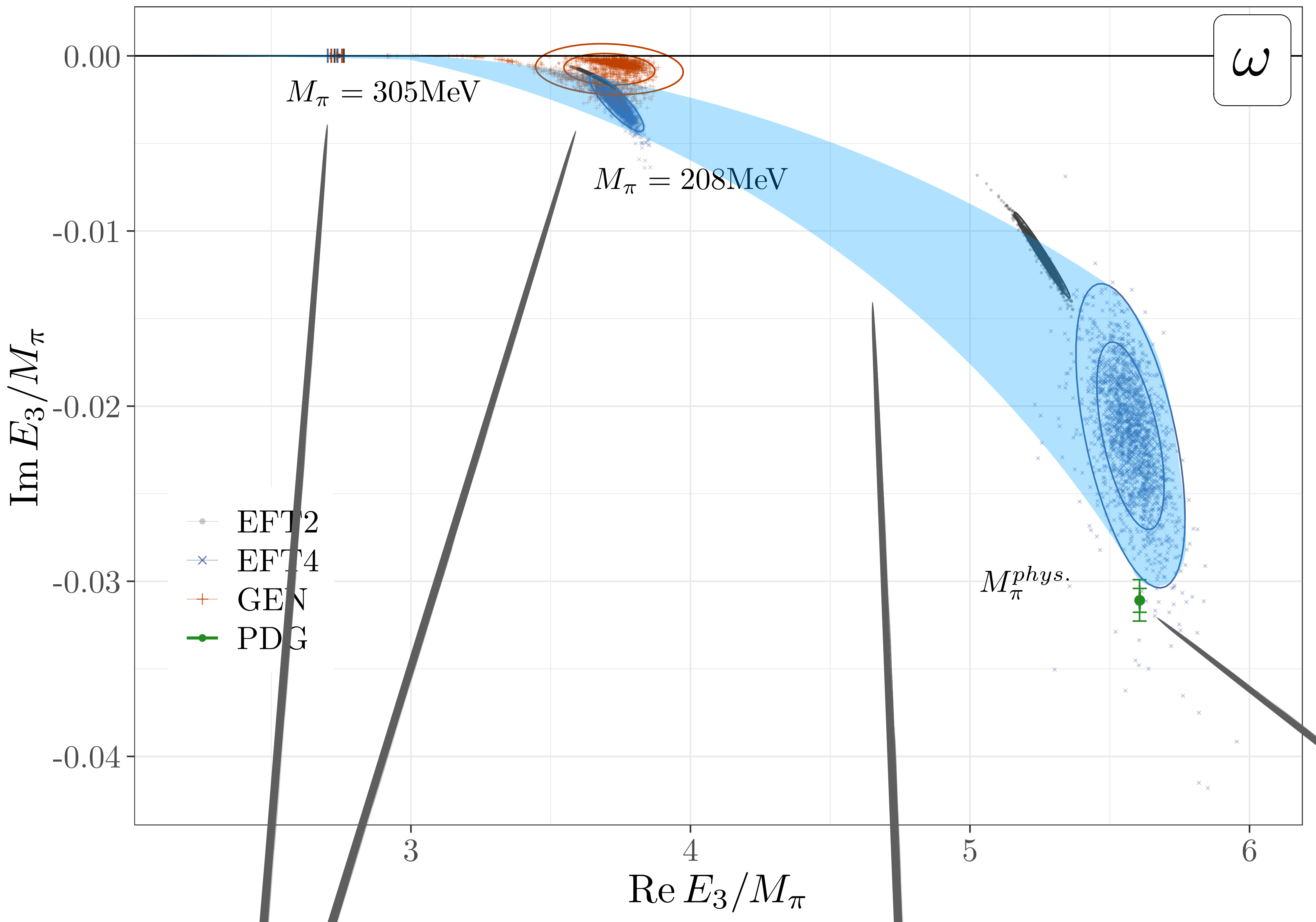
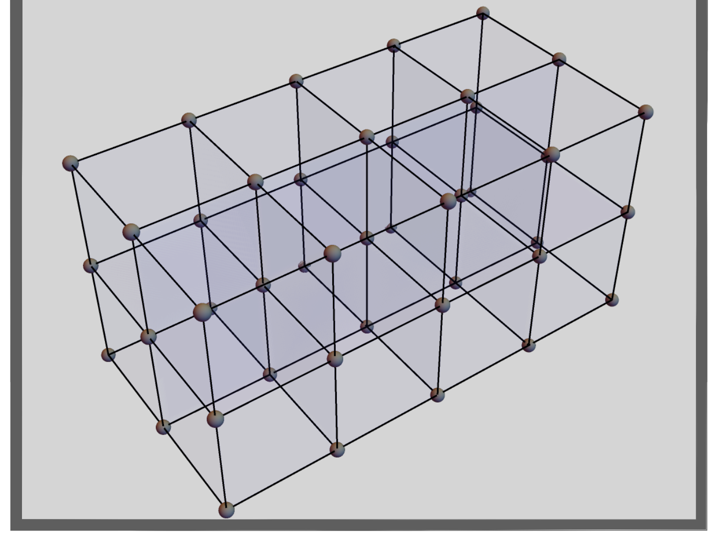
ω(782)

& other 3-hadron states from Lattice QCD and EFT



Lattice QCD



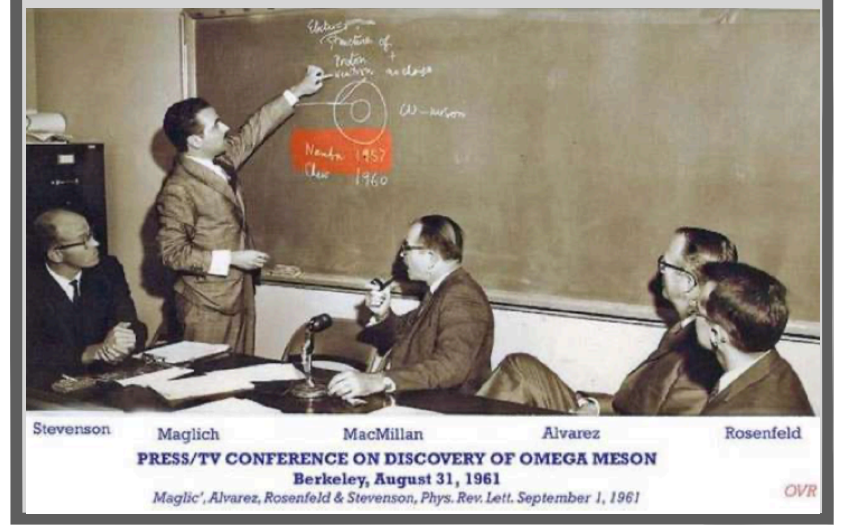
Effective Field Theory

$$\mathscr{L}_{\mathrm{VV\phi}} = g_{\mathrm{VV\phi}}\, \varepsilon_{\mu\nu\alpha\beta}\, \mathrm{Tr}[\partial^\mu V^\nu\, \partial^\alpha V^\beta \phi]\,,$$

$$\mathscr{L}_{\mathrm{V\phi\phi}} = (\mathrm{i}g/2)\, \mathrm{Tr}[V_\mu \phi\, \overleftrightarrow{\partial}^\mu \phi]\,,$$

$$\mathscr{L}_{\mathrm{V\phi\phi\phi}} = \mathrm{i}h\varepsilon_{\mu\nu\alpha\beta}\, \mathrm{Tr}[V^\mu\, \partial^\nu\phi\, \partial^\alpha\phi\, \partial^\beta\phi]$$

Experiment



MAXIM MAI

ALBERT EINSTEIN CENTER

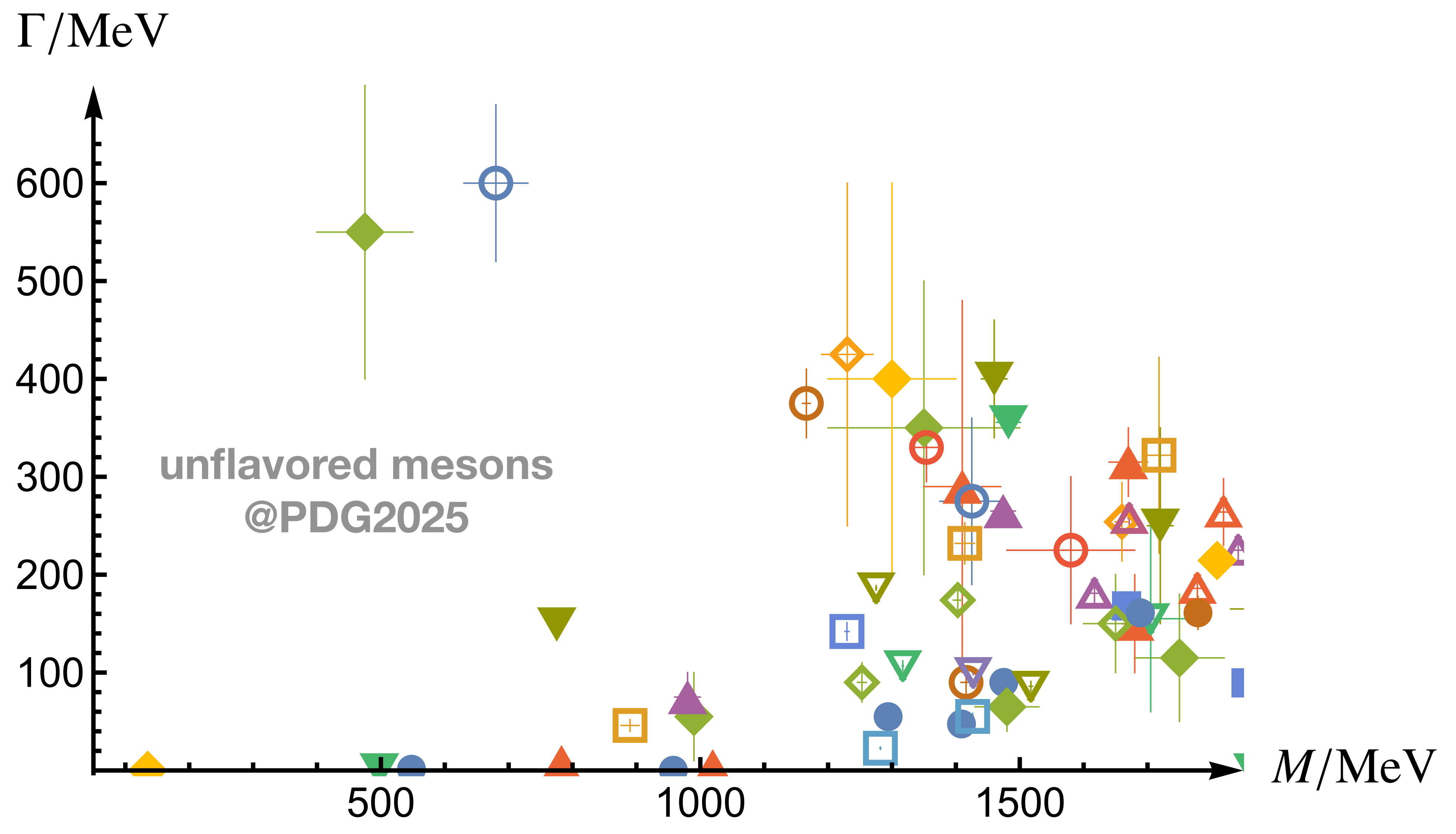
UNIVERSITY OF BERN

Hadron 2025 — Osaka 29.03.2025

DFG: Heisenberg Programme (project number: 532635001)

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

HADRON SPECTRUM



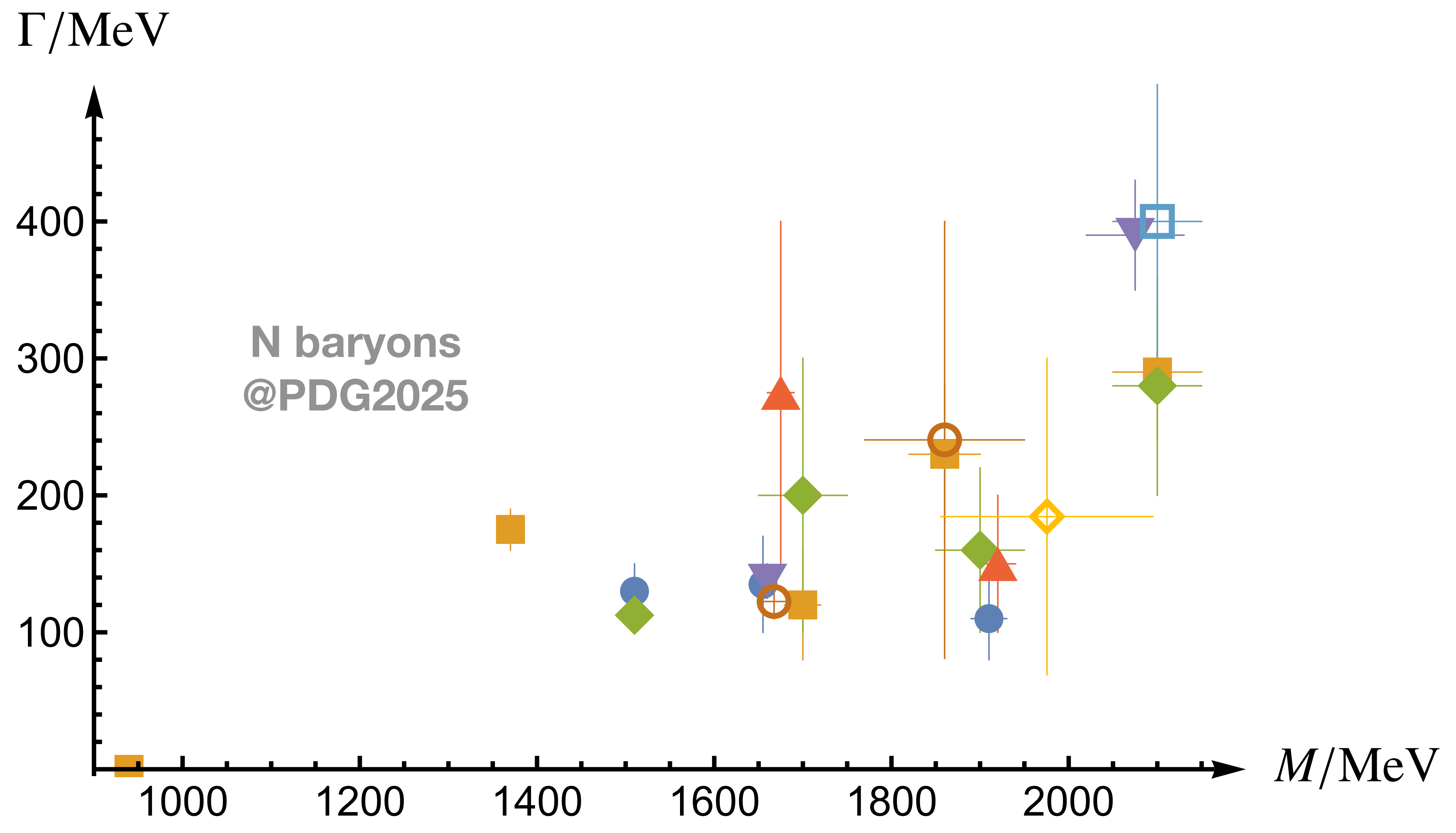
Experimental progress

- 70y research ($\Delta(1232), \rho(770), \omega(782), \ldots$)
- ongoing progress, new techniques and experiments

Talks: Pappagallo, Yelton, Meggiora, Küßner, Fabietti, Jude, Sakuma, Scordo, Shepherd…

- mostly excited states

≈ 100 mesons + 50 baryons (****)



Key questions

🦅 “what is the pattern of these states?“

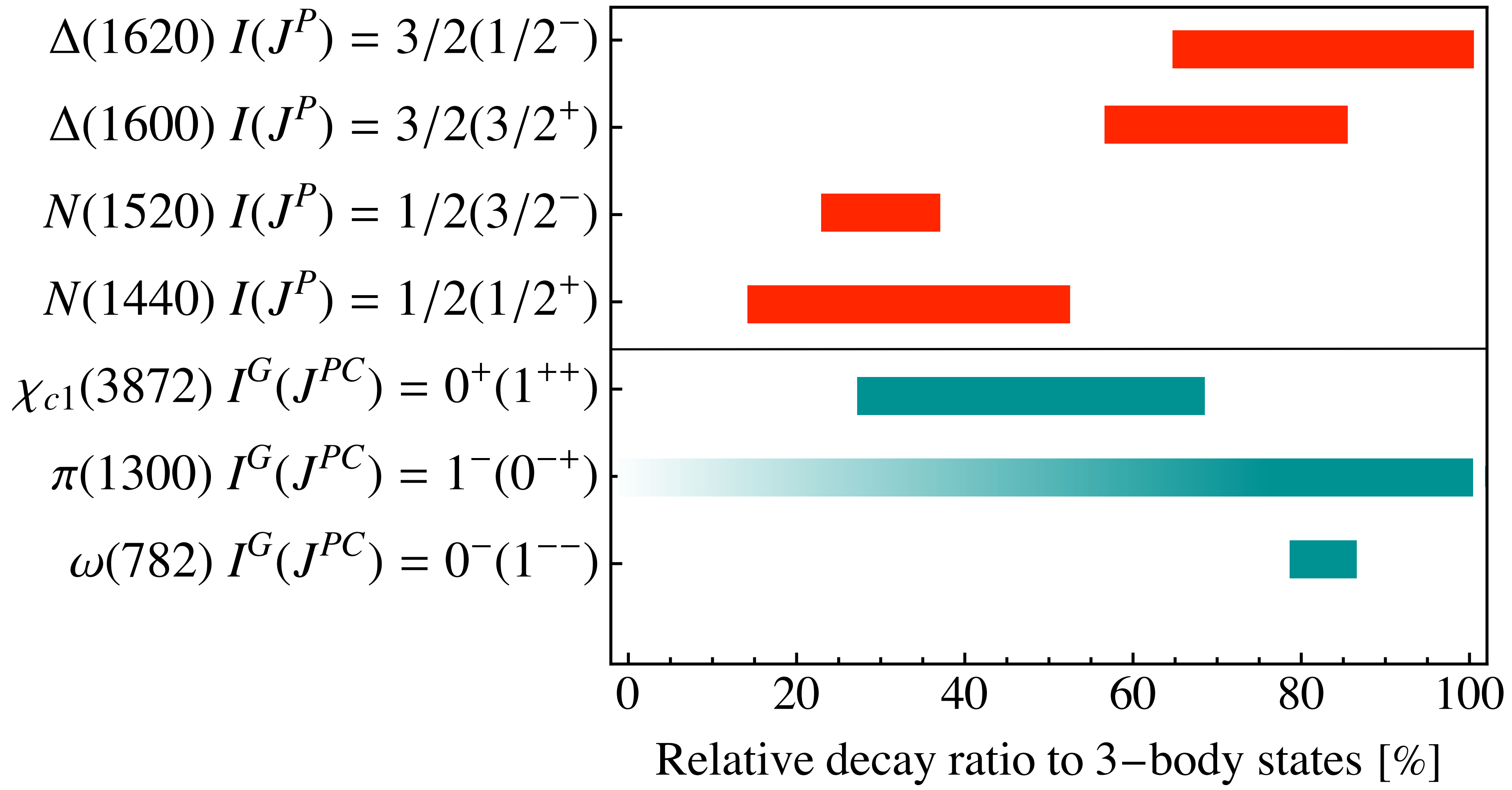
🐸 “how are they formed?”

Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6

HADRON SPECTRUM

Most known states have large 3-body content

- $\omega(782) \to \pi\pi\pi$
- $a_1(1260) \to \pi\pi\pi$
- $N(1440) \to \pi\pi N$
- $X(3872) \to DD\pi$



Beyond Standard Model searches (τ-EDM/…)

Belle@SuperKEKB, … Talk: Yelton, Yasaveev

Exotic states of matter

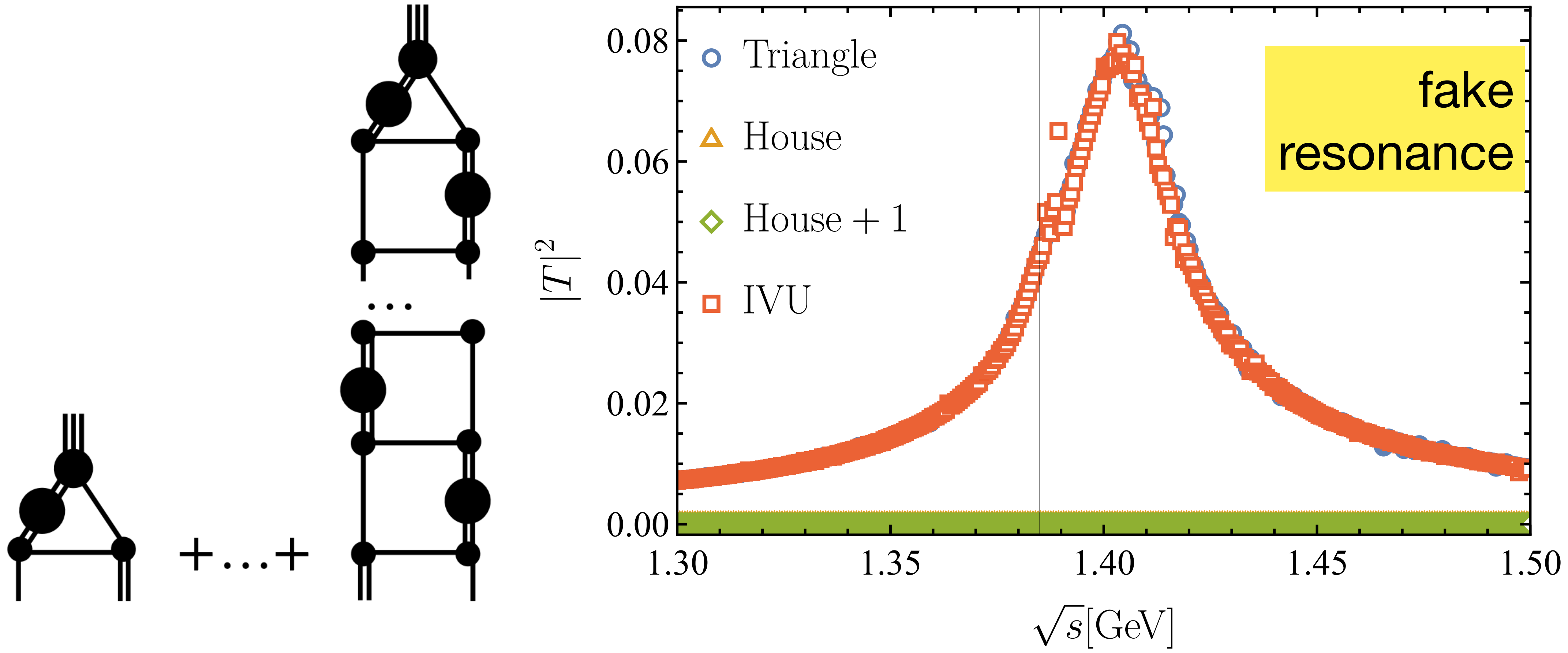
GlueX@JLAB, COMPASS@CERN, … Talks: Pappagallo

Singularity structure, long-range forces, left hand cuts…

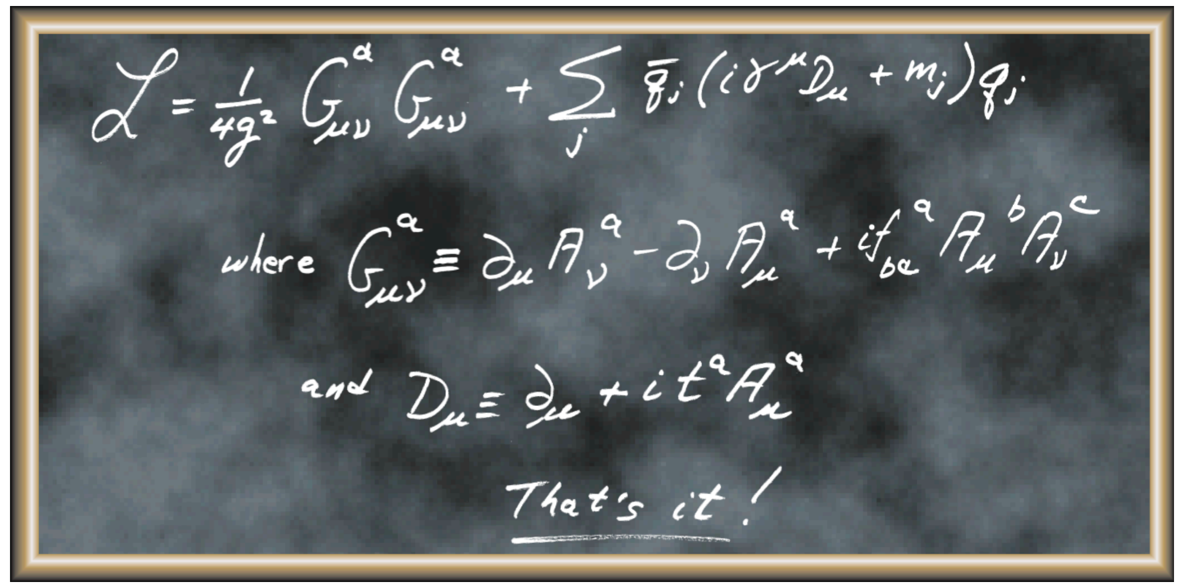
Talks: Hanhart, Guo, Wang, …

Mikhasenko:2015oxp Review: Guo:2019twa
Related: Dai:2018hqb, Dai:2018rra, Liang:2019jtr, Jing:2019cbw, Du:2021zdg, Duan:2023dky, Wang:2016dtb, Nakamura:2023obk, Zhang:2024dth, Achasov:2022onn, Nakamura:2023hbt, arXiv:1609.04133 [hep-ph].

POSTER SESSION TODAY — 🍷 🧀



THEORY



http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Low-energy regime of QCD = double trouble

- small relative momenta
- non-perturbative energy regime
- need to evaluate <u>infinitely</u> many diagrams

Further approaches: Functional methods, holography, K-matrix, dynamical models, …

Review: Eichmann/Sanchis-Alepuz/Alkofer/Fischer Prog.Part.Nucl.Phys. 91 (2016) 1-100
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Review: Döring/Haidenbauer/Sato/MM PPNP in progress

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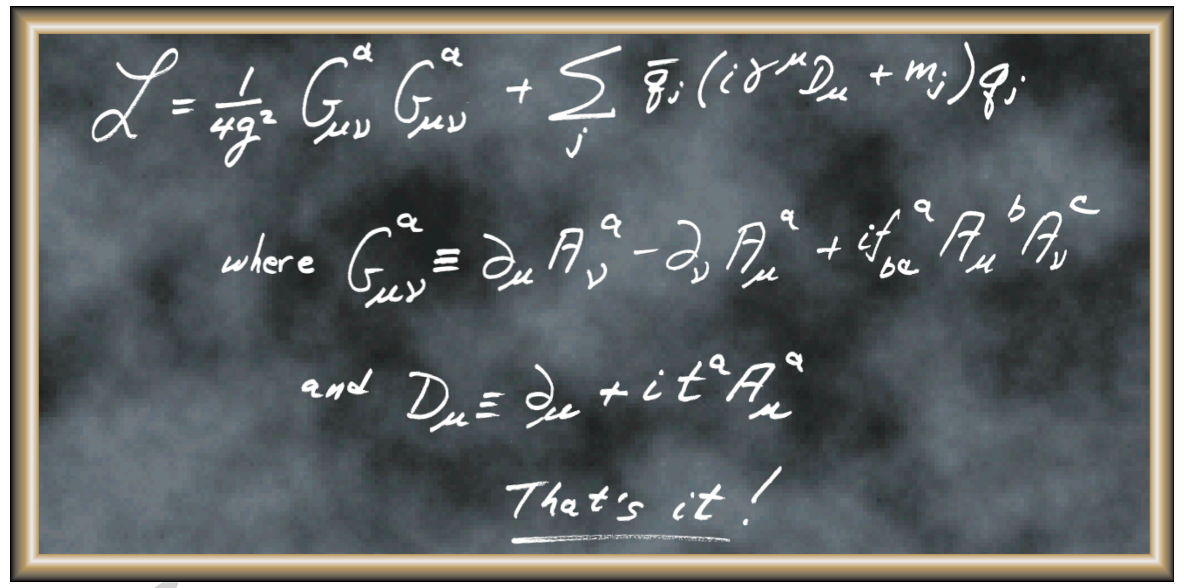

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$$Z[J] = \int [DU] e^{\int i d^4x \, \mathscr{L}_{\text{eff}}(U,v,a,s,p)}$$

Effective Field Theory (CHPT)

- Effective/Hadronic degrees of freedom
- <u>Infinitely</u> many low-energy constants
- Well-defined power counting
- Benchmark for many low-energy hadronic interactions

Weinberg (1979) Gasser, Leutwyler (1981), …
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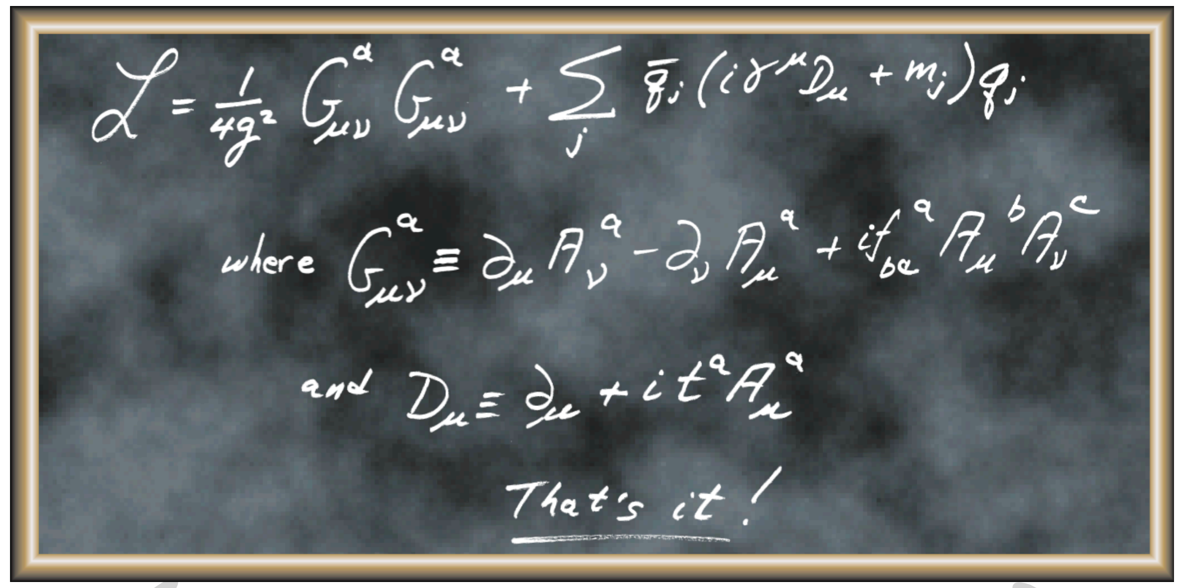

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$$Z[J] = \int [DU] e^{-S_E} \det[M[U]]$$

Lattice Gauge Theory

- QCD degrees of freedom
- discretized (Euclidean) space-time
- finite volume
- unphysical quark mass

Wilson, Phys. Rev. D10 (1974) 2445 , …
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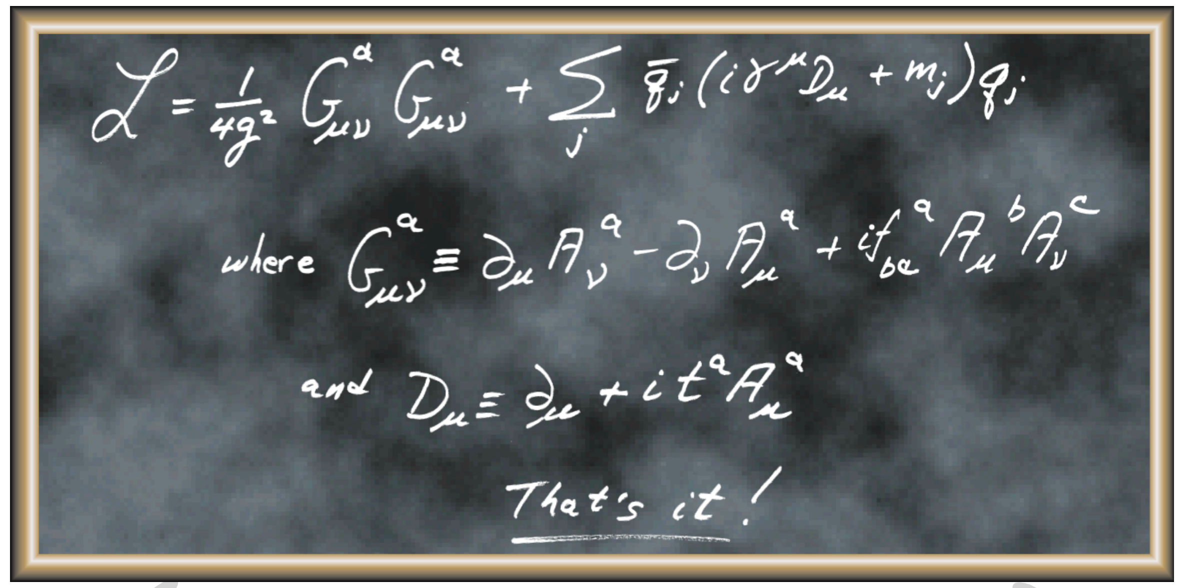

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quark mass dependence

first principle non-perturbative input

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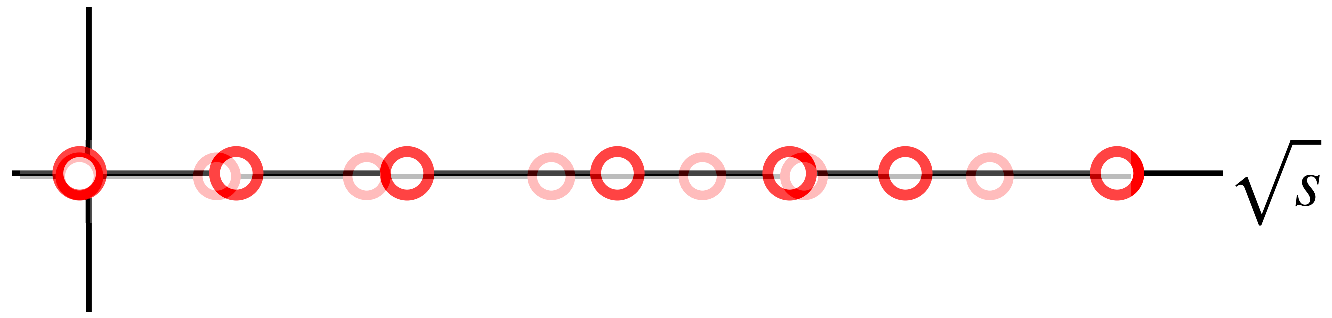
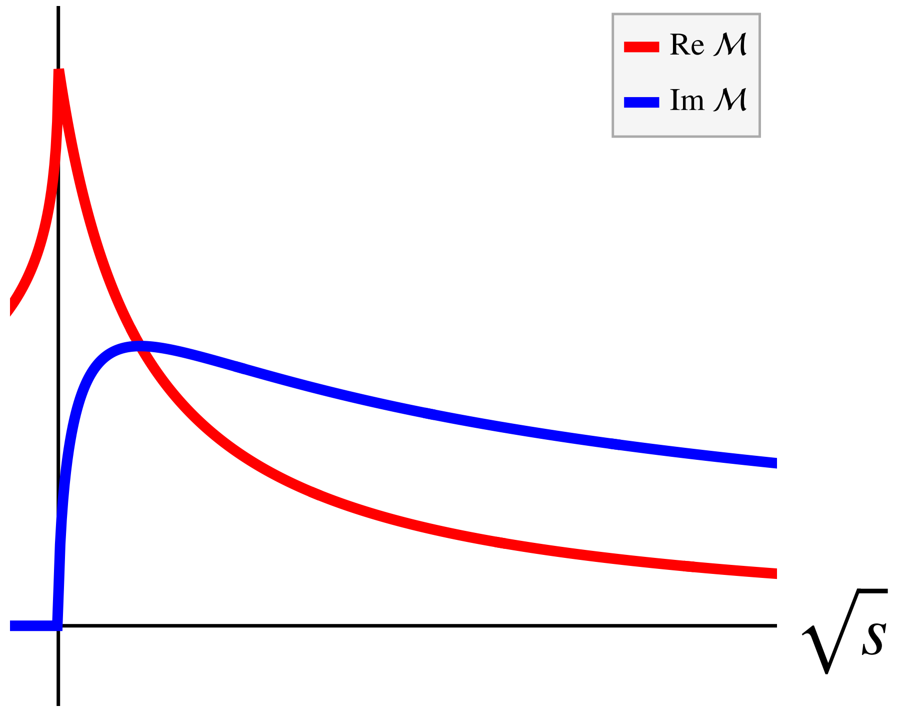
QUANTIZATION CONDITIONS

- Finite-volume calculations: no direct access to scattering quantities
- Real-valued energy eigenvalues
  - Shifted from free energies => physical information
  - **Relation to observables: Quantization condition**



Lattice QCD

continuum QFT



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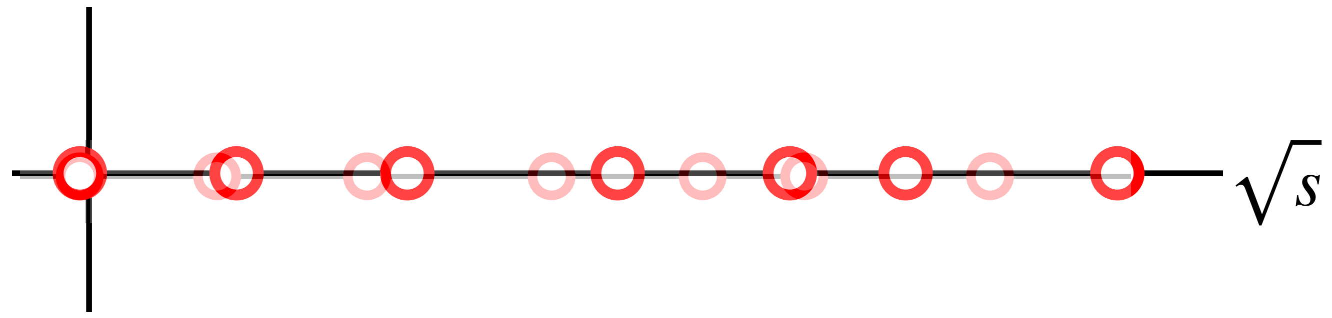

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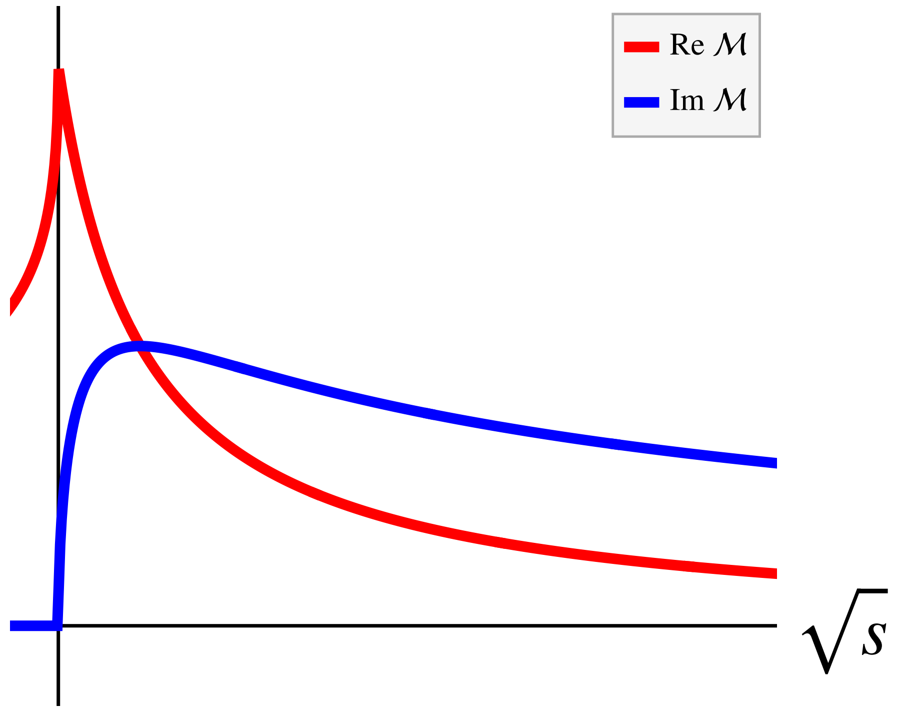
Lattice QCD

continuum QFT

- **one-way of thinking:**
  - on-shell states "feel" the box-size $\sim (ML)^n$
  - off-shell configurations decay exponentially $\sim e^{-ML}$

➡ separated by S-Matrix unitarity



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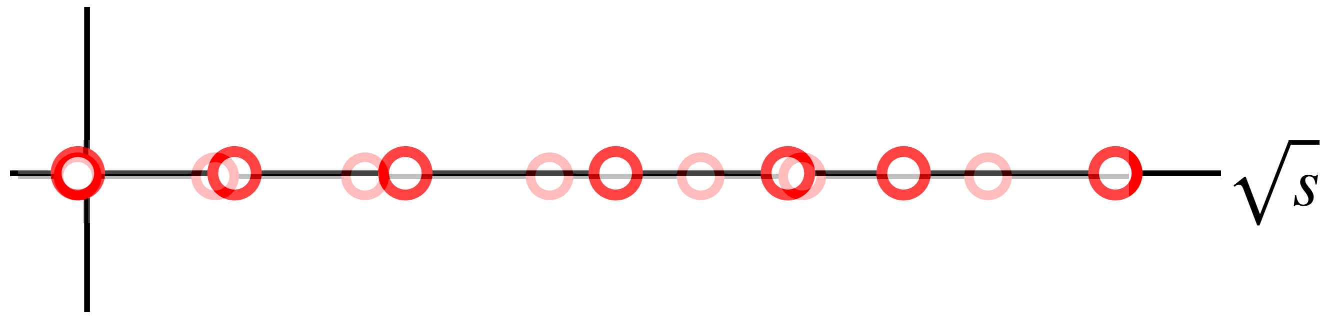

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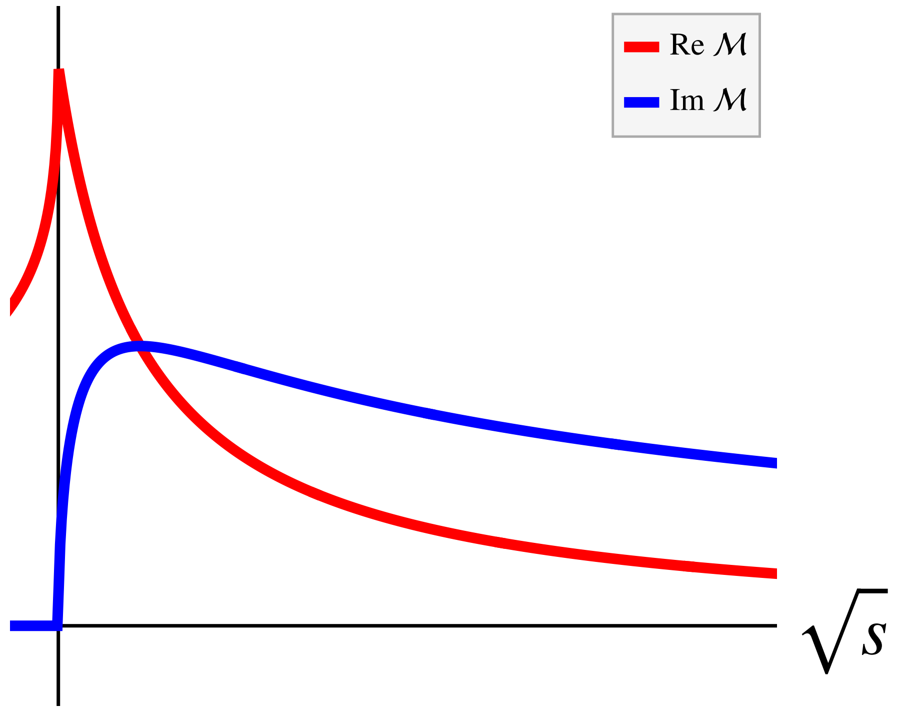
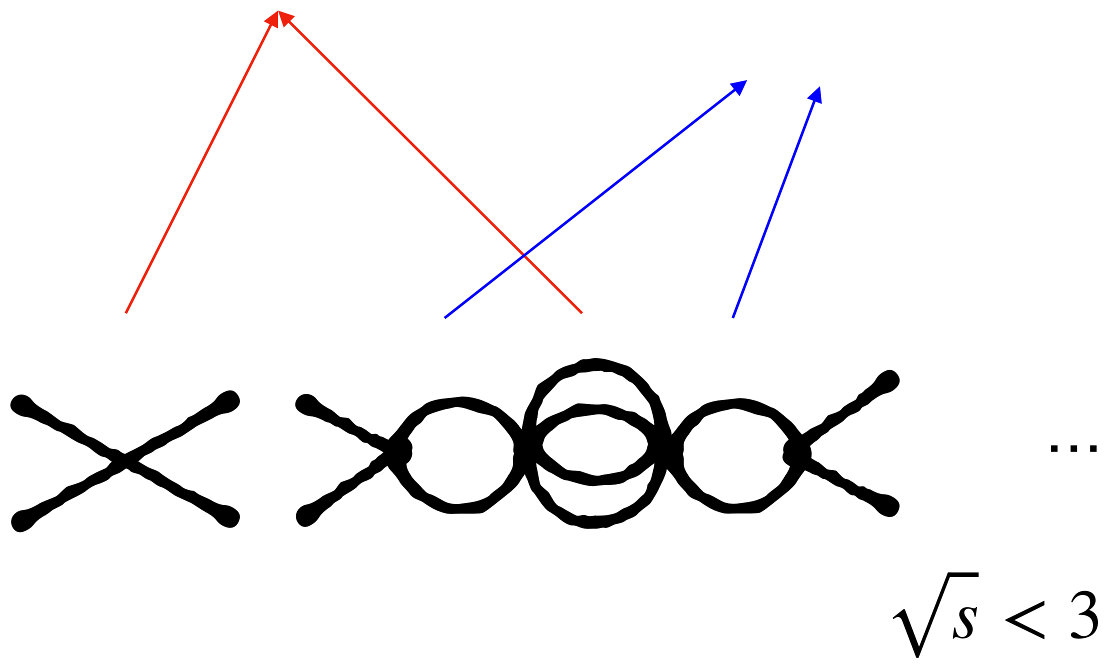
$$M_\infty^{-1} = \tilde{K}^{-1} - \int\frac{d^3l}{(2\pi)^3}\frac{1}{2E_l(s-4E_l^2+i\epsilon)}$$





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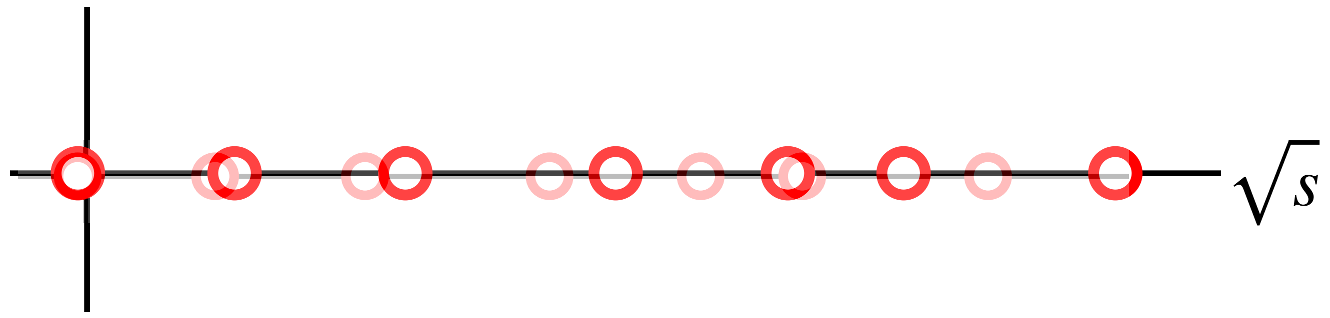

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Lattice QCD

continuum QFT

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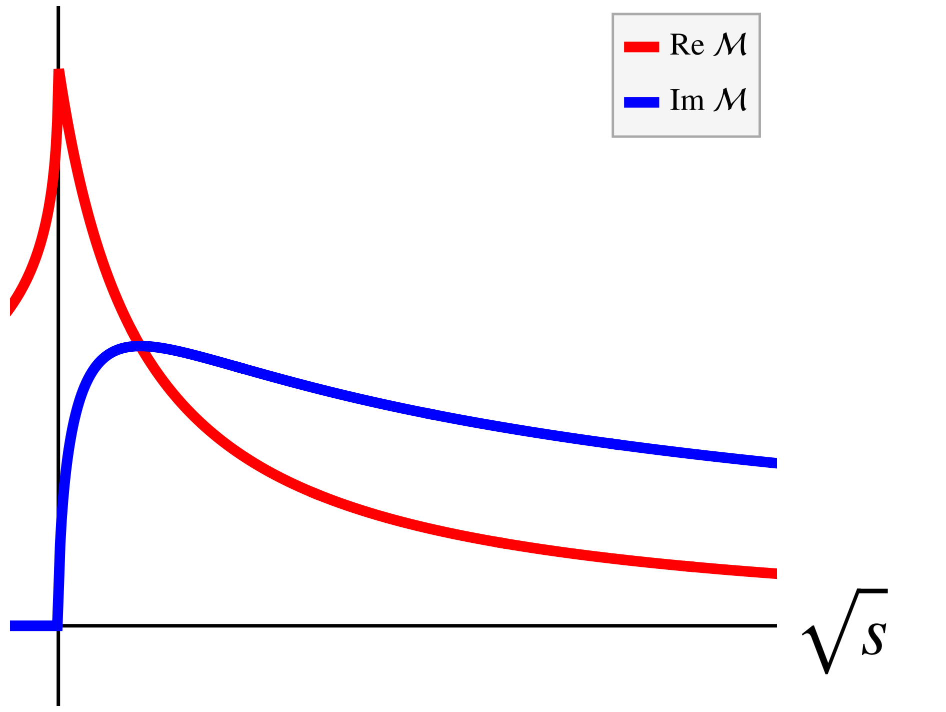

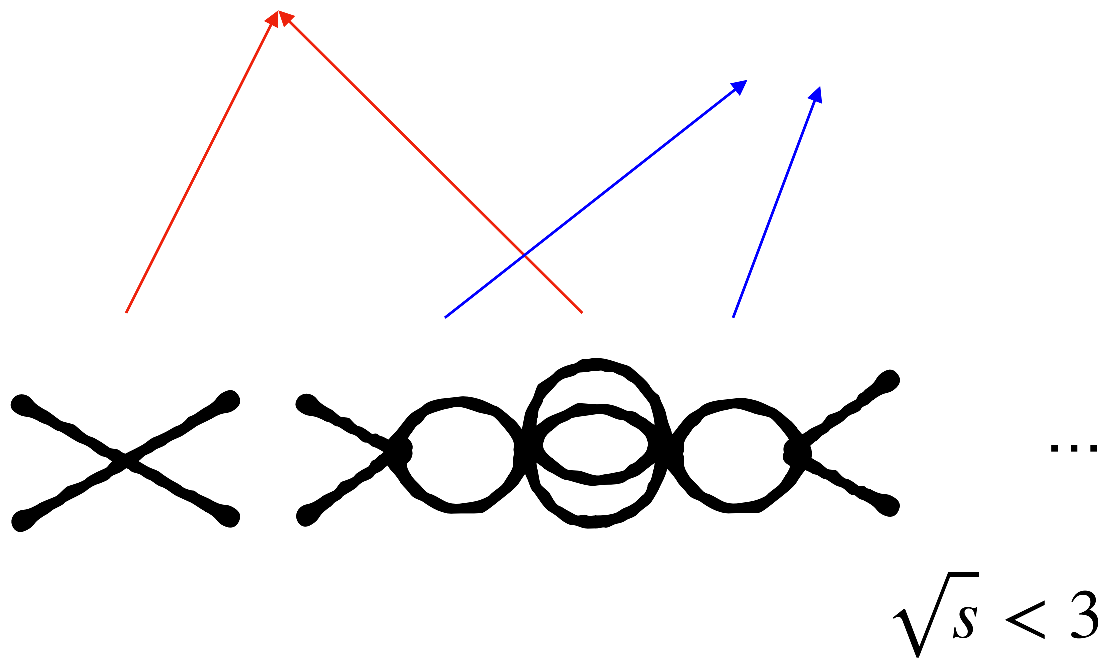

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3-HADRON SYSTEMS

Finite Volume Unitarity (FVU) approach

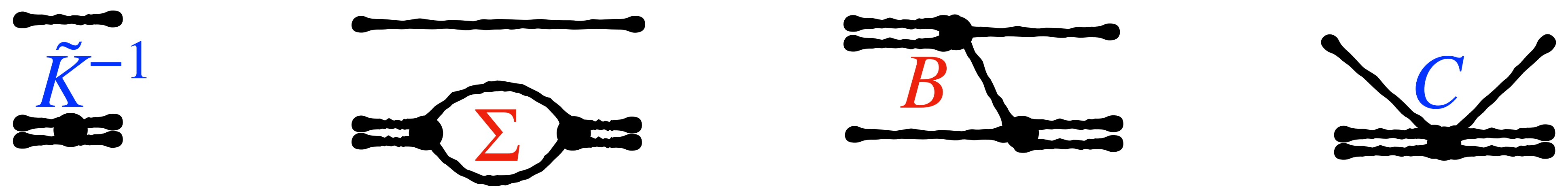
MM/Döring *Eur.Phys.J.A* 53 (2017) 12, 240

- 3-body unitarity accounts for all on-shell states
- genuine determinant condition
  - 2+1 spectator momentum integration

Faddeev, Schmid/Ziegelmann (1974), …

$\sqrt{s} < 4$

… more combinatorial possibilities



FVU Finite Volume Unitarity

$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1}-\Sigma^L\right)-B-C\,\right]_{\ell}^{\Lambda}\equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

IVU Infinite Volume Unitarity

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{\left(B+C\right)}{2E_l}\frac{1}{\tilde{K}^{-1}-\Sigma}T^c$$

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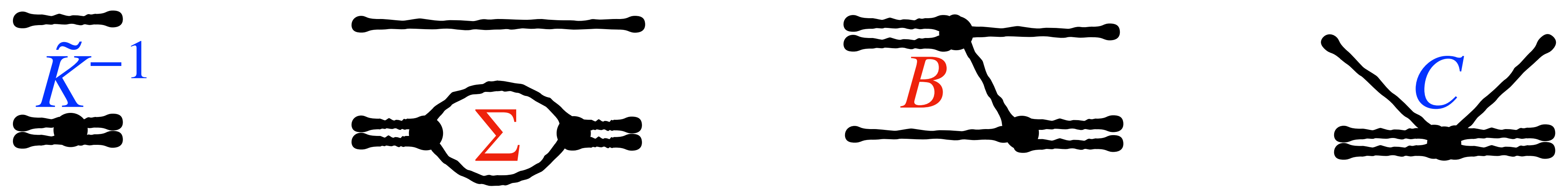

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MM/Döring
***Eur.Phys.J.A* 53 (2017) 12, 240**

Alternatives: RFT, NREFT

RFT(Hansen/Sharpe 2014) NREFT(Rusetsky/Hammer/Pang 2017)

- physically equivalent, not equal
  Jackura et al. *Phys.Rev.D* 100 (2019) 3, 034508, Garofalo et al. *JHEP* 02 (2023) 252

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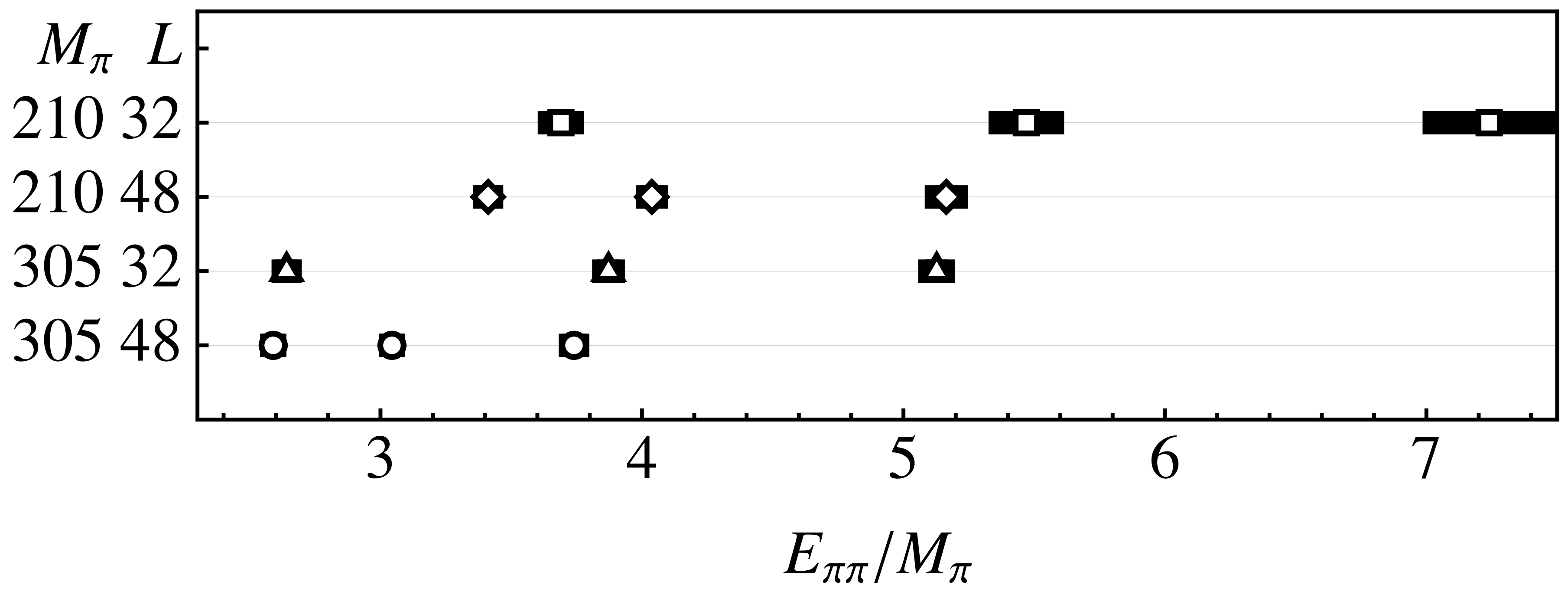
Many new applications

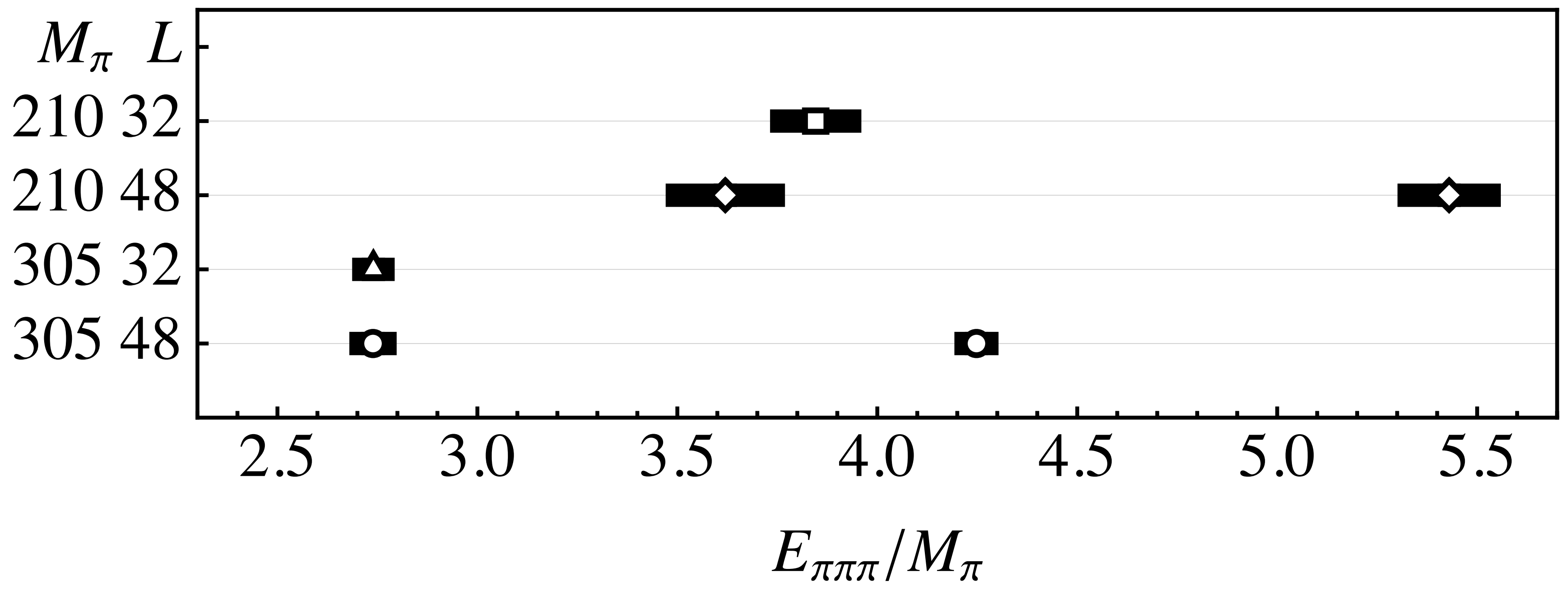
- proof of concepts and spin-less repulsive systems
  MM/Doring Phys.Rev.Lett. 122 (2019), Fischer et al. Eur.Phys.J.C 81 (2021), Blanton, Lopez, Hansen, Briceno, …
- systems with left-hand cut
  Hansen et al. *JHEP* 06 (2024), Dawid et al. *JHEP* 01 (2025), Rusetsky, Guo, …**Talks Guo, Hanhart, …**
- 3-body resonant systems
  MM/Culver Phys.Rev.Lett. 127 (2021)
  Yan et al. Phys.Rev.Lett. 133 (2024)

$\omega \to \pi\pi\pi$

Lattice QCD (more details next talk)

- 2+1 Flavors, 2/3 particle operators
- 2 pion masses, 2 volumes





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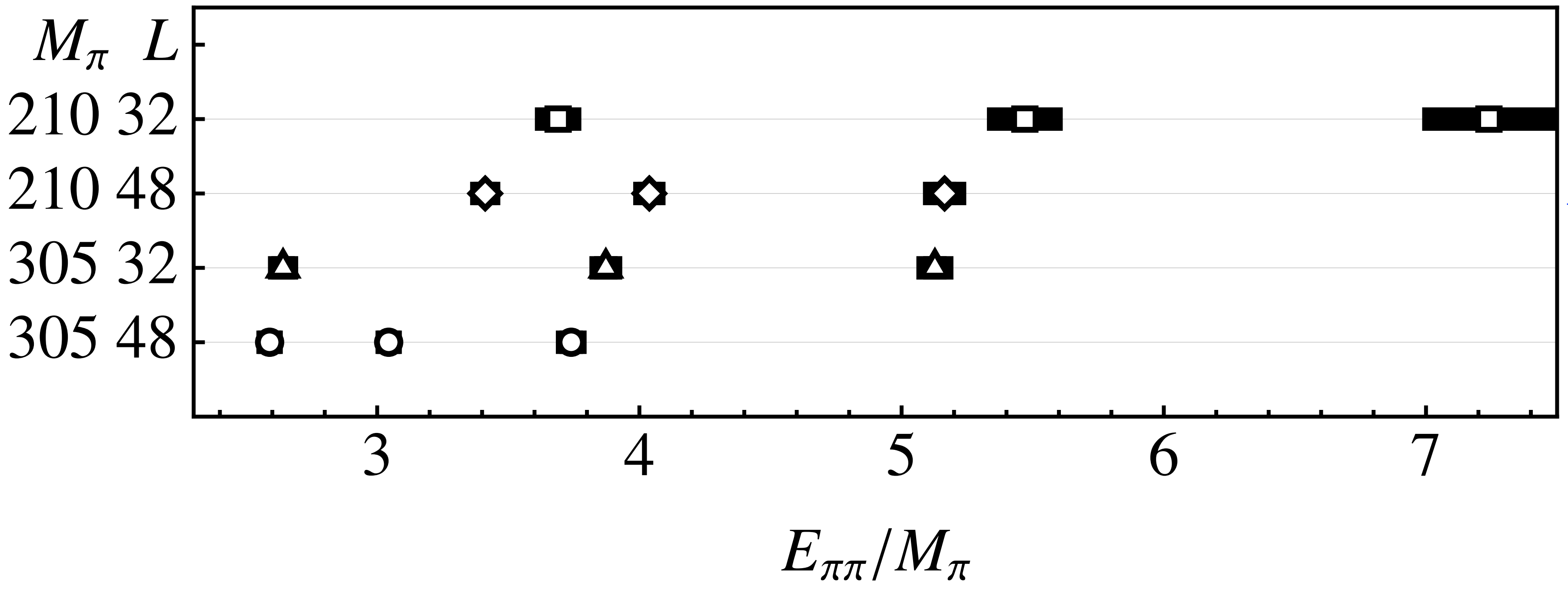
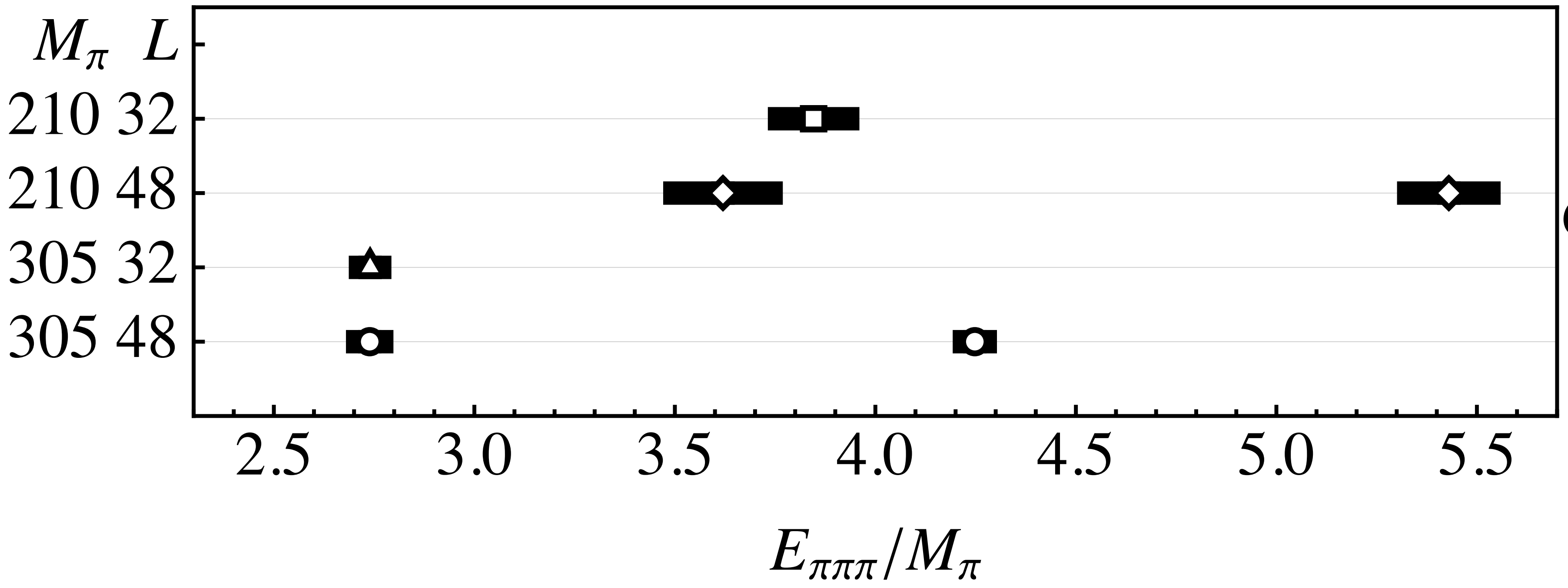


$$\tilde{K}^{-1} - \Sigma^L \equiv 0$$

explicitly known

Parametrize and fit



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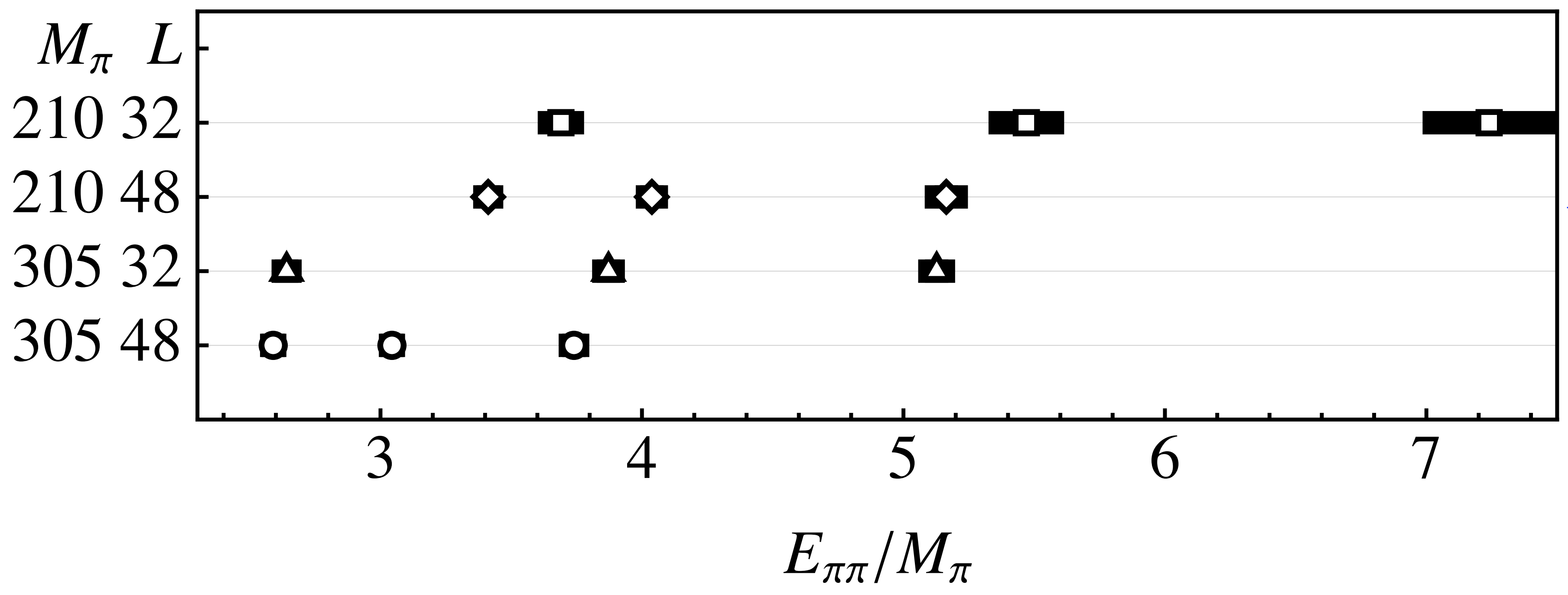
$$C = \frac{c_0}{s - M_\omega^2} + c_1 + \dots \qquad \tilde{K}^{-1} = \delta_{\lambda'\lambda}\delta_{\mathbf{p}'\mathbf{p}} \sum_{i=0}^{N} a_i \sigma_p^i$$

GENeric

$$C = \frac{6s(M_\rho^2 - \sigma_q + 6g^2 f_\pi^2)(M_\rho^2 - \sigma_p + 6g^2 f_\pi^2)}{64 g^2 \pi^3 f_\pi^6 (s - M_\omega^2)} \qquad \tilde{K}^{-1} = \delta_{\lambda'\lambda}\delta_{\mathbf{p}'\mathbf{p}} \frac{\sigma_p - M_\rho^2}{2g^2}$$

EFT



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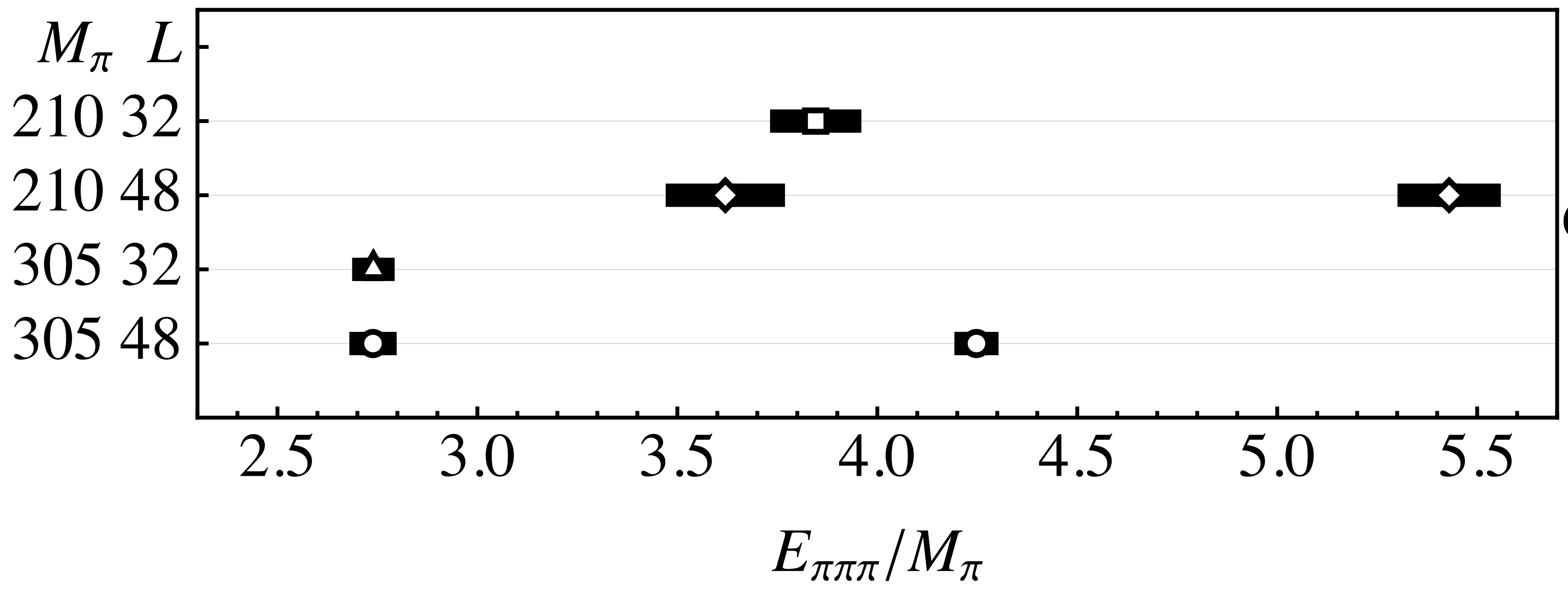

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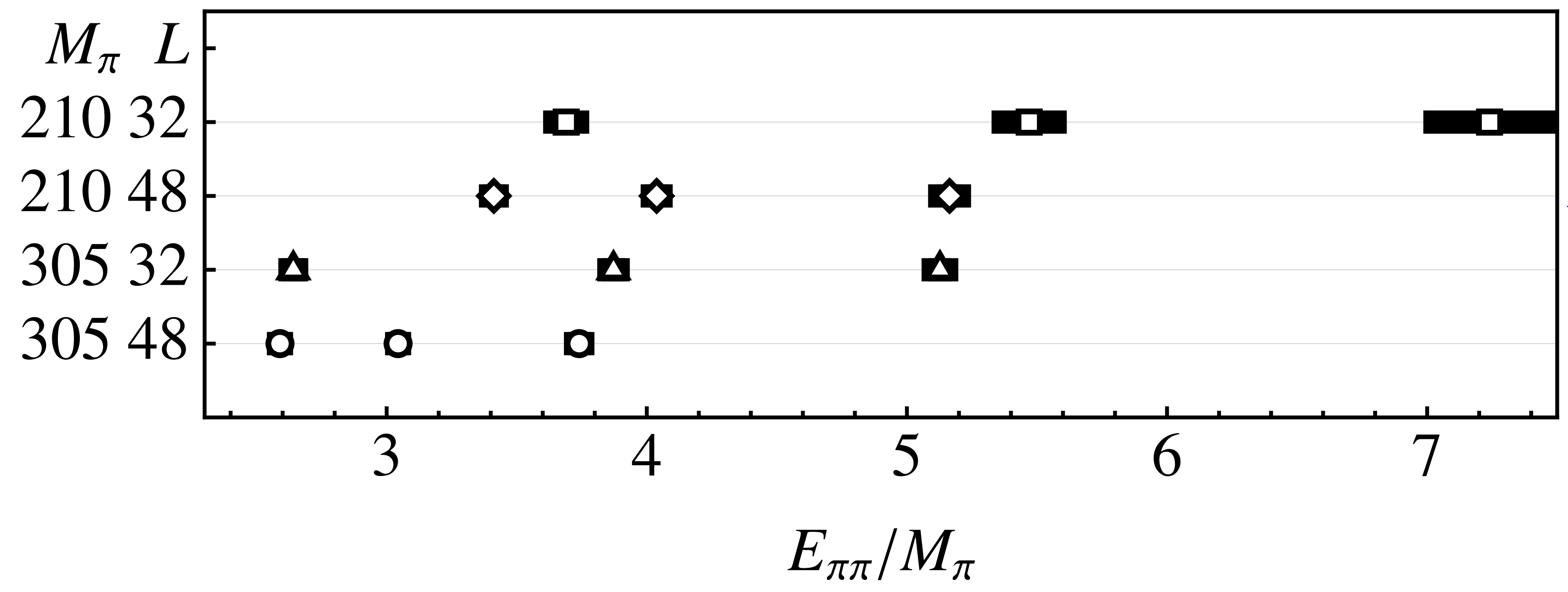

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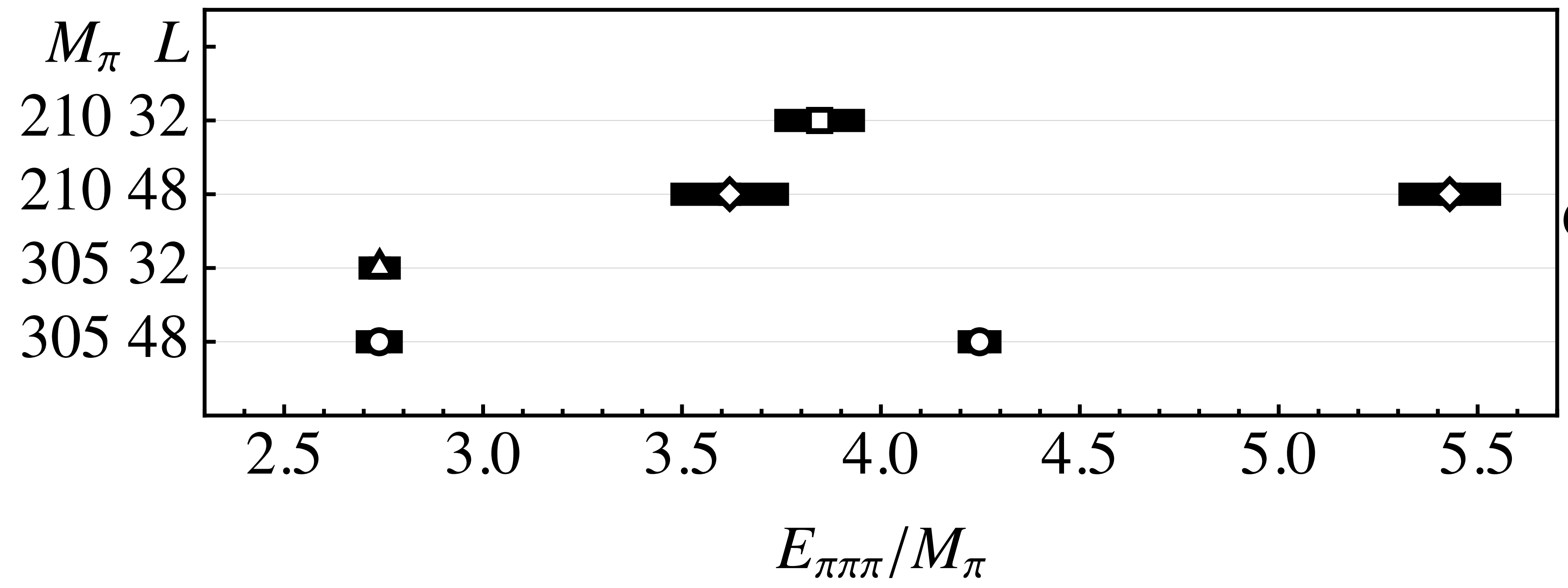
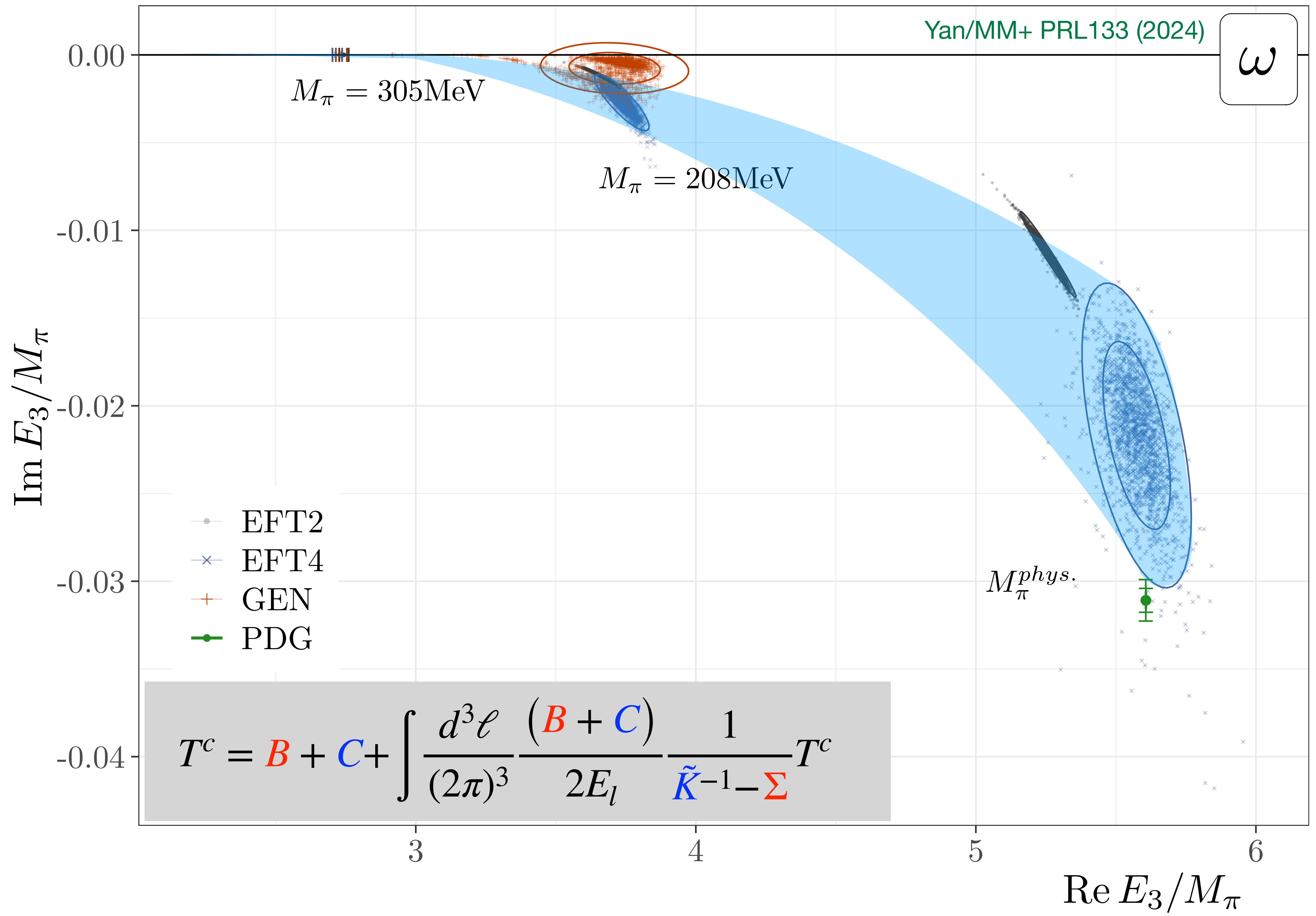


$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$



$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}^{-1} - \Sigma} T^c$$

... similar program carried out for $a_1(1260)$
MM/Culver+ PRL127 (2021)

SUMMARY / OUTLOOK

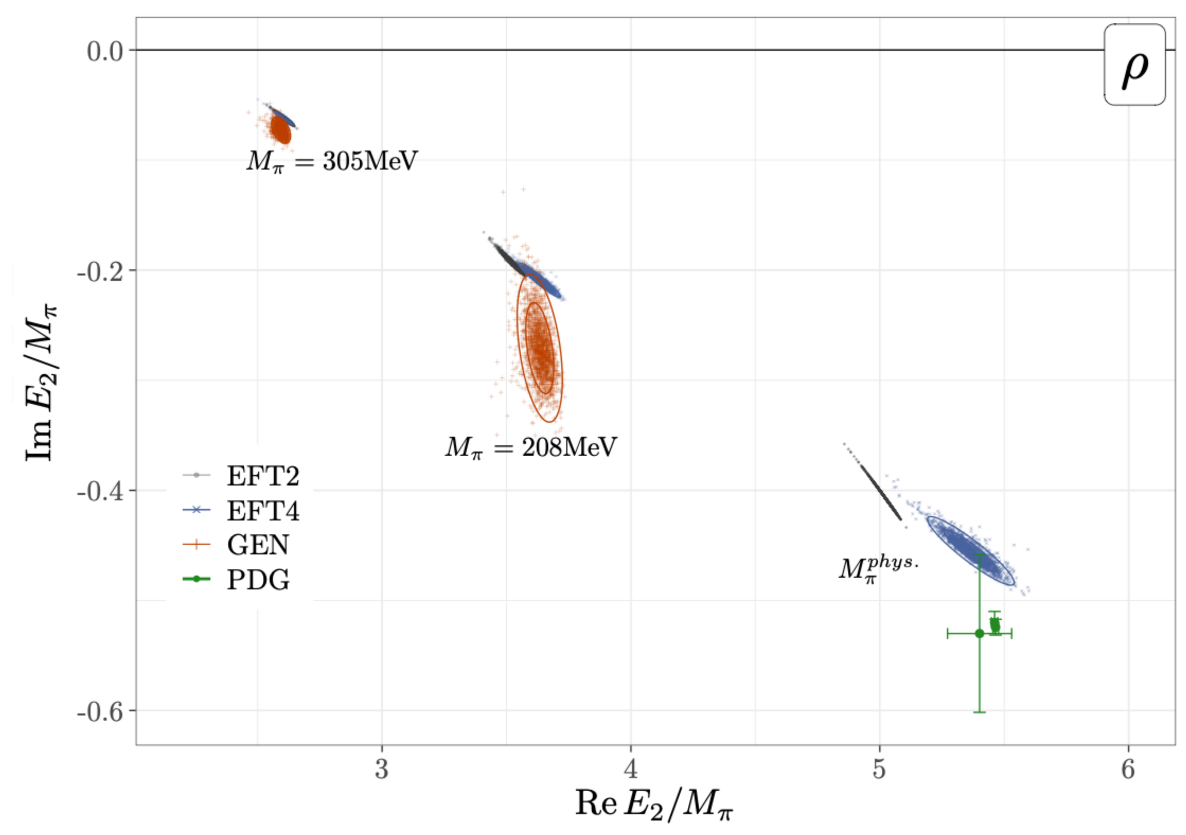
2body EFT+LQCD many new insights

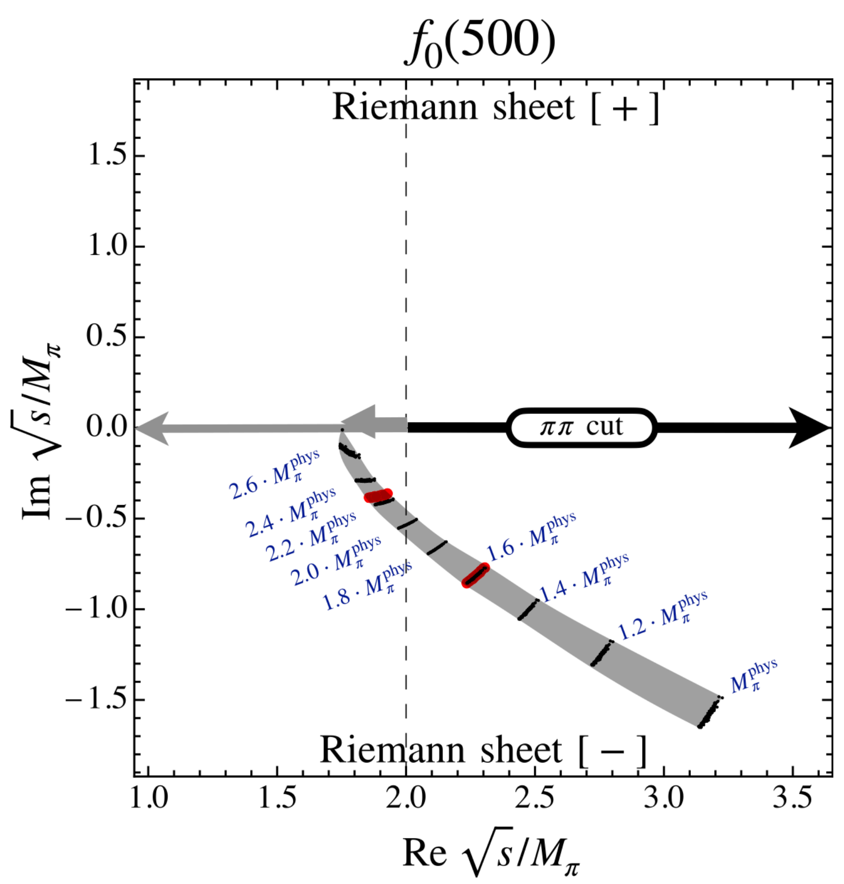
- ☑ $f_0(500), \rho(770), \ldots$ well established quark-mass dependence
- ☑ Two-pole structure: $\Lambda(1405), \Lambda(1380)$ **discovered/confirmed**

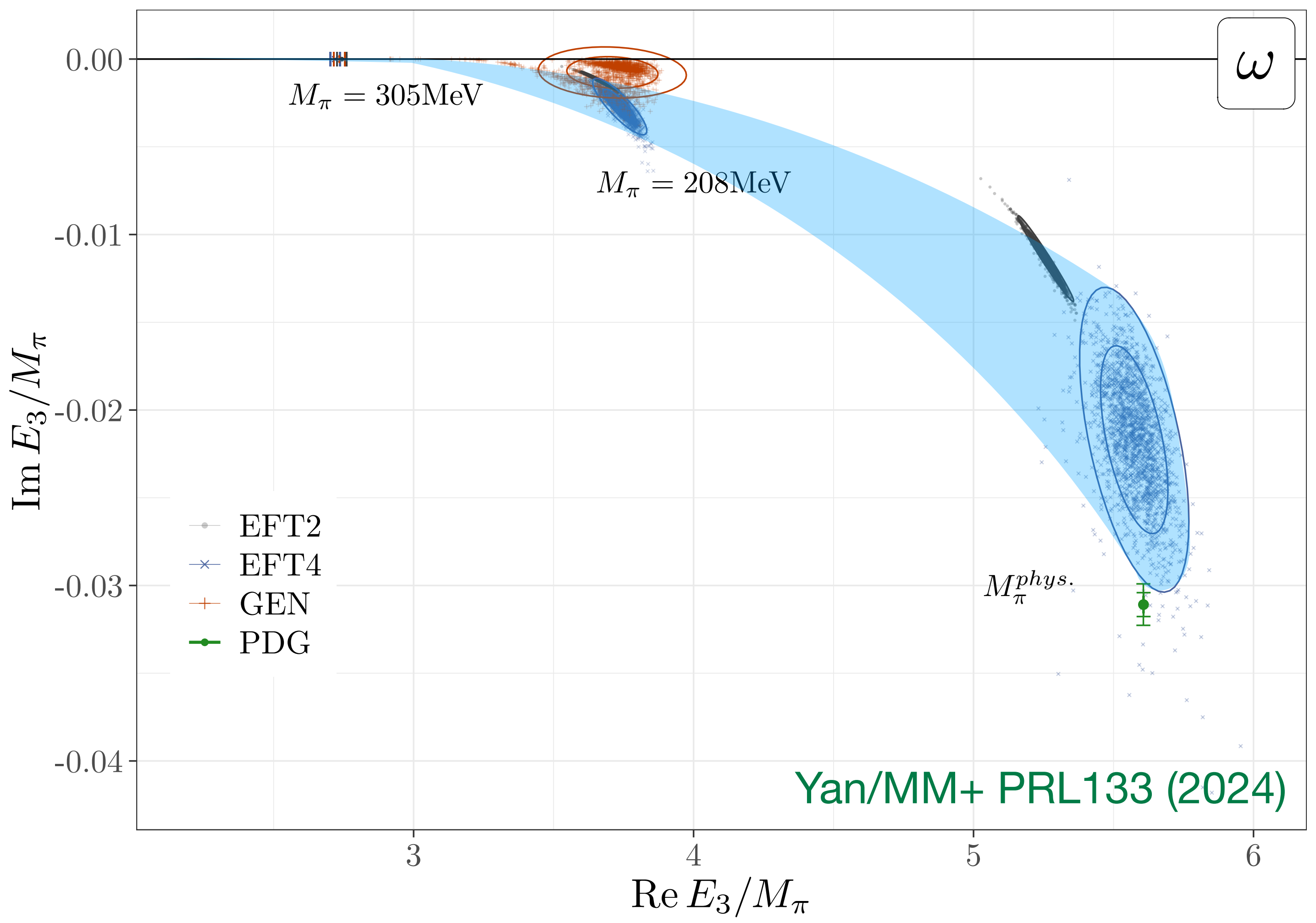
  [BaSc] Bulava et al. PRL 132 (2024) 5; 2307.13471,
  UCHPT Guo+ Phys.Lett.B 846 (2023) 138264, Zhuang+ 2405.07686 [hep-ph] —>Talk Zhuang

Novel FVU 3b Quantization Condition

- ☑ pilot results on $3\pi(I=3{,}2..), a_1(1260), \phi^4, \ldots$
- ☑ **Re-discovered $\omega(782)$ from QCD**
  - pole and chiral trajectories







Outlook

- ☐ $DD\pi$, spin-exotics, $N(1440)$, …

  Talk Guo, Hanhart, …
- ☐ Triangles: $a_1(1420)$

  first steps: JHEP 10 (2024) 246
- ☐ strangeness channels…

  Feng+ *Phys.Rev.D* 110 (2024)

POSTER TODAY —