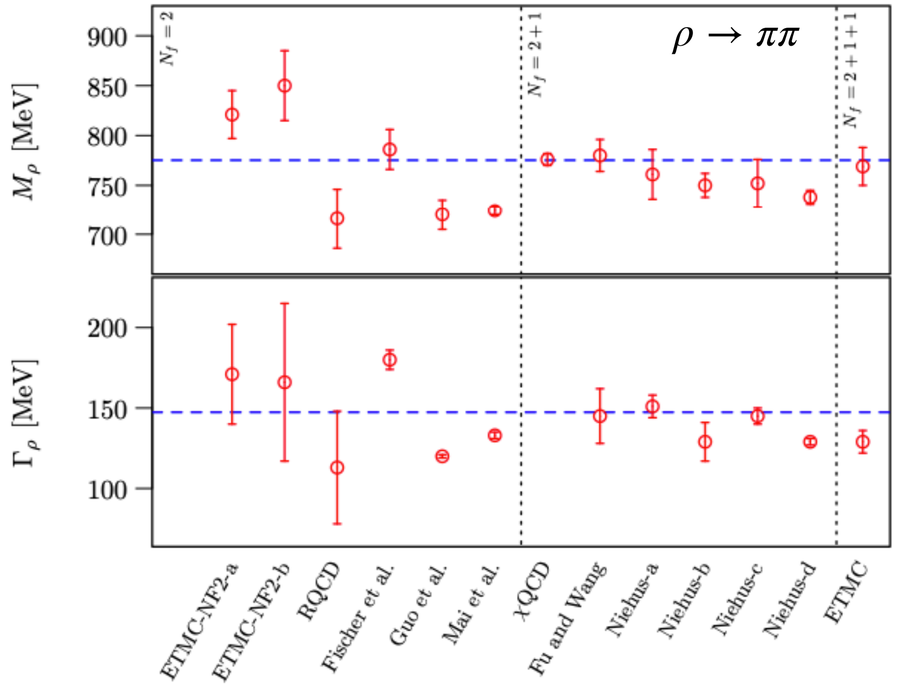
  - > *Quantization Condition*

Many two-body cases are studied




Reviews:
MM/Meißner/Urbach 2206.01477 review in Phys. Rept.
Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018)

# LATTICE HADRON SPECT.

## Current frontier: three-body dynamics from LQCD

- three-body Quantization Conditions[1]

1) REVIEWS: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021)
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3-body force

2-body interaction

one-particle exchange

2-body self-energy

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  > many perturbatively interacting systems are studied[2]

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# RESONANT 3-BODY SYSTEMS

- related to many physical systems

  > *$a_1(1260)$, $N^*(1440)$, $X(3872)$, ...*

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- challenges:

  > *finite-volume spectrum*

  > *quantization condition adaptation*

  > *pole-extraction in the infinite volume*

# RESONANCES FROM LQCD: #1

**Example[1]: $a_1(1260)$ $I^G(J^{PC}) = 1^-(1^{++})$**

- finite-volume spectrum @ $M_\pi = 224$ MeV
- FVU update:

  > new d.o.f.: spin -- helicity index $\lambda$

$$0 = \det \Big[ \; 2L^3 E_{\mathbf{p}} \left( \tilde{K}_2^{-1} - \Sigma_2^L \right) - B - C \; \Big]^{\Lambda = T_{1g}}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

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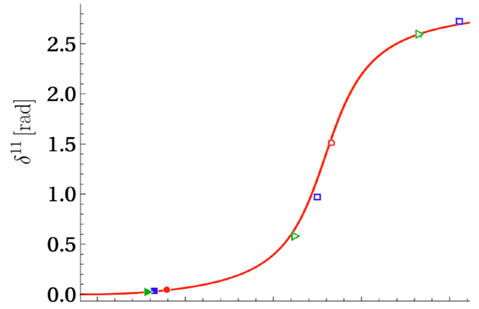
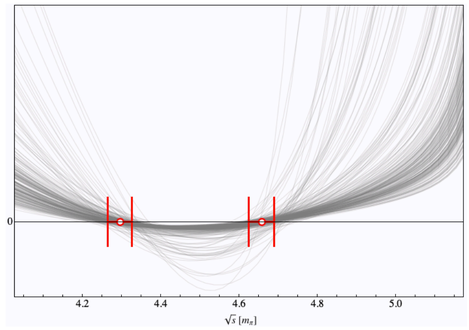
- attractive 2-body interaction $\pi\pi^{I=1}$ ($\rho$)




- fit to GWQCD levels[2]

$$\frac{c_0}{s - m_{a_1}^2} + c_1 + c_2 s$$

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2) Guo et al PRD94 (2016); MM et al. PRD100 (2019)

# RESONANCES FROM LQCD: #1

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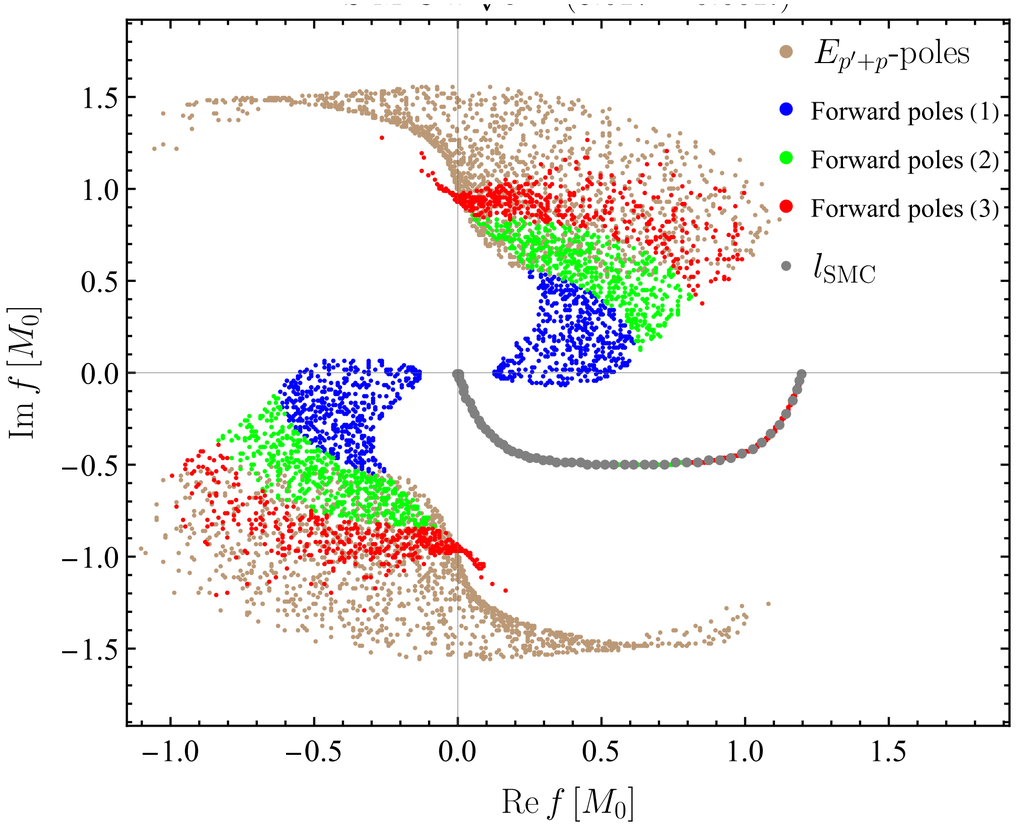
- Infinite-volume amplitude (FVU ➠ IVU)

  > solution for $s \in \mathbb{C}$ per complex momentum contour (SMC)

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

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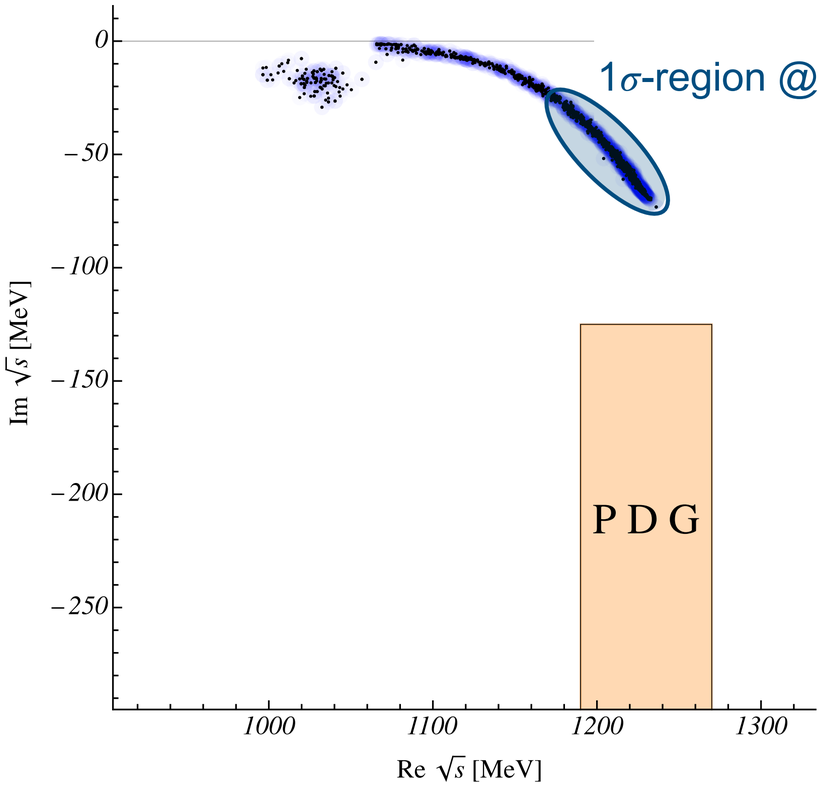


1σ-region @ $m_\pi$=224 MeV

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# RESONANCES FROM LQCD: #2

## Example: $\varphi^4$ theory with explicit three-body state[1]

$$S = \int dx \sum_{i=0,1} \left[ \frac{1}{2} \partial_\mu \varphi_i^\dagger \partial_\mu \varphi_i + \frac{1}{2} m_i \varphi_i^\dagger \varphi_i + \lambda_i (\varphi_i^\dagger \varphi_i)^2 \right] + \frac{g}{2} \varphi_1^\dagger \varphi_0^3 + h.c.$$

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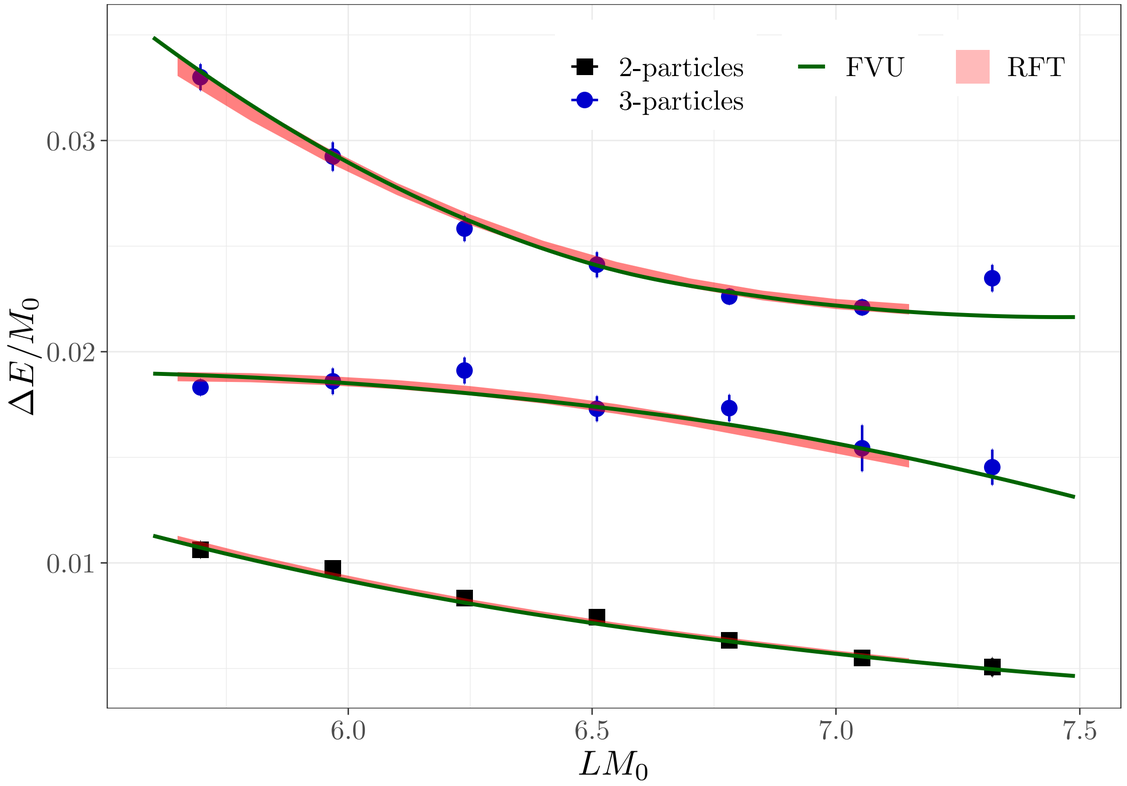
> map out $LM_0$ dependence ⇒ avoided level crossing

> RFT and FVU equally good for same data

*g=20*

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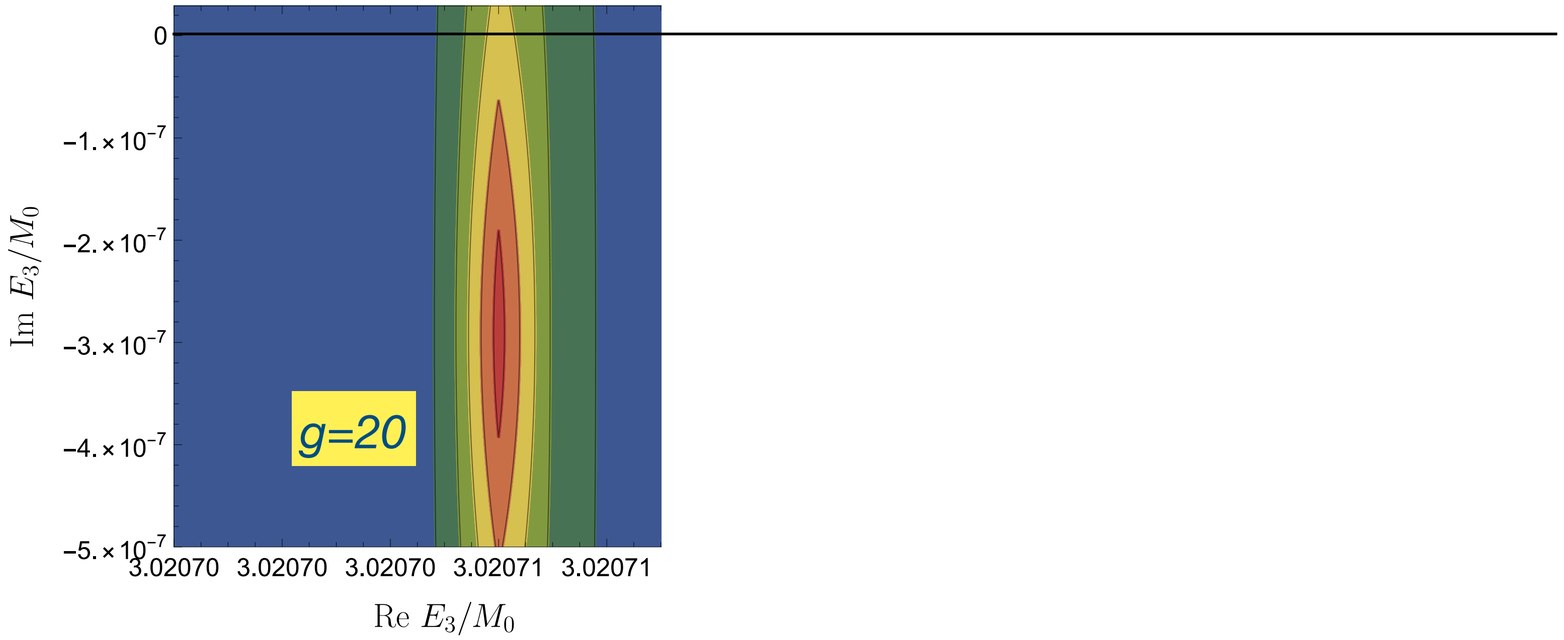


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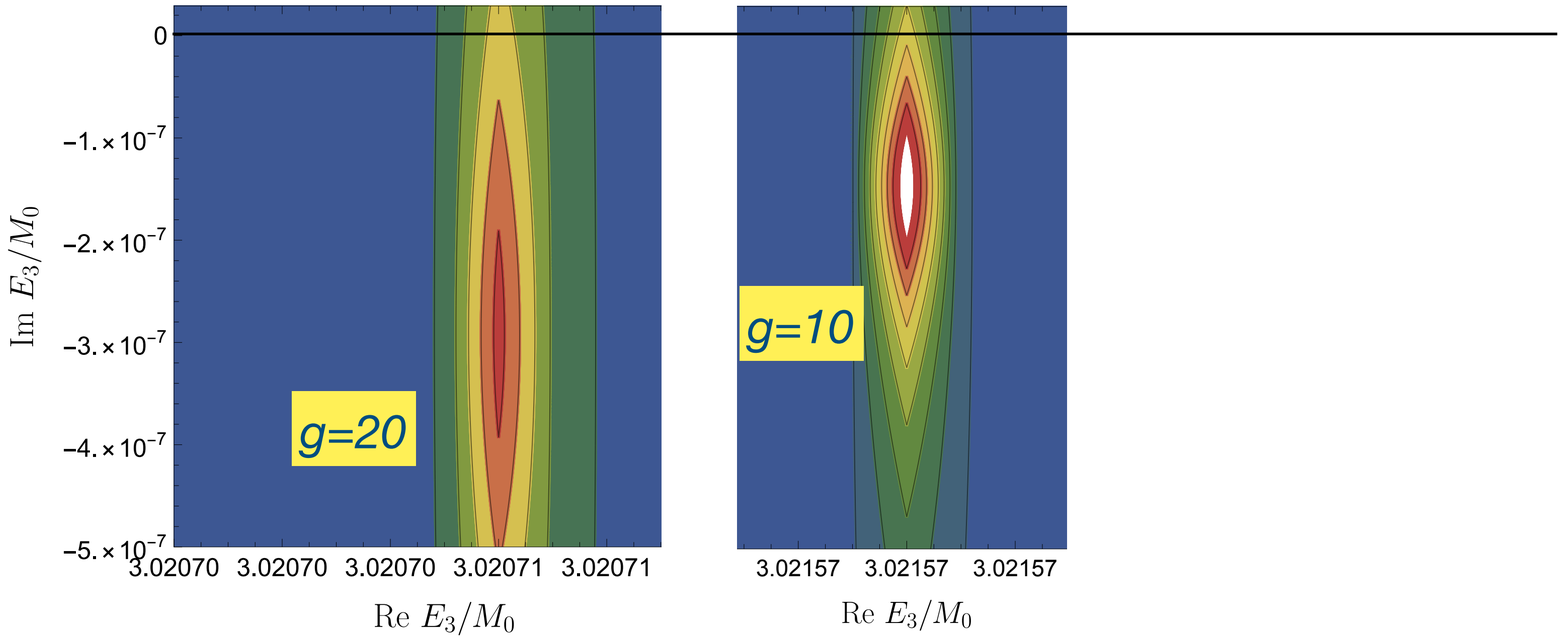


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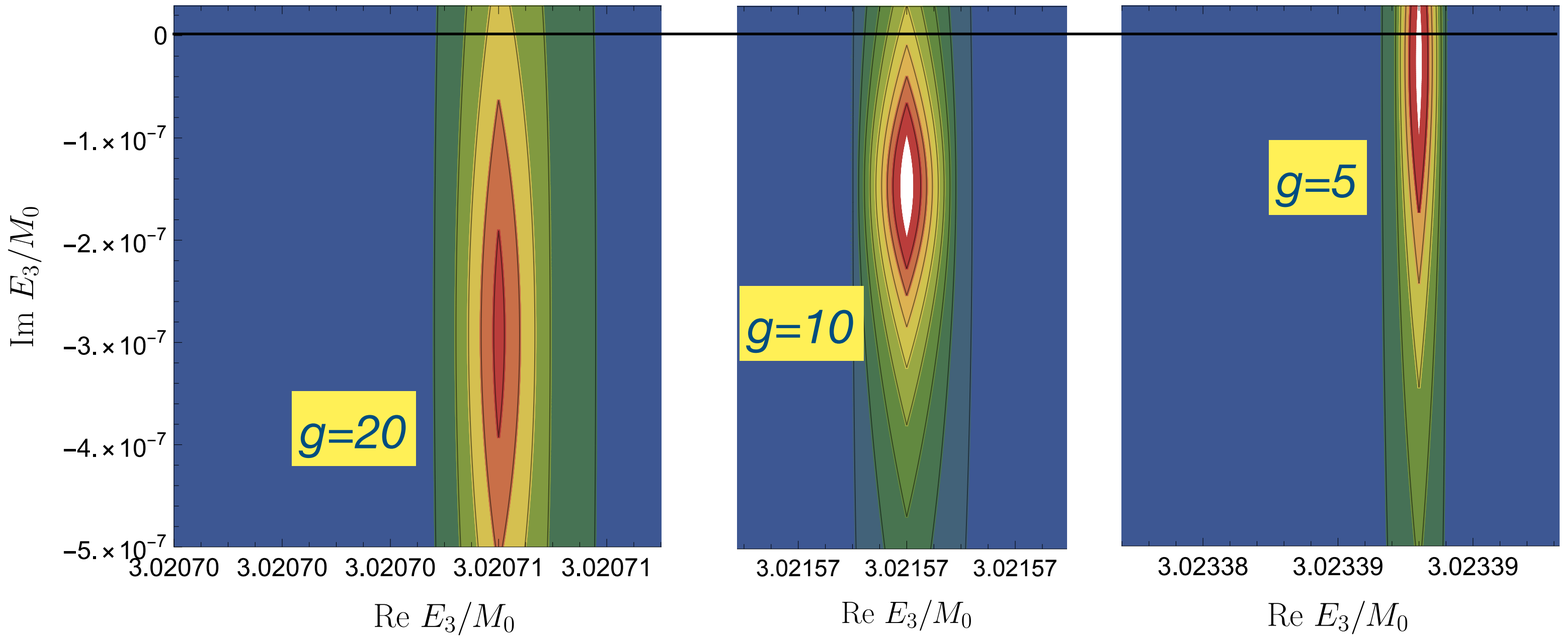


1) TALK: Marco Garofalo

# SUMMARY

*Resonant three-body systems from Lattice QCD become accessible*

- $a_1(1260)$ extracted from Lattice

  > universal parameters extracted (chiral trajectory on the way)

- $\varphi^4$ theory as benchmark:

  > consistency between RFT and FVU for the same set of data

  > avoided level crossing occurs as expected ( ≠ *resonance width !)*

  > *poles extracted (FVU ➟ IVU)*

# OUTLOOK

- Avoided level crossing in physical systems: $a_1(1260)$ ...
- Alternative access to finite-volume spectrum for resonant systems (?)

➥ **TALK: Daniel Severt**

Deutsche Forschungsgemeinschaft DFG

SPARES

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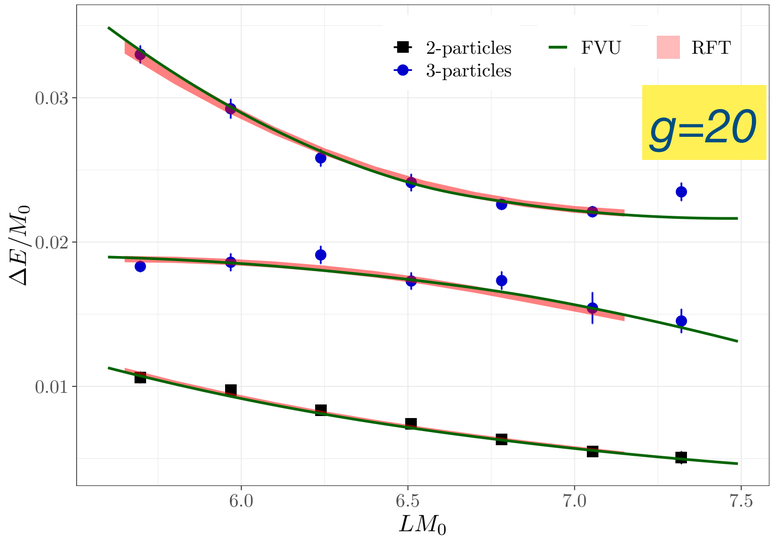
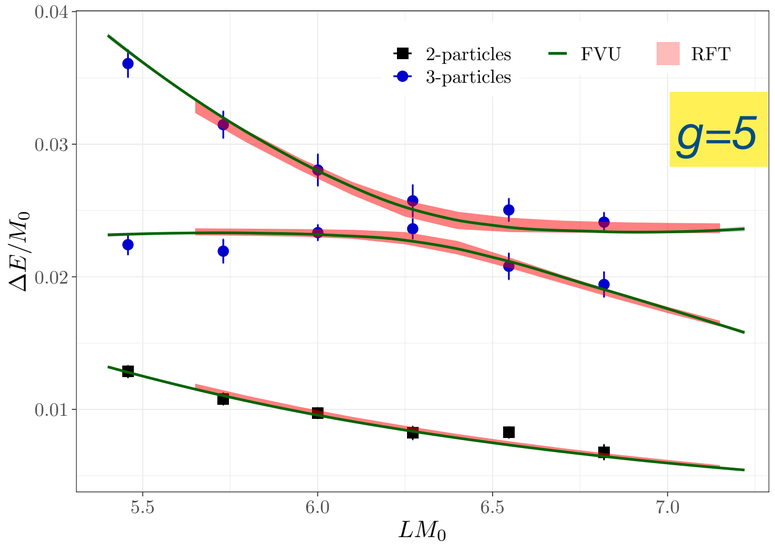



1) TALK: Marco Garofalo

# 3-BODY QUANTIZATION CONDITION (FVU)

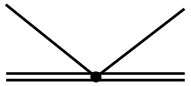
$$0 = \det \Big[ B(s) + C(s) - 2L^3 E_{\mathbf{p}} \left( \tilde{K}_2^{-1}(s) - \Sigma_2^L(s) \right) \Big]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

**one-particle exchange**

- fixed by 3b-unitarity
- no free parameters

**three-body force**

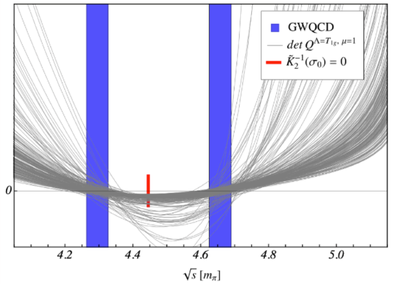
- dynamics of $\rho\pi$ system



- regular function ⇒ Laurent series

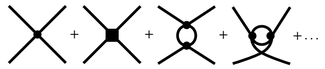
$$C_{\ell'\ell}(s,\mathbf{p}',\mathbf{p}) = \sum_{i=-1}^{\infty} c^{(i)}_{\ell'\ell}(\mathbf{p}',\mathbf{p})(\mathbf{s} - \mathbf{m}^2_{\mathbf{a_1}})^{\mathbf{i}}$$

- fit to 3-body levels




**two-body kernel**

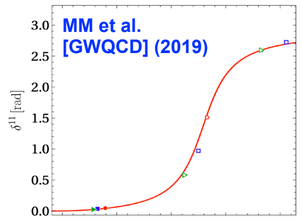
- dynamics of *I=1* $\pi\pi$ system



- regular function ⇒ polynomial

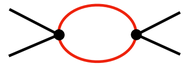
$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- parameters *($a_0$,$a_1$)* from cross-channel fit to $\pi\pi$ GWQCD levels




**two-body self-energy**

- fixed by 2b-unitarity



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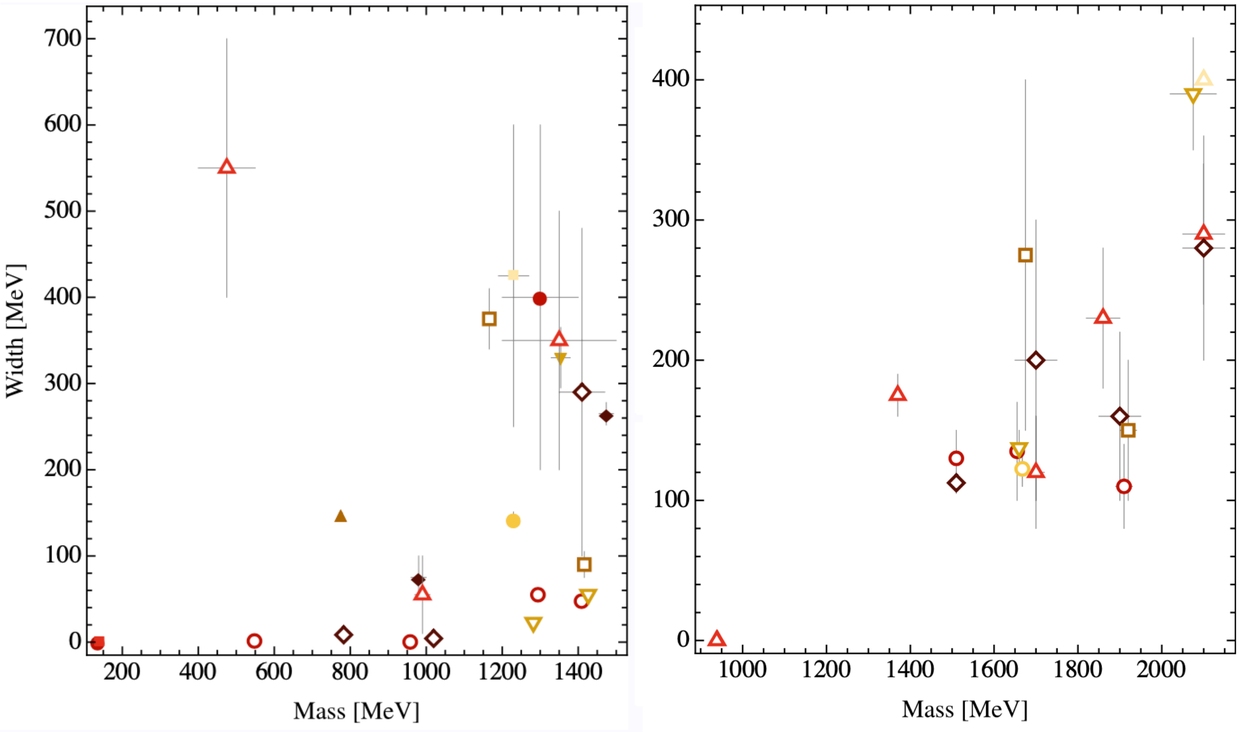
- govern intermediate energy-regime of QCD. Link to perturbative regime




Data: PDG 2021
Plot: MM/Meißner/Urbach 2206.01477 review in Phys. Rept.