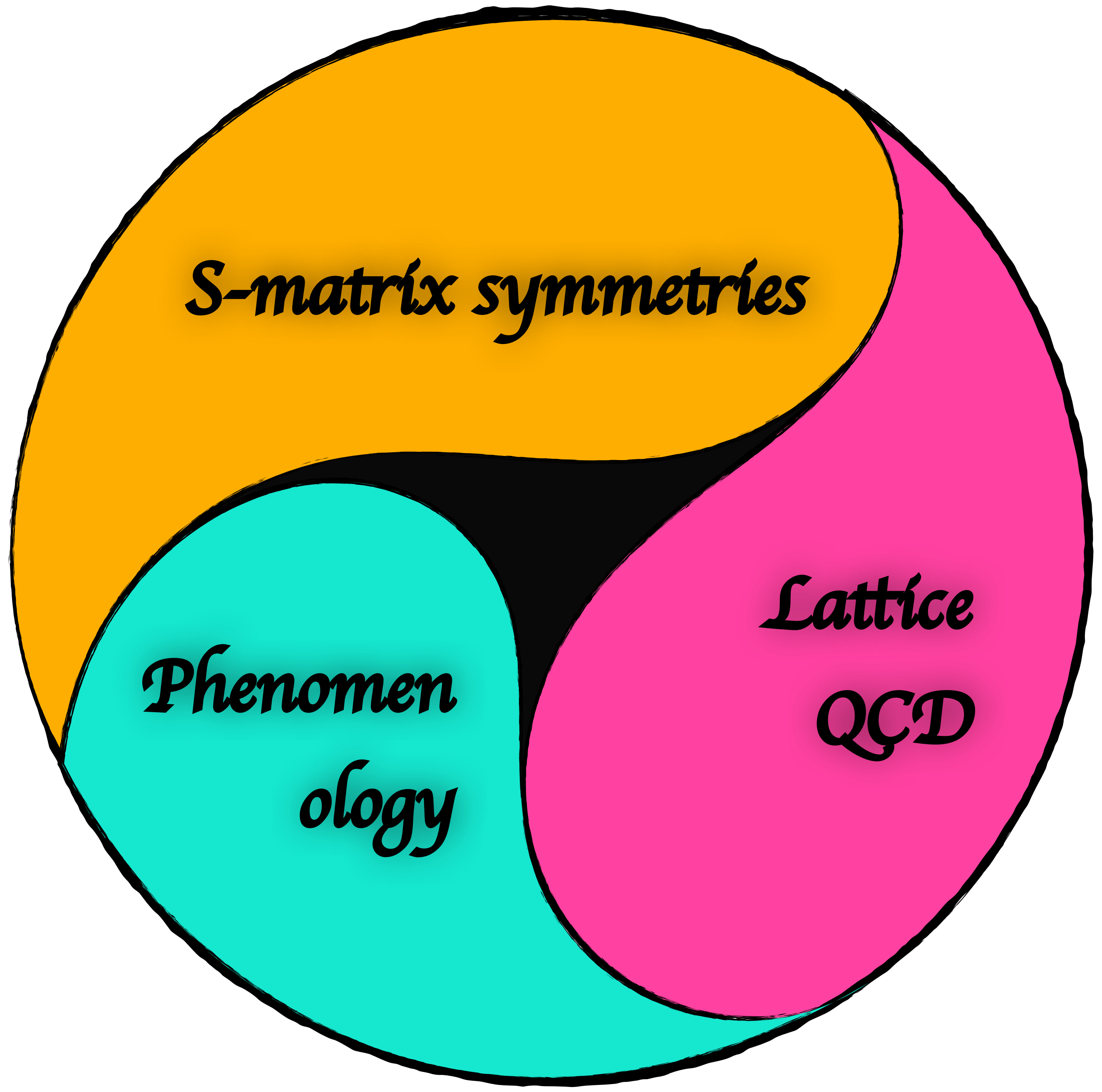
RESONANCE PARAMETER FROM LATTICE QCD



MAXIM MAI

... with J.-X. Lu, L.-S. Geng, M.Döring [Phys.Rev.Lett. 130 (2023) 7]
... with C.Culver, A.Alexandru, D.Sadasivan, M Döring [Phys.Rev.Lett. 127 (2022)]
... with M.Garofalo, F. Romero-López, A.Rusetsky, C.Urbach [JHEP 02 (2023) 252]
... with D.Severt, Ulf-G. Meißner [2212.02171 [hep-lat]]

FUNDING: DFG CRC 110 (GERMANY); DOE DE-SC0016582/DE-SC0016583 (USA); NSF PHY 2012289 (USA)

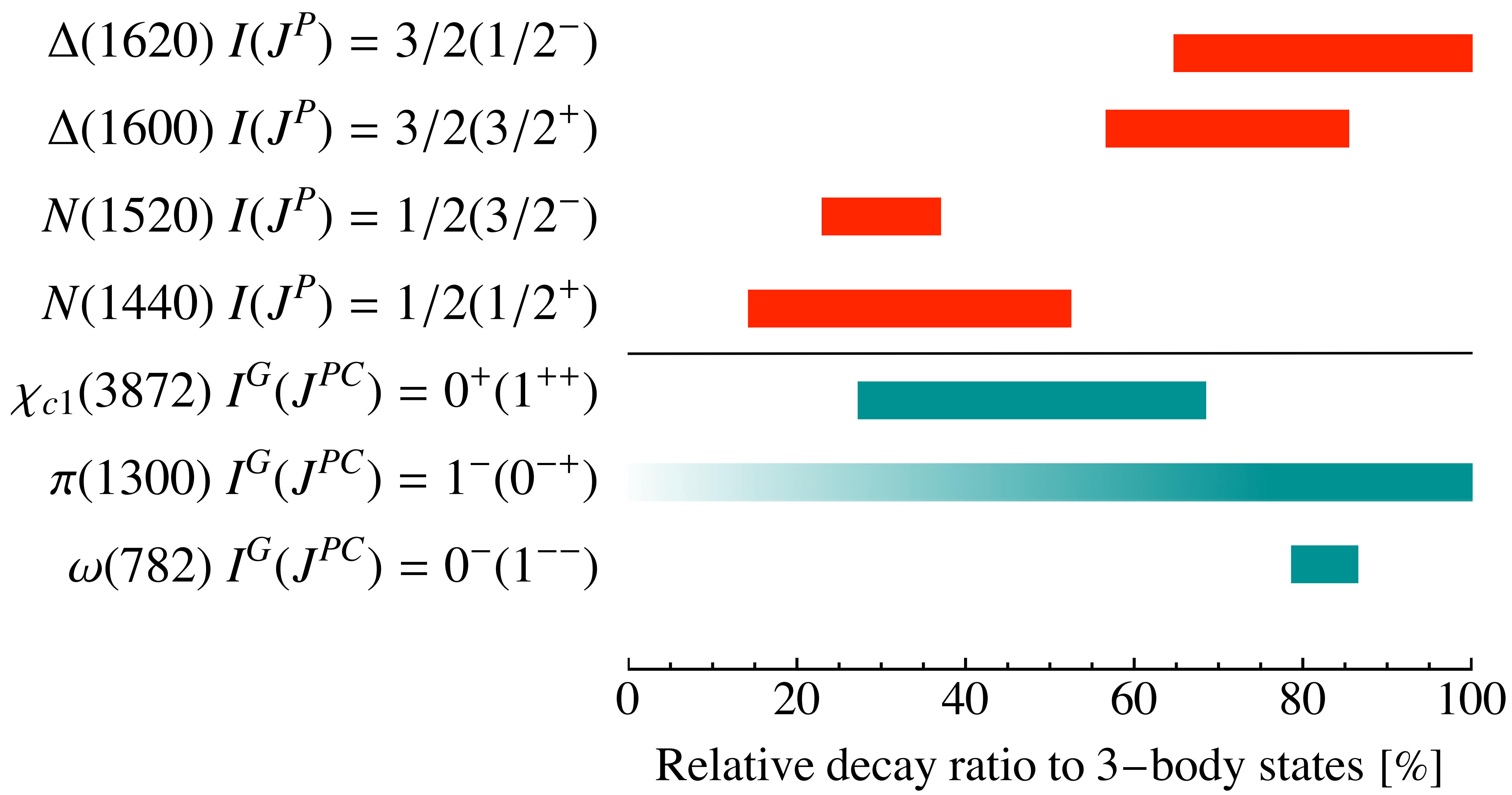
HADRON SPECTRUM

- Mostly unstable states:

  ≈100 mesons

  ≈50 baryons (****)

- Many states have considerable but not well known three-body content



[Particle Data Group] Workman et al. PTEP 2022 (2022)
MM/Meißner/Urbach Phys.Rept. 1001 (2023)

UNIVERSAL PARAMETERS

- Analyticity of the S-matrix (complex energy)
  - poles on unphysical Riemann sheets
- Physical information (real energy)
  - experiment
  - theory (Lattice QCD)

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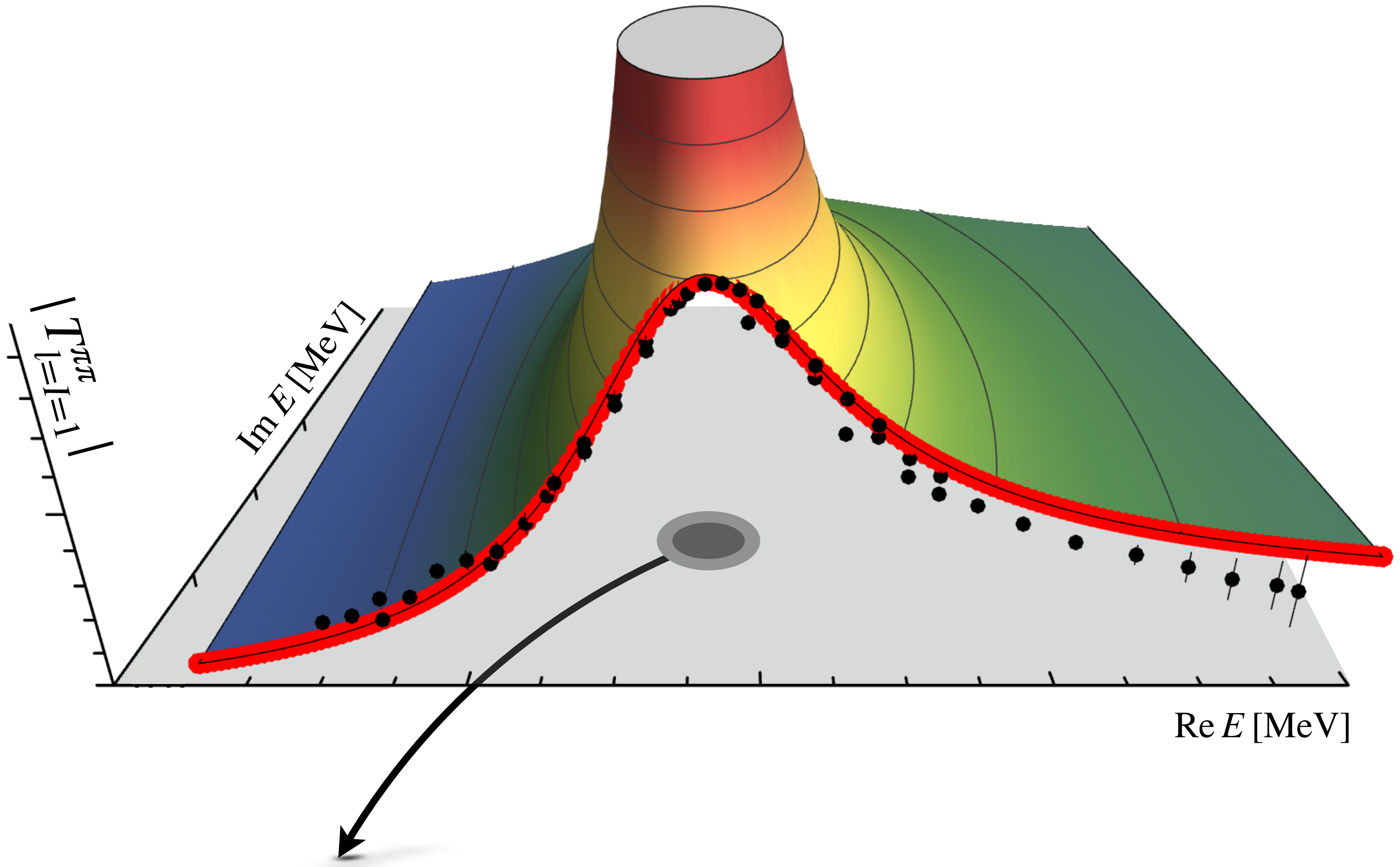


$M^* = (750 - i60)$ MeV

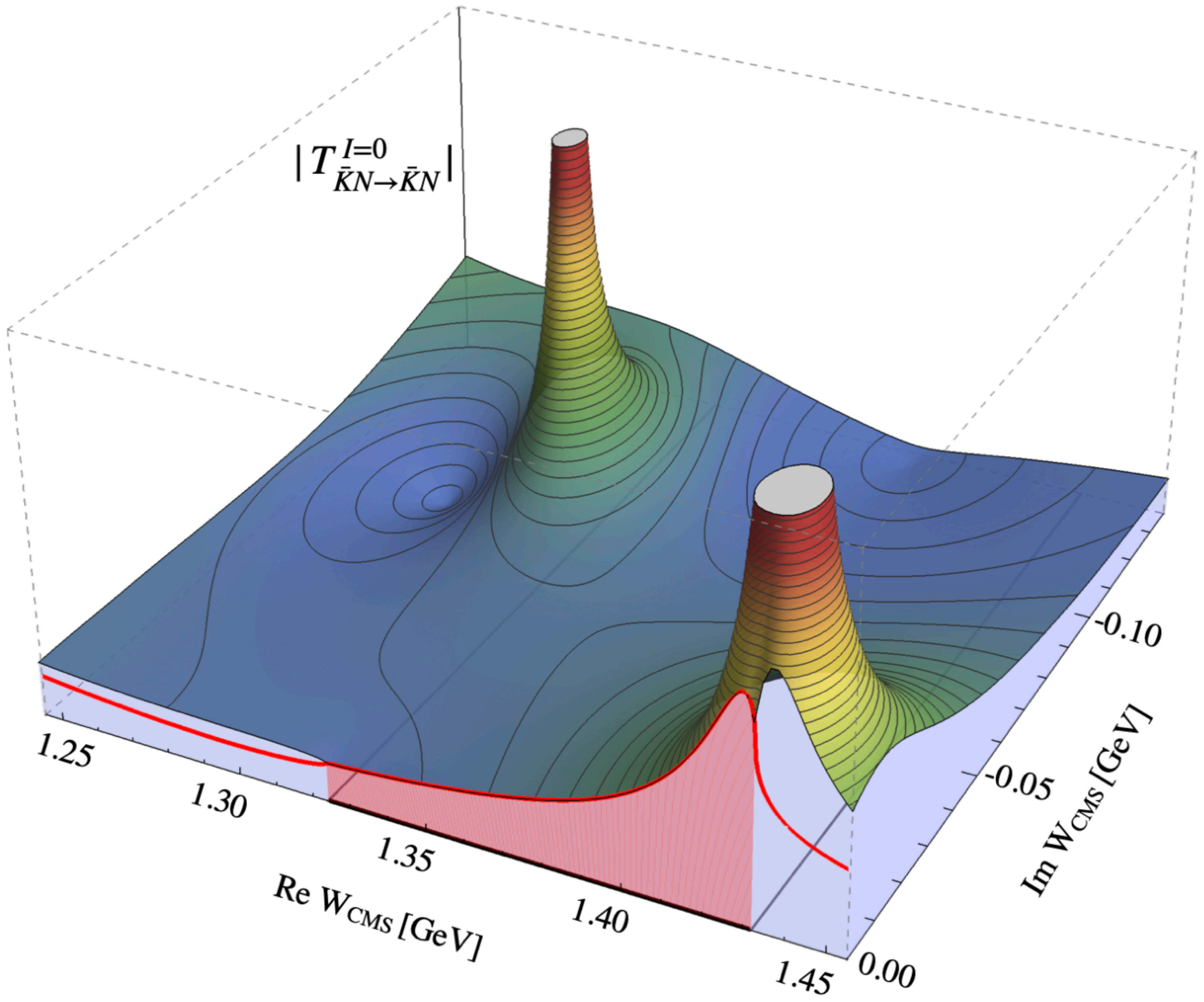
e.g., Breit-Wigner parameter: $M_\rho - i\Gamma_\rho/2$

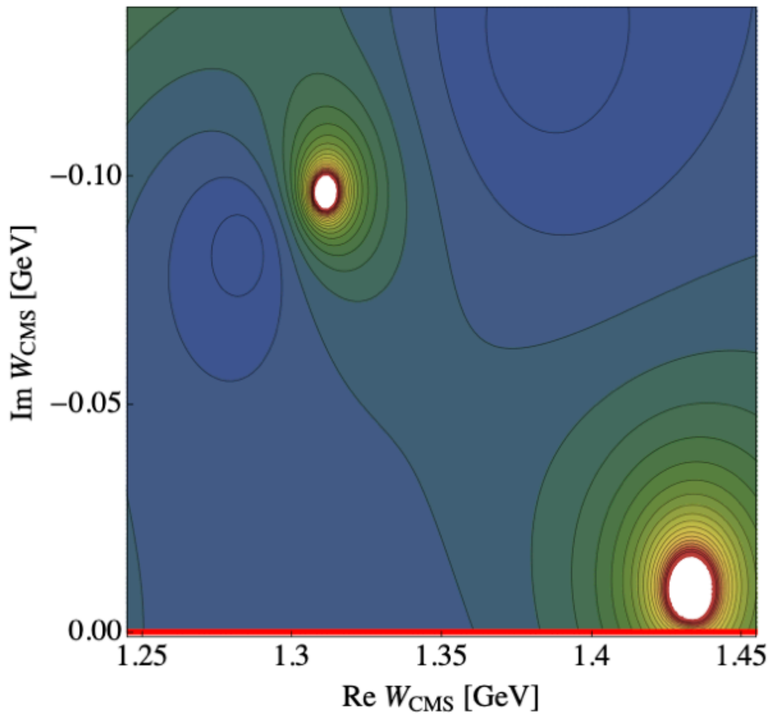
Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

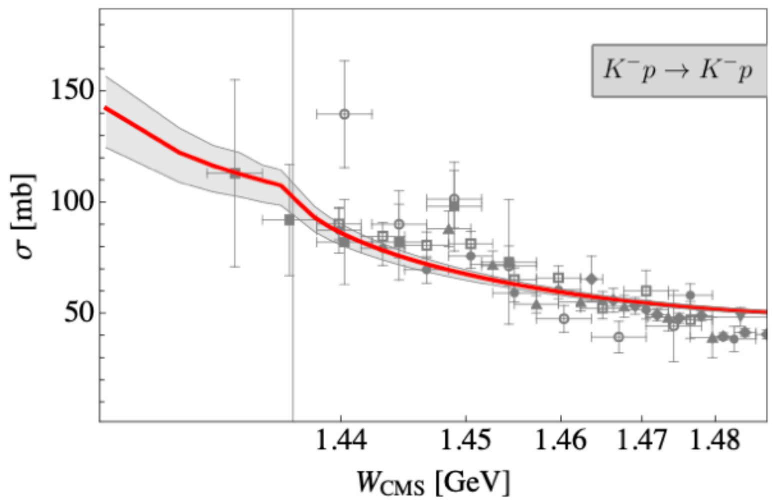
EXAMPLE: Λ(1405)

QCD ➟ CHPT ➟ UCHPT (*S=-1, I=0, J^P=1/2$^-$*)

- Formalises established state: **Λ**(1405)
- Predicts[1] a new state: **Λ**(1380)
  - stable to many tests[2]







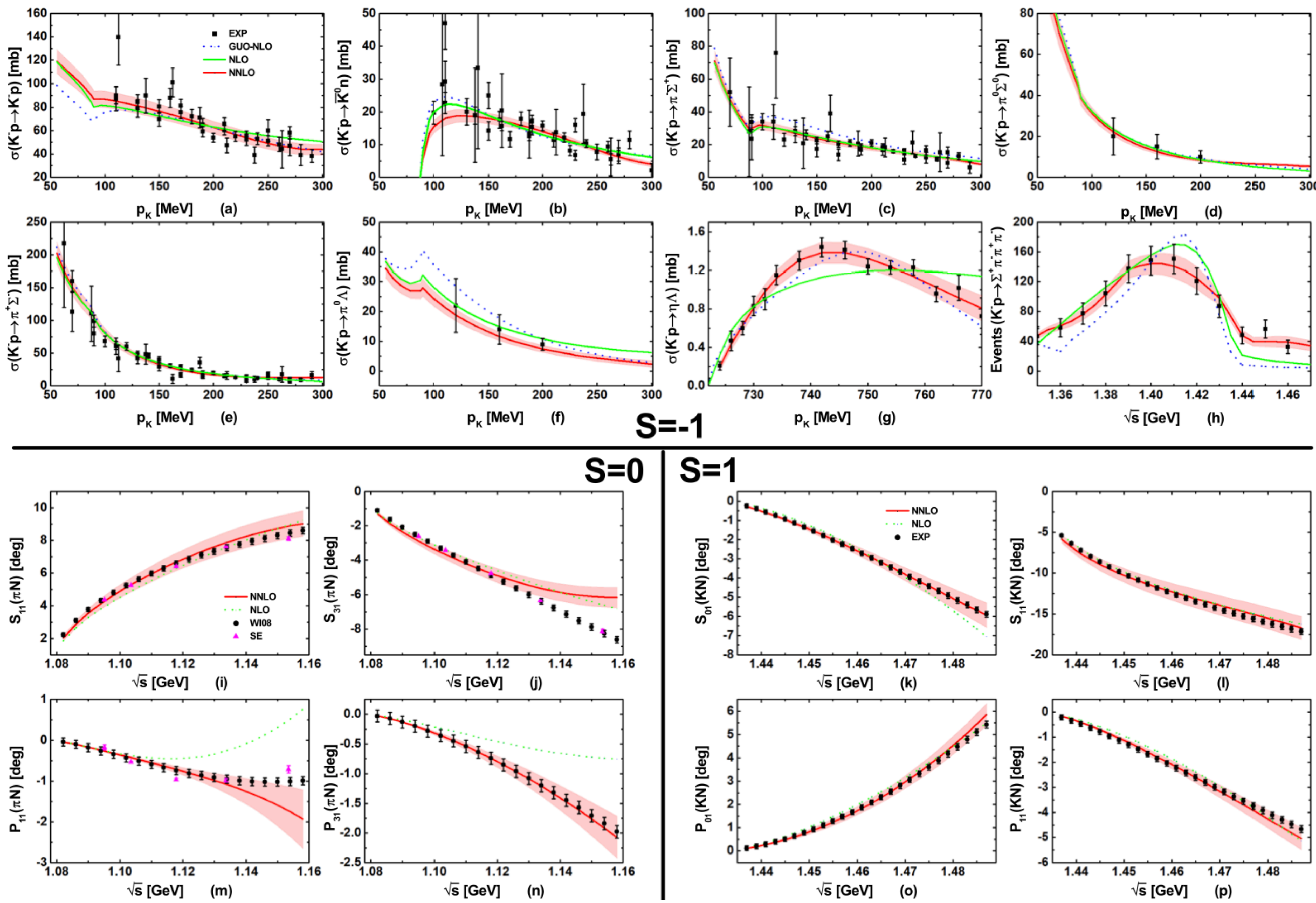
1) Oller/Meißner(2001), Ikeda/Hyodo/Weise(2011), MM/Meißner(2013), …
2) Anisovitch et al.(2018), Cieply/Bruns(2022), Sadasivan/MM/Döring/…(2018/2022)

Review: MM Eur.Phys.J.ST 230 (2021)

EXAMPLE: Λ(1405)

QCD ➟ CHPT ➟ UCHPT (*S=-1, I=0, $J^P=1/2^-$*)

- **model update**
  - NNLO CHPT kernel[1]
  - Unifies ***πN***, ***KN*** and ***KbarN*** interactions
  - 2-pole structure confirmed... again!



1) Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023) 7

EXAMPLE: Λ(1405)

QCD ➟ CHPT ➟ UCHPT ($S=-1$, $I=0$, $J^P=1/2^-$)

- **input updates:**
  - Motivated new experiments[1]
  - Lattice QCD[2] (?)

1) CLAS, GlueX, SIDDHARTA2, JPARC, AMADEUS, KLOE, Klong, etc..
2) **TALK**: Mohler

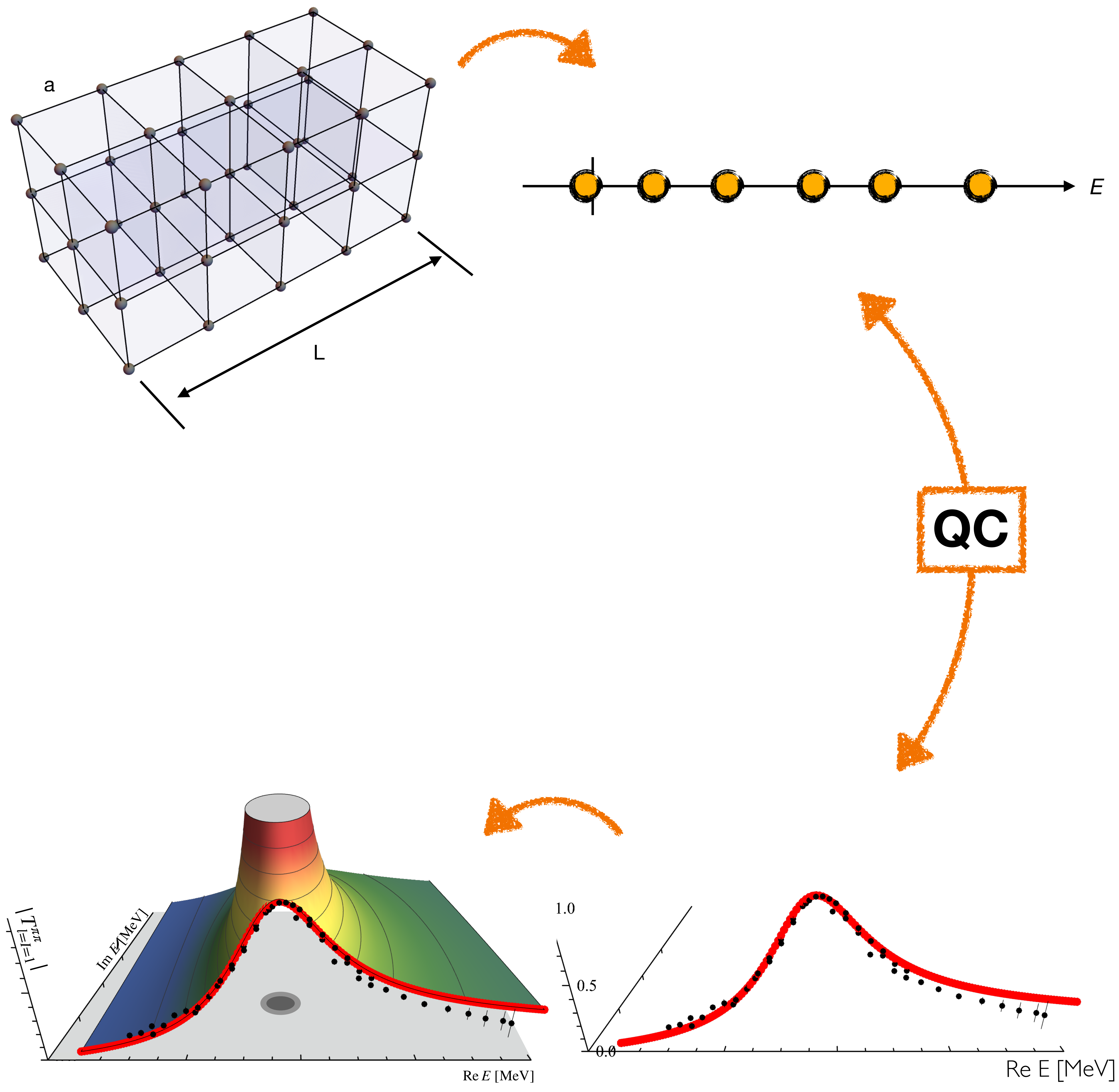
RESONANCE PARAMETERS FROM LATTICE QCD

GENERAL WORKFLOW

- QCD Green's functions on discretized Euclidean space-time in finite volume

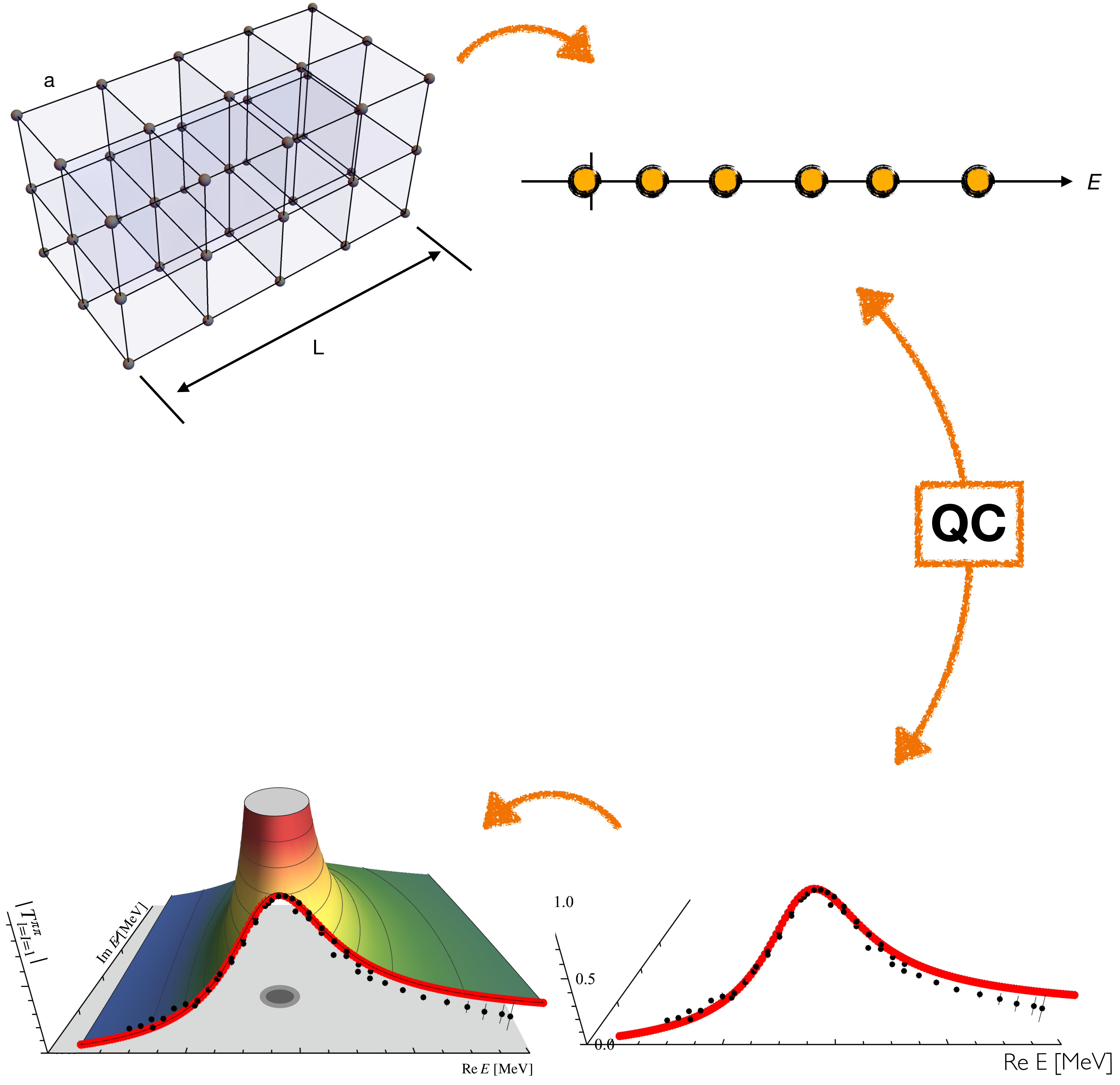
Quantization conditions (QC):

discrete finite-volume spectrum ➟ infinite-volume quantities



GENERAL WORKFLOW

- New progress in the 3-body sector[1]
  - RFT/NREFT/FVU 3b-quantization conditions
  - many new applications



1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo, Briceño….

Reviews:
Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);
MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
TALKS: Romero-López; Döring; Rusetsky; Sharpe; Draper

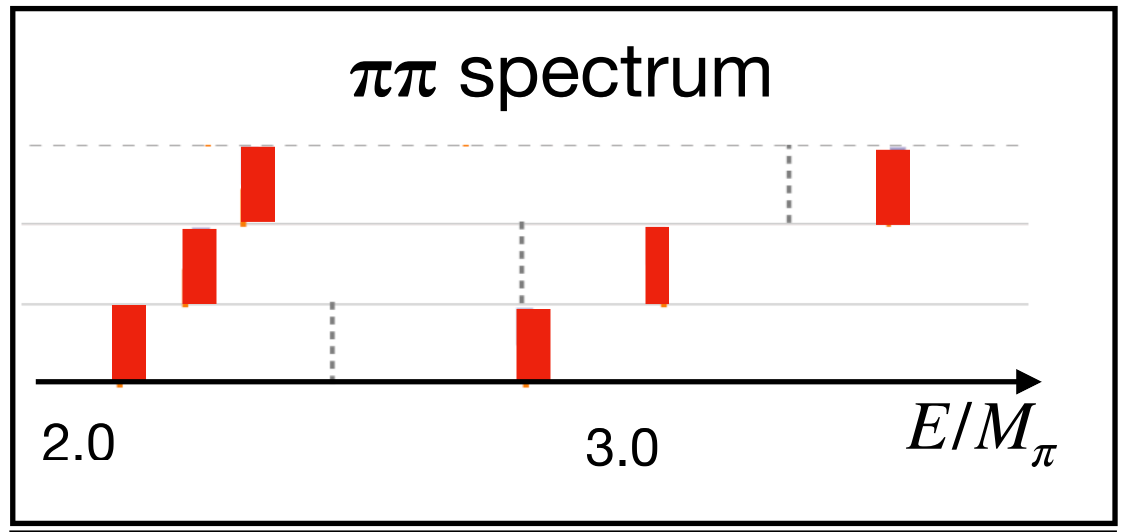
EXAMPLE $a_1(1260)$

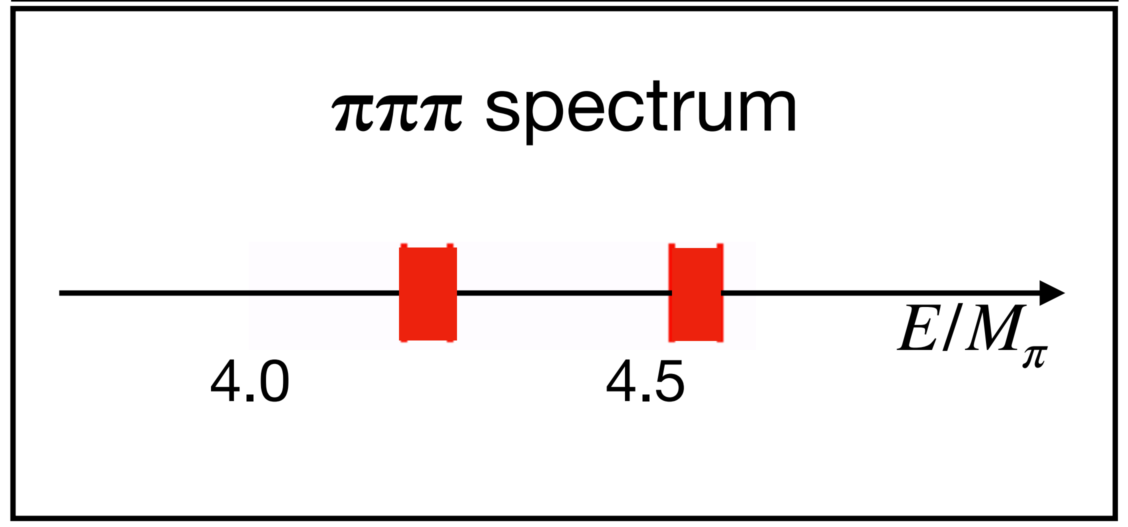
- 2- and 3-body lattice results with multi-hadron operators
- FVU identifies infinite-volume quantities
- Poles via 3b-integral equation
  - complex contour deformation[1]

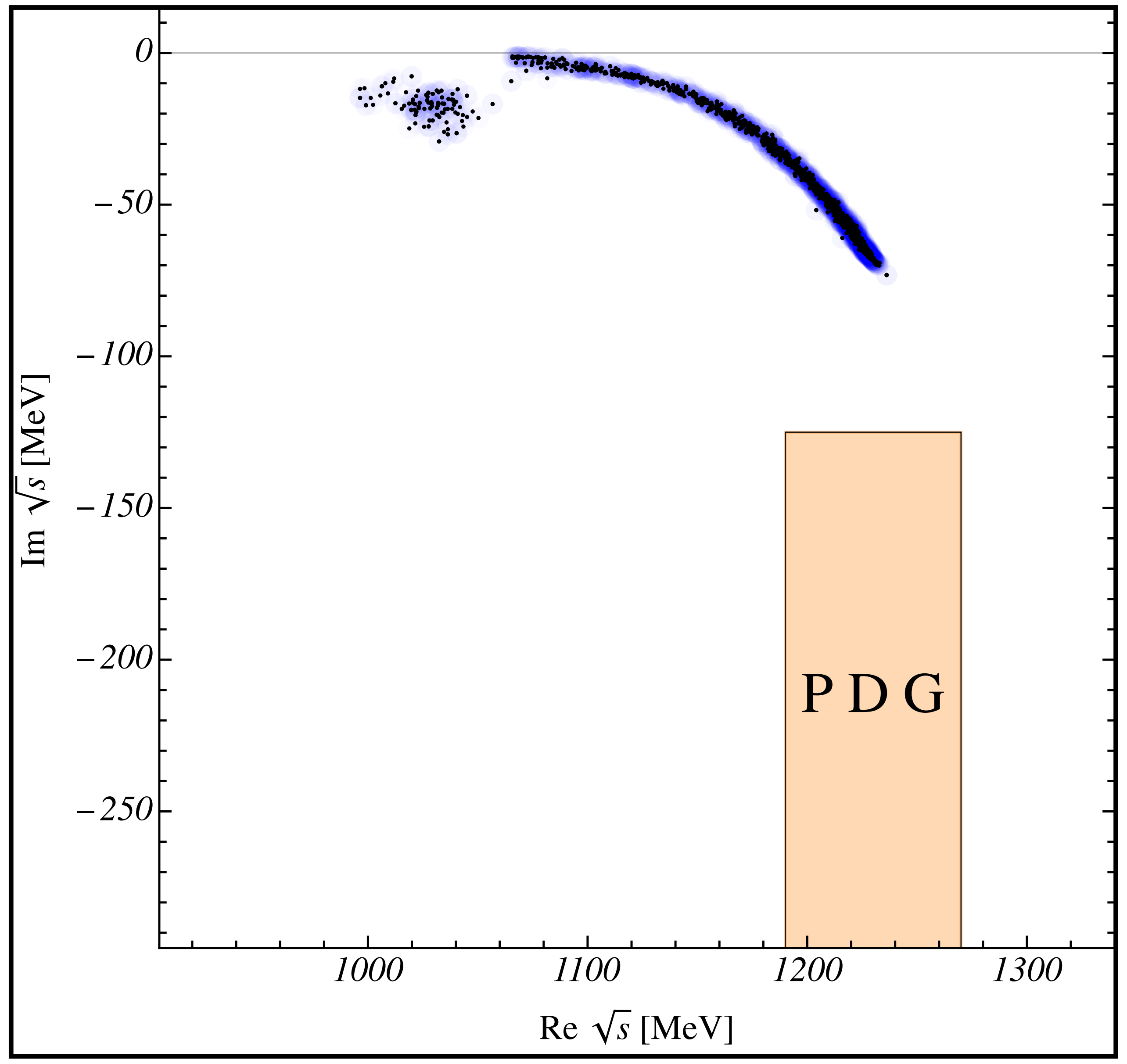
$$0 = \det\left[\, 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1} - \Sigma_2^L\right) - B - C \,\right]^{T_{1g}}$$

FVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\,\frac{(B+C)}{2E_l}\,\frac{1}{\tilde{K}_n^{-1} - \Sigma_n}\,T^c$$







1) TALK Döring

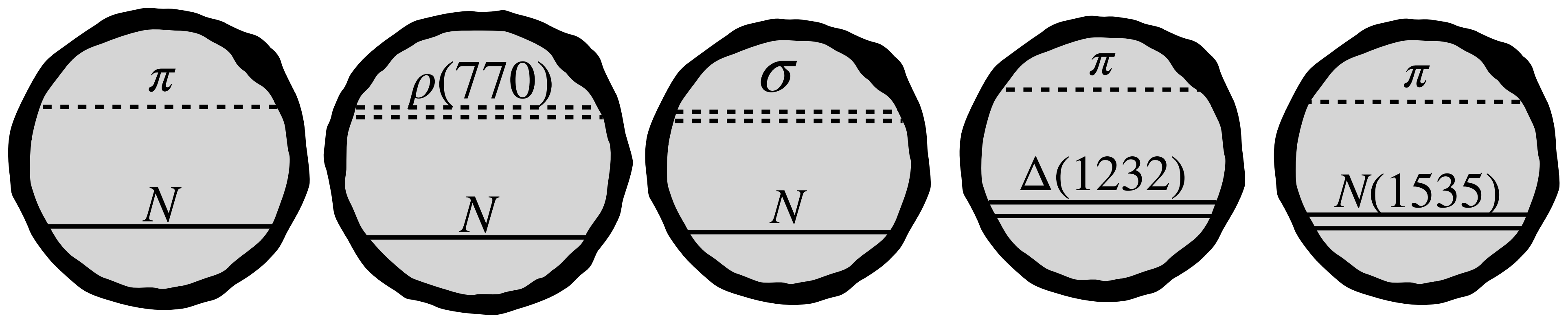
1) [GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)
2) Sadasivan/MM/Akdag/Döring PRD 101 (2020)
3) MM/Culver/Sadasivan/Brett/Döring/Alexandru/Lee [GWQCD] PRL 127 (2022)

EXAMPLE N(1440)J^P=1/2$^+$

- Unusual line-shape[1] (large decay-ratios to three-body channels)
- Many interaction channels
- FVU/RFT/NREFT predictions are matter of time (mod. interest and resources)

Key questions for now:

Is it realistic to fix all free parameters form the lattice? What precision do we require?



... and more in SU(3)

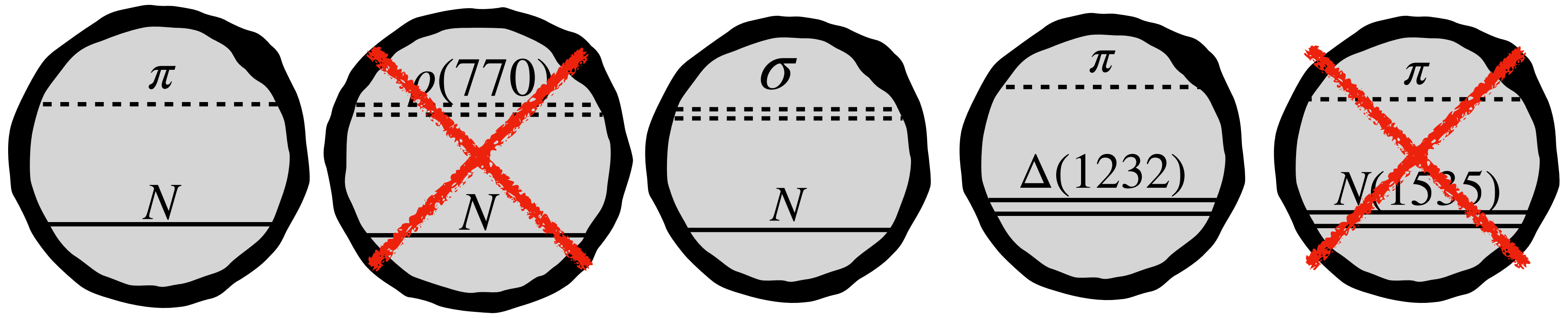
1) Arndt et al. (2006); Döring et al. (2009); Alvarez-Ruso et al. (2010)

EXAMPLE N(1440) $J^P = 1/2^+$

Pilot study[1]

- self-energy formalism from a particle-dimer Lagrangian
- 3-hadron configurations in self-energy formalism

⚠️ no particle-exchange diagrams



~~... and more in SU(3)~~

1) Severt/MM/Meißner 2212.02171 [hep-lat]

EXAMPLE N(1440) $J^P=1/2^+$

Pilot study[1]

- self-energy formalism from a particle-dimer Lagrangian
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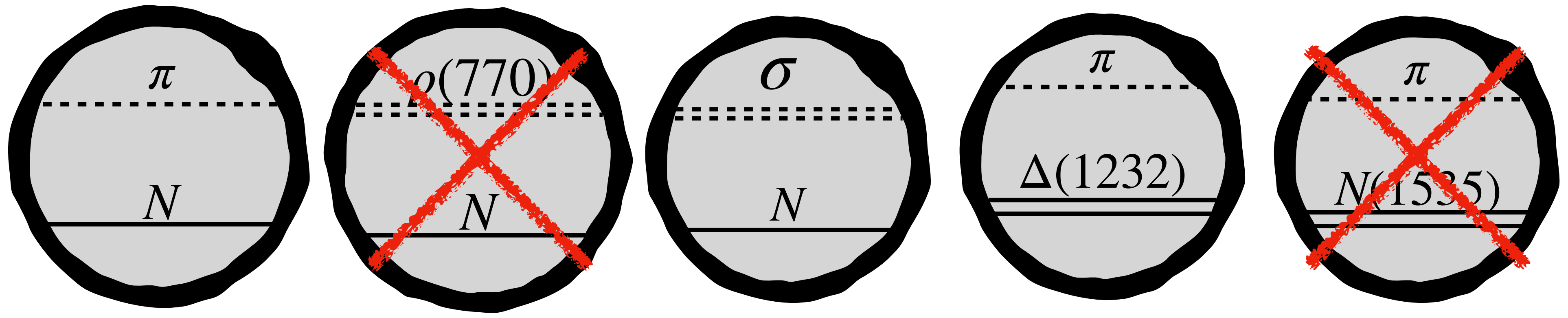
⚠️ no particle-exchange diagrams

Finite-volume spectrum for fixed parameters

- energy shifts very small
- opposing effects of $N\sigma$ and $\Delta\pi$ channels



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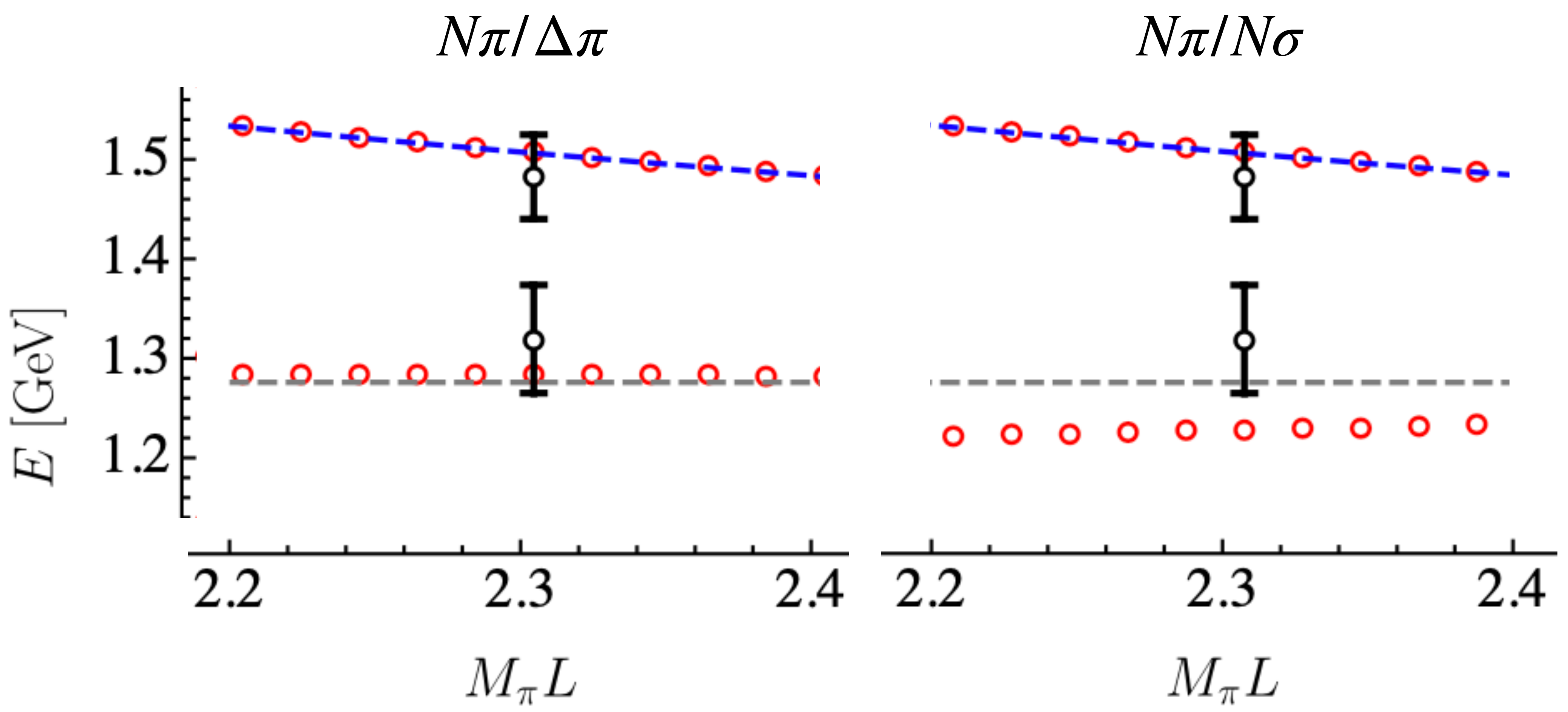

1) Severt/MM/Meißner 2212.02171 [hep-lat]

Lattice values (black dots) Lang et al. Phys.Rev.D 95 (2017) 1

CRITICAL TESTS OF FINITE-VOLUME FORMALISMS

with M.Garofalo, MM, F. Romero-López, A.Rusetsky, C.Urbach

JHEP 02 (2023) 252

SETUP

Complex φ^4 theory with an explicit 3-body state

$$\boxed{\mathcal{L} = \sum_{i=0,1} \left[\frac{1}{2}\partial^\mu \varphi_i^\dagger \partial_\mu \varphi_i + \frac{1}{2} m_i^2 \varphi_i^\dagger \varphi_i + \lambda_i (\varphi_i^\dagger \varphi_i)^2\right] + \frac{g}{2}\varphi_1^\dagger \varphi_0^3 + \mathrm{h.c.}\,.}$$

- implemented on the lattice[1]
- similar to pilot 2-body studies[2]

1) https://github.com/HISKP-LQCD/ Z2-phi4/tree/complex-ising
2) Gattringer and C.B. Lang, Phys. Lett. B 274 (1992) 95 ;Rummukainen/Gottlieb, Nucl. Phys. B 450 (1995) 397

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- implemented on the lattice[1]
- similar to pilot 2-body studies[2]

Key questions:

How well do RFT/FVU perform on the same data?

How does the avoided level crossing appear in 3-body systems?

1) https://github.com/HISKP-LQCD/ Z2-phi4/tree/complex-ising
2) Gattringer and C.B. Lang, Phys. Lett. B 274 (1992) 95 ;Rummukainen/Gottlieb, Nucl. Phys. B 450 (1995) 397

SUMMARY: RFT/FVU

RFT[1]/FVU[2]

- same building blocks
- formal equivalence and relations exist[4]
- particular scheme may be advantageous in different circumstances

RFT[1] $$0=\det\left(L^3\left(\tilde{F}/3-\tilde{F}(\tilde{K}_2^{-1}+\tilde{F}+\tilde{G})^{-1}\tilde{F}\right)^{-1}+K_{\mathrm{df},3}\right)$$

FVU[3] $$0=\det\left(B_0+C_0-E_L\left(K^{-1}/(32\pi)+\Sigma_L\right)\right)$$

3-body force

2-body interaction

one-particle exchange

2-body self-energy

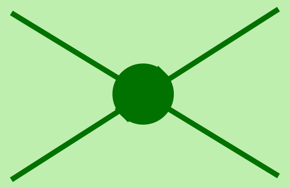
1) Hansen/Sharpe (2014) ...
2) MM/Döring EPJA 53 (2017) ...
3) Brett et al.Phys.Rev.D 104 (2021) 1; Jackura et al. Phys.Rev.D 100 (2019) 3

FINITE-VOLUME SPECTRUM

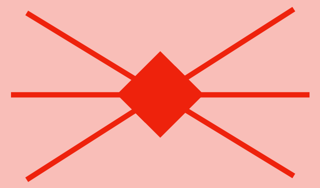
RFT and FVU fits

$$0 = \det\left(L^3\left(\tilde F/3 - \tilde F(\tilde K_2^{-1} + \tilde F + \tilde G)^{-1}\tilde F\right)^{-1} + K_{\mathrm{df},3}\right)$$

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$



$$q^* \cot\delta = \frac{1}{aM_0}$$

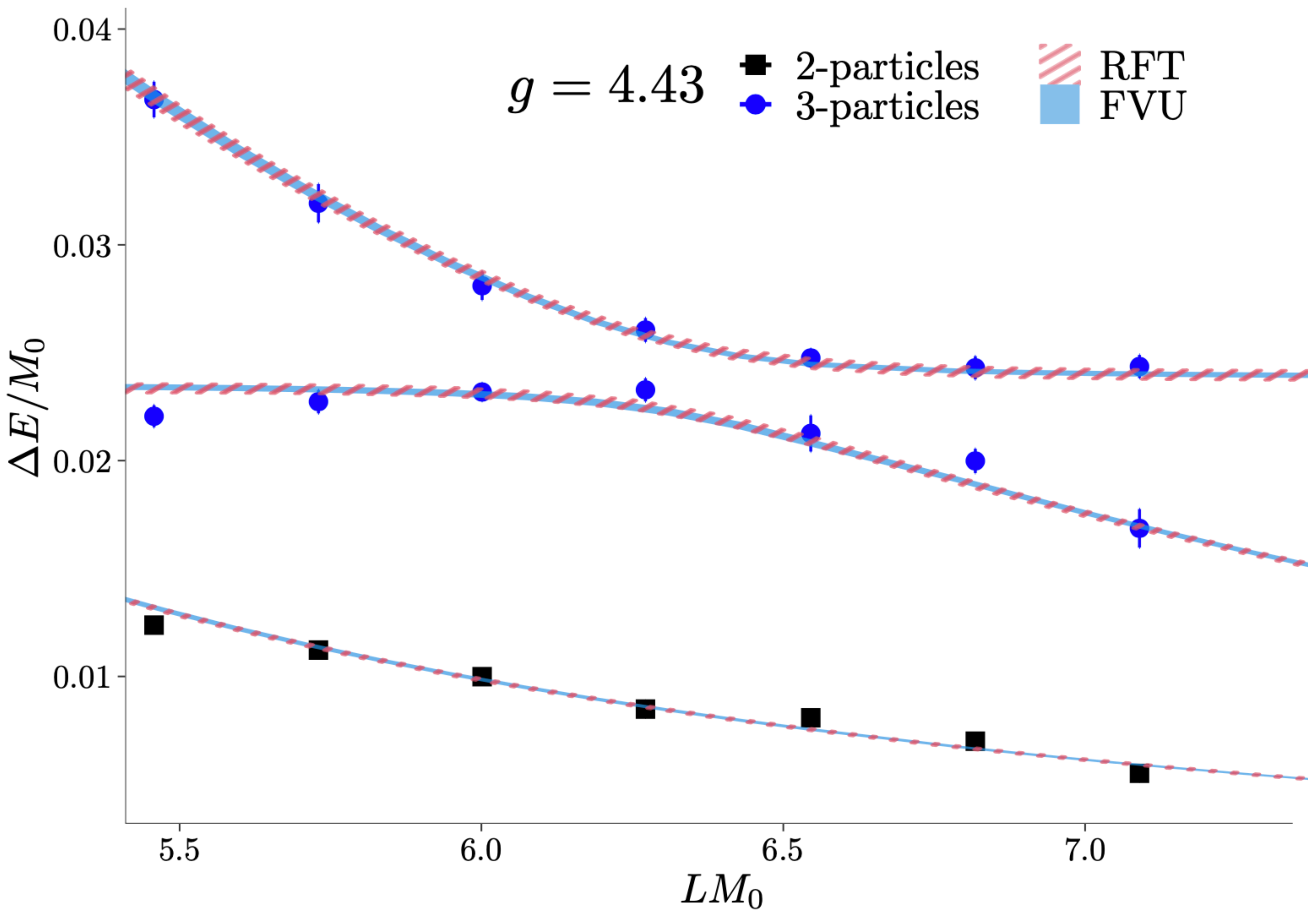


$$\frac{c_0}{E_3^3 - m_R^2} + c_1$$

FINITE-VOLUME SPECTRUM

RFT and FVU fits

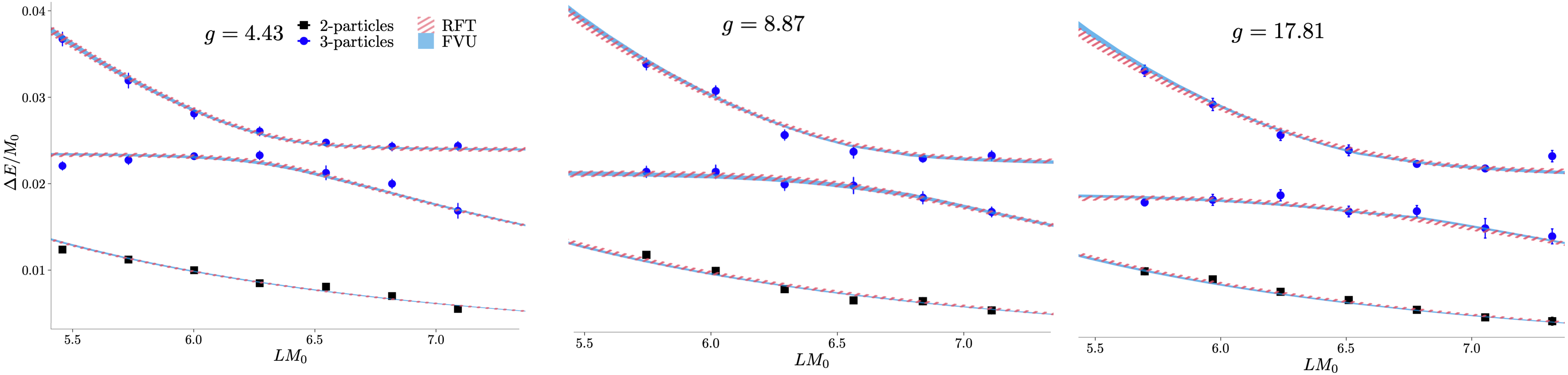
- 3-parameter fits are preferable
- fit quality RFT/FVU very similar
- observable quantities (a) consistent



	$a\,M_0$	m_R/M_0	c_0	$c_1 M_0^2$	m'_R/M_0	c'_0	$c'_1 M_0^2$	χ^2_{dof}
FVU	$-0.1512(09)$	$3.0229(1)$	$-0.0188(35)$	–	–	–	–	2.9
RFT	$-0.1522(12)$	–	–	–	$3.0232(2)$	$-31.6(8.4)$	–	2.5
FVU	$-0.1569(12)$	$3.0233(2)$	$-0.0297(57)$	$2.29(38)$	–	–	–	1.5
RFT	$-0.1571(10)$	–	–	–	$3.0237(2)$	$-37.6(9.0)$	$2789(540)$	1.5

AVOIDED LEVEL CROSSING

Increase g($\varphi_1 \rightarrow 3\varphi_0$) coupling ⇒ avoided level crossing becomes wider



COMPLEX POLES

- Analytic continuation of RFT/FVU scattering amplitudes to the complex energy plane
- Methods are different (so far):

	RFT	FVU
real kinematics	**calculates**	extrapolates
complex kinemaitcs	extrapolates	**caculates**

BUT: pole positions are consistent

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- Analytic continuation of RFT/FVU scattering amplitudes to the complex energy plane
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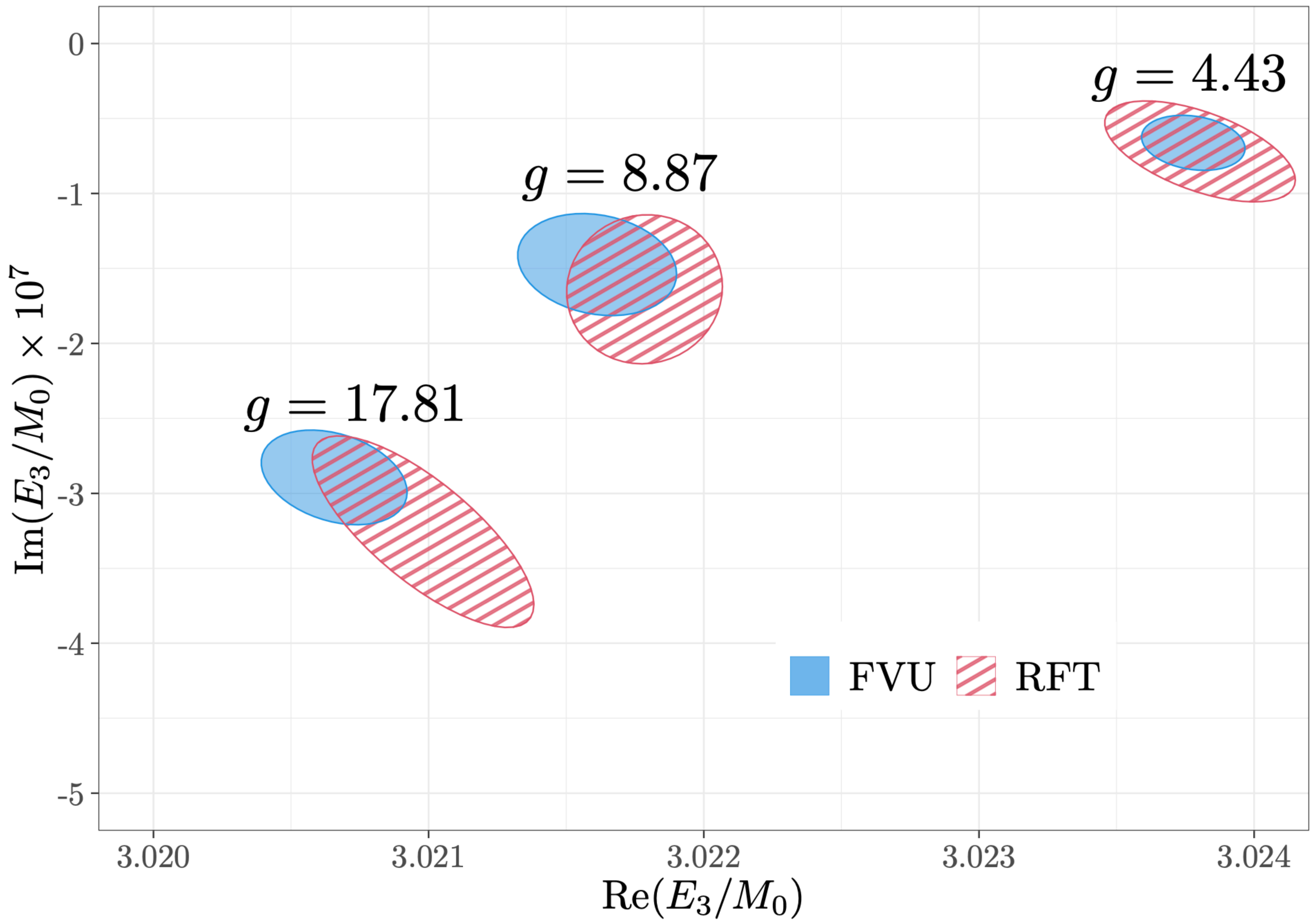


SUMMARY

- Universal parameters of a new class of resonant systems become accessible
- Important progress from lattice and theory side

  ... 3-body quantization conditions perform similarly well on the same inputs

  ... using various approaches ➟ systematics assessment
- Parameters of $a_1(1260)$ already accessible from lattice

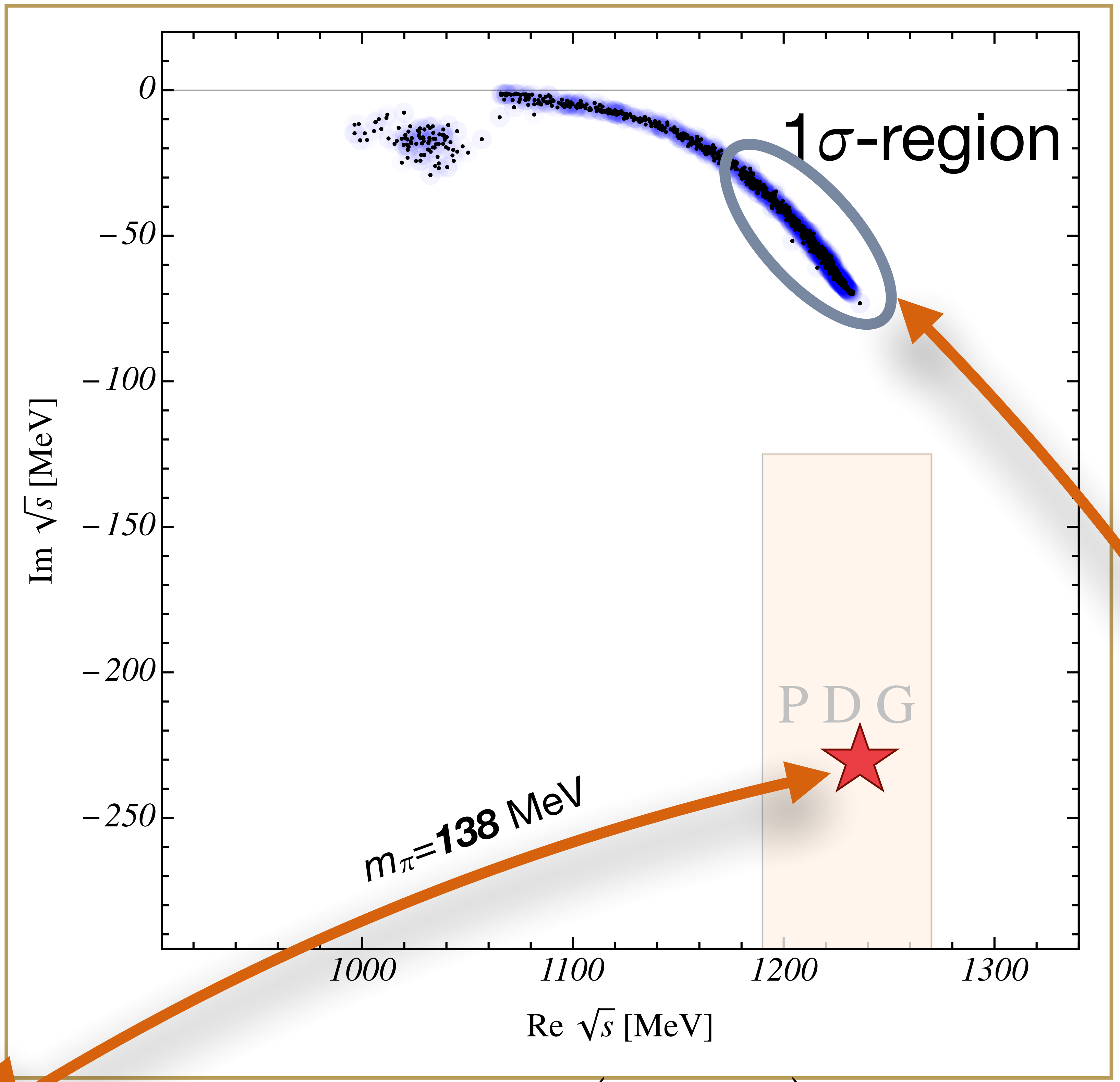
  ... in a heavy universe
- Pilot studies of the Roper N(1440) finite-volume spectrum

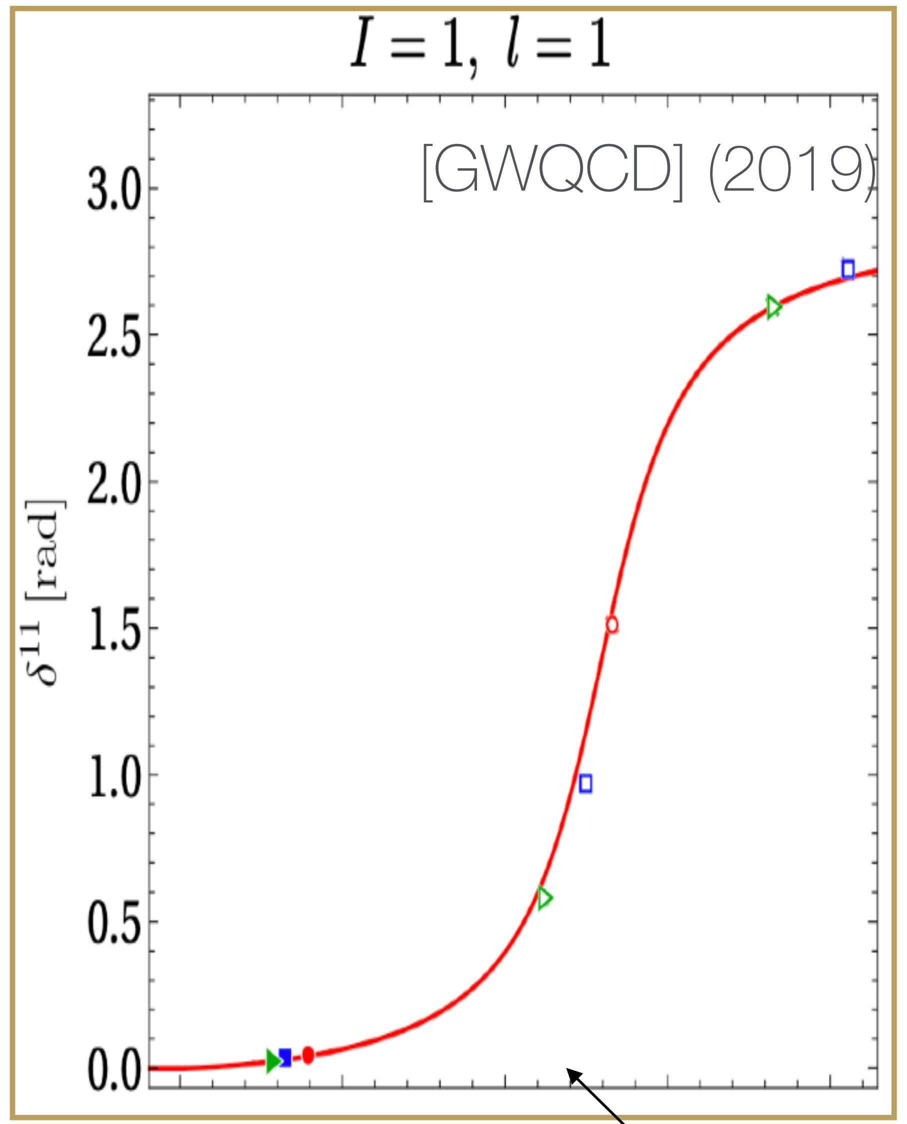
  ... large cancellations call for more (precise) inputs

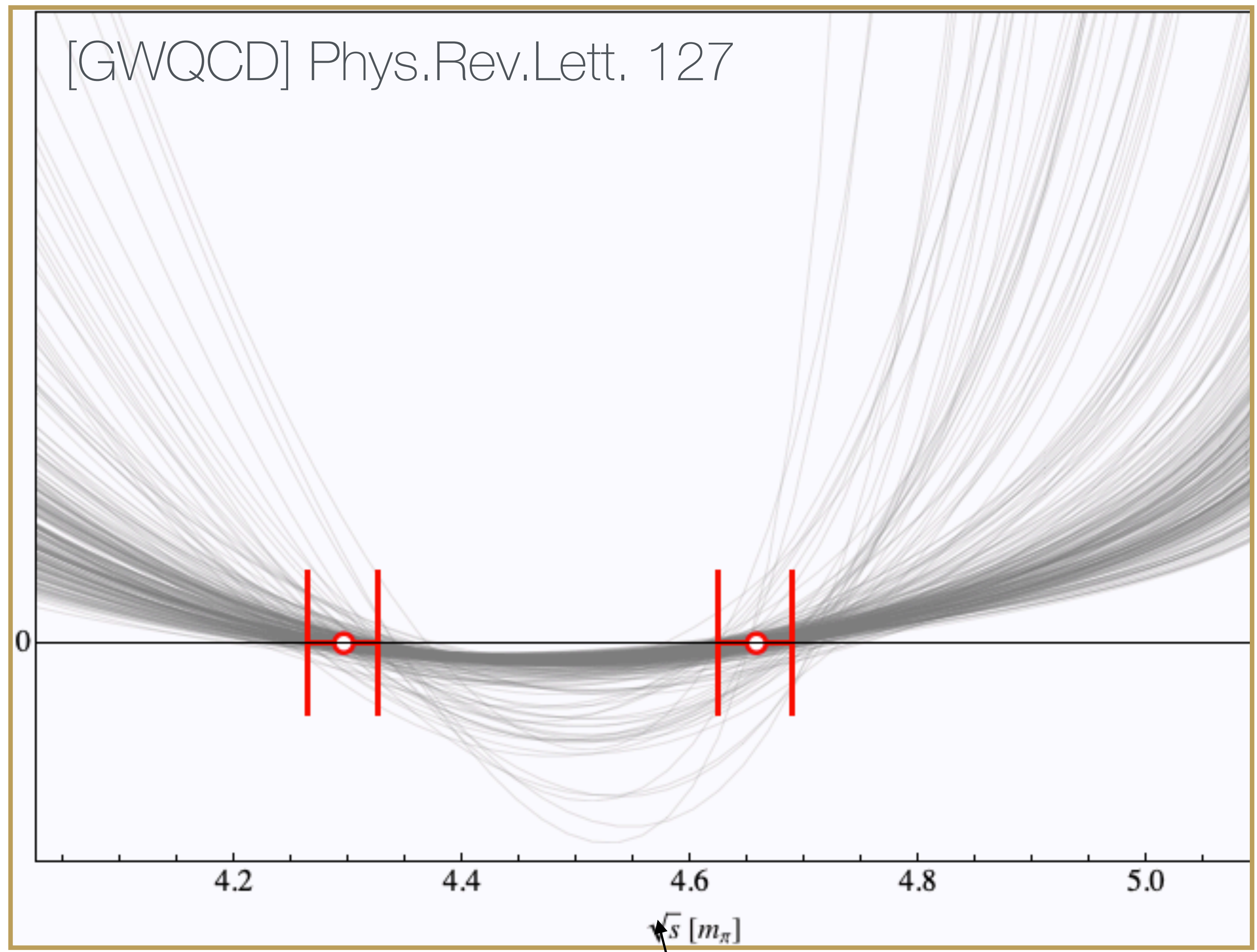
- The full dimer Lagrangian:

$$\begin{aligned}
\mathcal{L}_T = {} & R^\dagger 2W_R\left(i\partial_t - W_R\right)R + f_1 R^\dagger\phi^\dagger\phi R - f_2[R^\dagger\phi\psi + R\phi^\dagger\psi^\dagger] \\
& - f_3[R^\dagger\phi\Delta + \Delta^\dagger\phi^\dagger R] - f_4[R^\dagger\sigma\psi + \psi^\dagger\sigma^\dagger R] \\
& + \alpha_\Delta m_\Delta^2\Delta^\dagger\Delta + g_1\Delta^\dagger\phi^\dagger\phi\Delta - g_2[\Delta^\dagger\phi\psi + \Delta\phi^\dagger\psi^\dagger] \\
& + \alpha_\sigma M_\sigma^2\sigma^\dagger\sigma + h_1\psi^\dagger\sigma^\dagger\sigma\psi - h_2[\sigma^\dagger\phi\phi + \sigma\phi^\dagger\phi^\dagger] \\
& - G_{R\sigma}[R^\dagger\phi^\dagger\sigma\psi + \psi^\dagger\sigma^\dagger\phi R] - G_{R\Delta}[R^\dagger\phi^\dagger\phi\Delta + \Delta^\dagger\phi^\dagger\phi R] \\
& - G_{\Delta\sigma}[\Delta^\dagger\phi^\dagger\sigma\psi + \psi^\dagger\sigma^\dagger\phi\Delta]
\end{aligned}$$

A1(1260) FROM LATTICE QCD

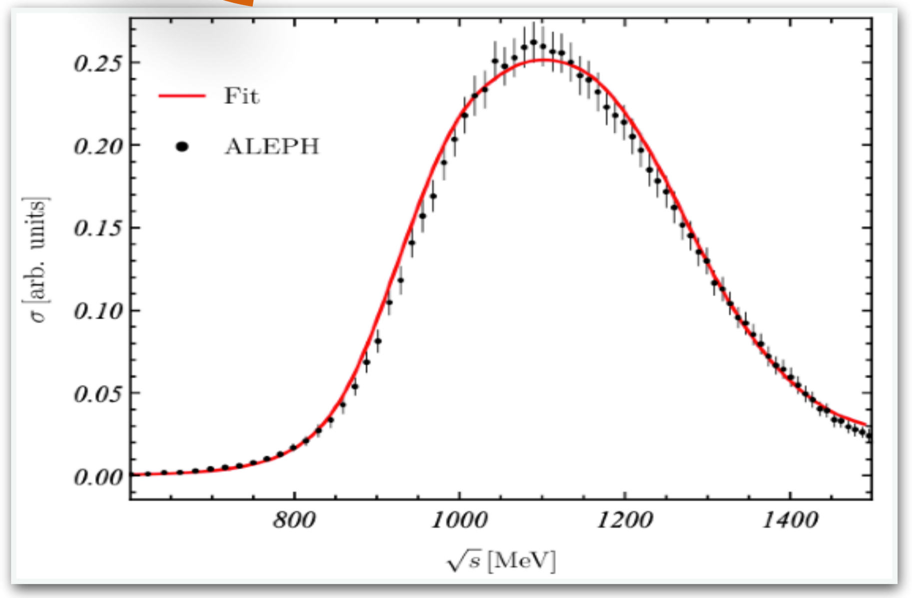


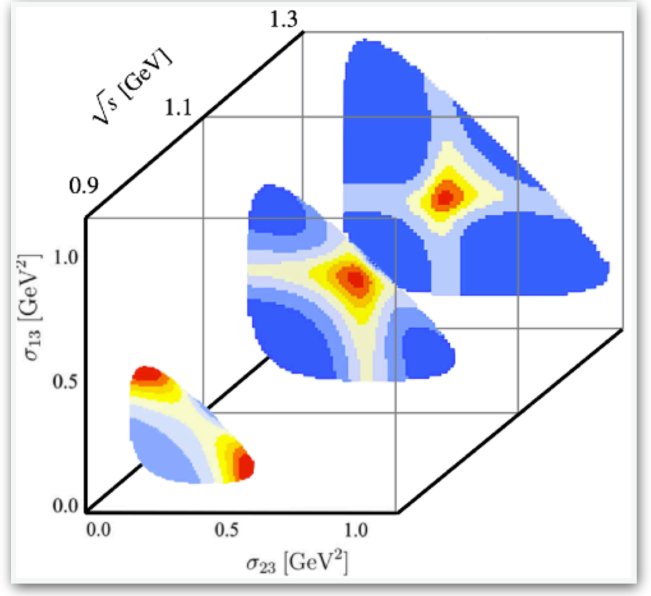




$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1}-\Sigma\right)-B-C\right]_{\mathbf{p}'\mathbf{p}}$$





Our Universe

"Heavier Universe"