NUCLEAR FUSION

PRINCIPLES AND PROGRESS

Maxim Mai

University of Bonn

08.03.2024
Habilitationskolloquium

“black board with many formulas blending into a nuclear fusion reactor”
MM@ChatGPT4.0

OUTLINE

I. Motivation

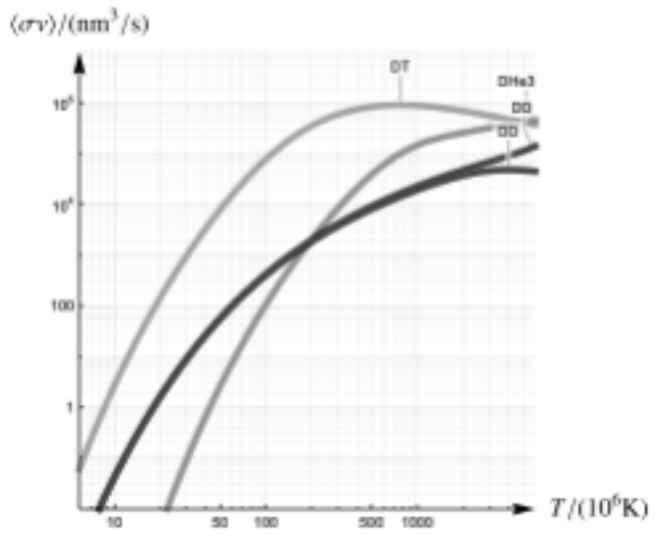
II. Fundamental principles

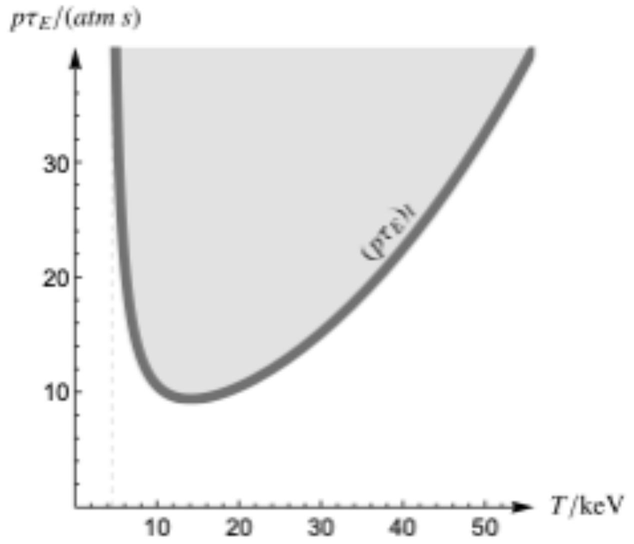
nuclear reactions,

electrostatic repulsion,

reaction rates,

plasma, …





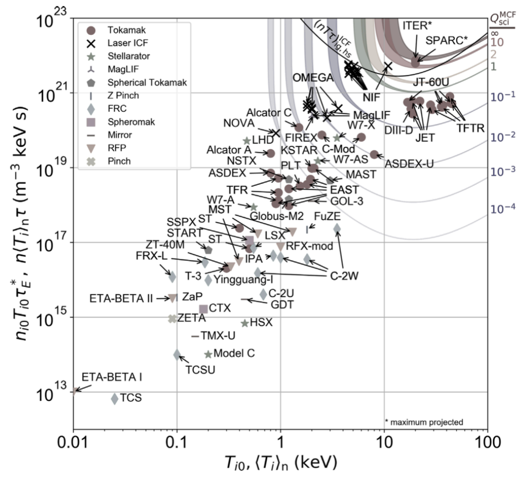
III. Technical implementation

Lawson criterium,

ignition,

confinement,

fuel self-sufficiency, …



IV. Summary

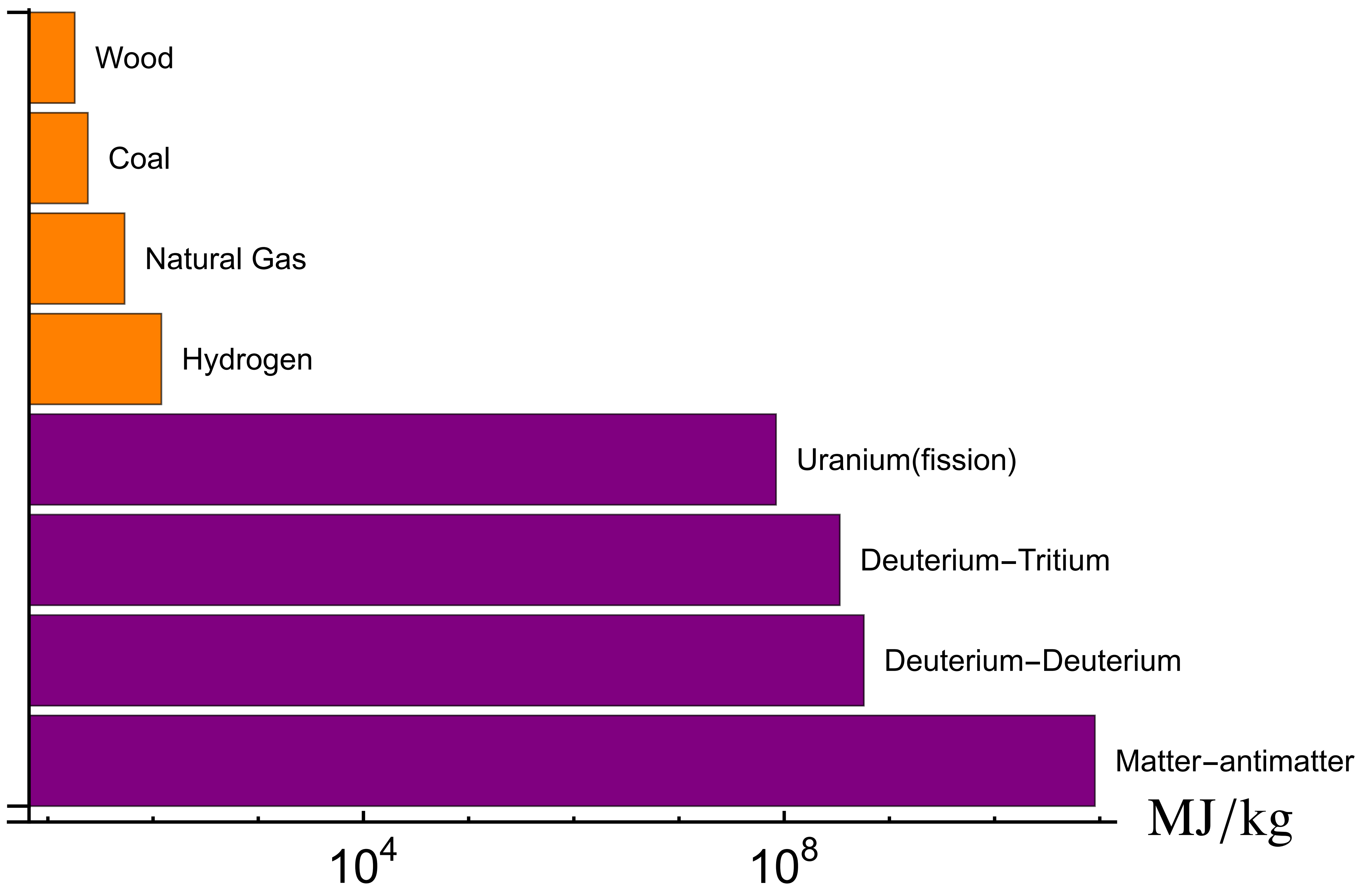
“black board with many formulas blending into a nuclear fusion reactor”
MM@ChatGPT4.0

MOTIVATION

Electricity production

	known since	availability	impact on nature
fossil fuels	~1880s	flexible (60-900y)	CO^{2} pollution
natural	~1900s	local	areal changes
renewable	~1890s	stochastic	areal changes
nuclear fission	~1950s	flexible (200y+)	nuclear waste
nuclear fusion	?	flexible(?)	~ 0(?)

Energy density[1]



---

[1] International Energy Agency (IEA), Department of Energy (DOE), Nuclear Energy Agency (NEA)

MOTIVATION

Nuclear fusion in “action”

1. stars
   - gravity: high density, temperature $T \sim 10^7\,\mathrm{K}$
   - ${}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + e^+ + \nu \qquad \Delta E = 0.42\,\mathrm{MeV}$
   - CNO and tripple-alpha processes — fine-tuning[1]
2. fission/fusion weapons
   - fission stage: high density, temperature $T \sim 10^8\,\mathrm{K}$
   - fusion stage: deuterium-tritium reaction

fission weapons ⇜⇝ commercial reactors = $\mathcal{O}(10y)$

… can one de-weaponise fusion? *[2]*

[1] Adams, Phys. Rept. 807 (2019) 1; Lähde et al. Eur.Phys.J.A 56 (2020) 3
[2] Dittmar, Energy 37 (2012) 35-40

II. FUNDAMENTAL PRINCIPLES

Power balance

electrostatic repulsion

Ignition

Lawson criterion

…

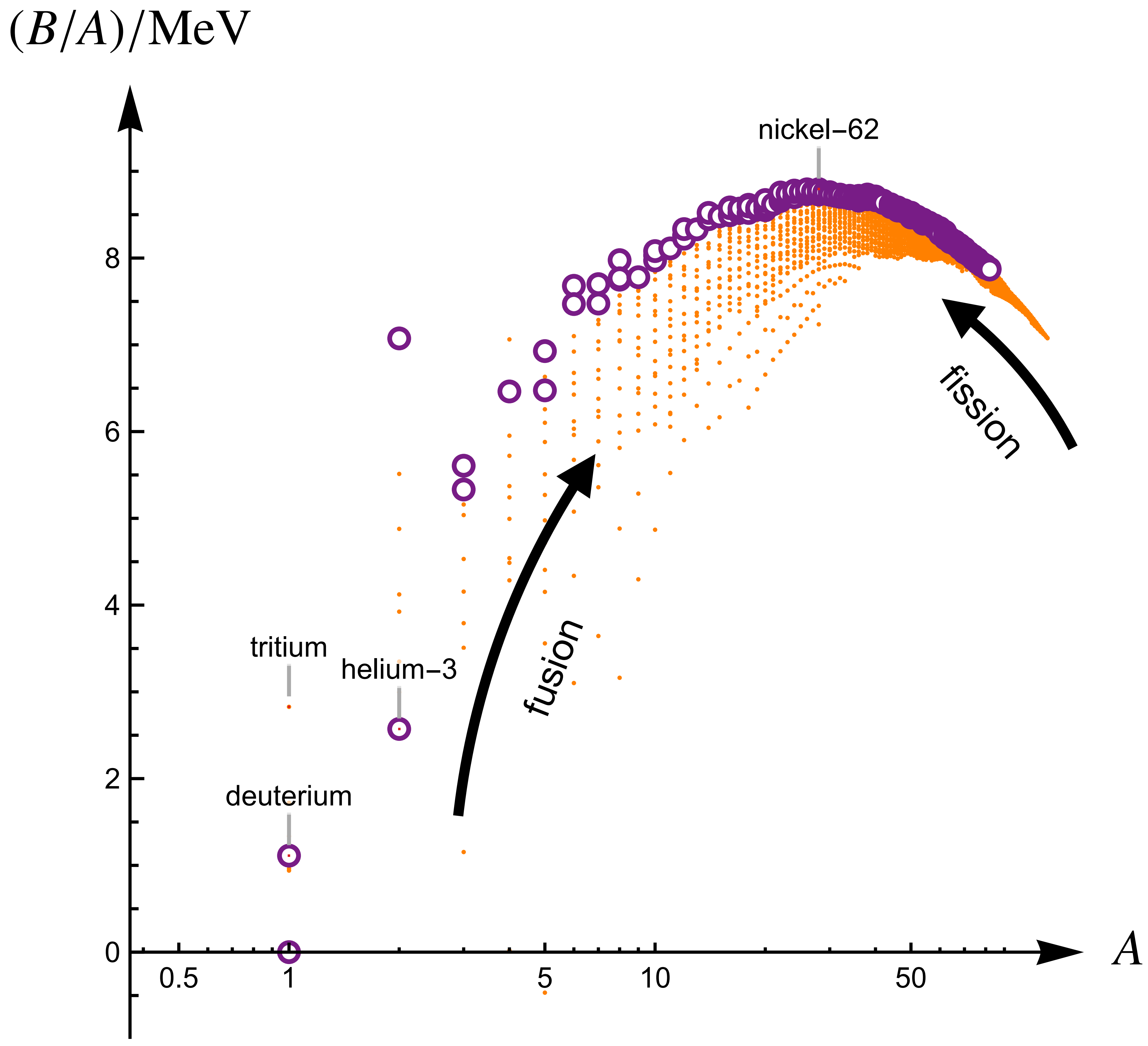
“black board with many formulas blending into a nuclear fusion reactor”
MM@ChatGPT4.0

BASIC PRINCIPLE

Nuclear fusion

- $X(B_X) + Y(B_Y) \rightarrow Z(B_Z)$
- Excess in energy $\Delta E = B_Z - B_Y - B_X > 0$ for small A
- Example[1]:

$$\underbrace{{}^{2}_{1}\mathrm{H}}_{B=2.2\,\mathrm{MeV}} + \underbrace{{}^{2}_{1}\mathrm{H}}_{B=2.2\,\mathrm{MeV}} \rightarrow \underbrace{{}^{3}_{2}\mathrm{He}}_{B=7.8\,\mathrm{MeV}} + {}^{1}\mathrm{n} \quad (\Delta E = 3.3\ \mathrm{MeV})$$



Strong(short-) vs. EM(long-range) force

---

[1] https://periodictable.com/Isotopes/010.20/index.p.dm.html

COULOMB BARRIER

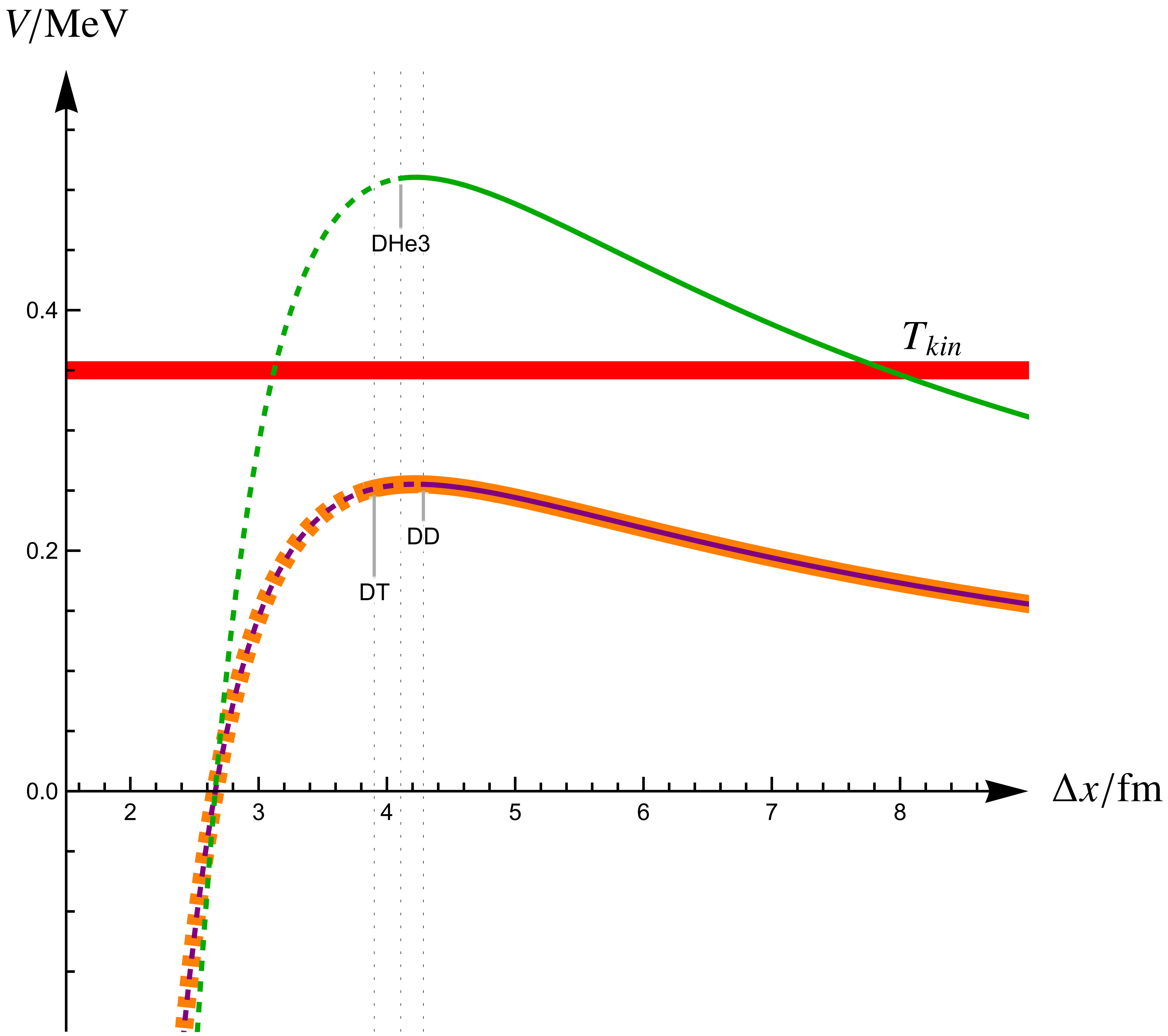
Reactants are charged!

- Electrostatic repulsion

1. deuterium-deuterium $^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + ^{1}\mathrm{n} + 3.3\ \mathrm{MeV}$
2. deuterium-deuterium $^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} + 4.0\ \mathrm{MeV}$
3. deuterium-tritium $^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}\mathrm{n} + 17.6\ \mathrm{MeV}$
4. deuterium-helium3 $^{2}_{1}\mathrm{H} + ^{3}_{2}\mathrm{He} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{1}\mathrm{H} + 18.3\ \mathrm{MeV}$

- Tunnelling: larger distances (lower temperature)

… through WKB[1]



---

[1] Liu et al. Phys Rev C 104, 044614 (2021);
[2] Bosch/Hale 1992 Nucl. Fusion 32 611

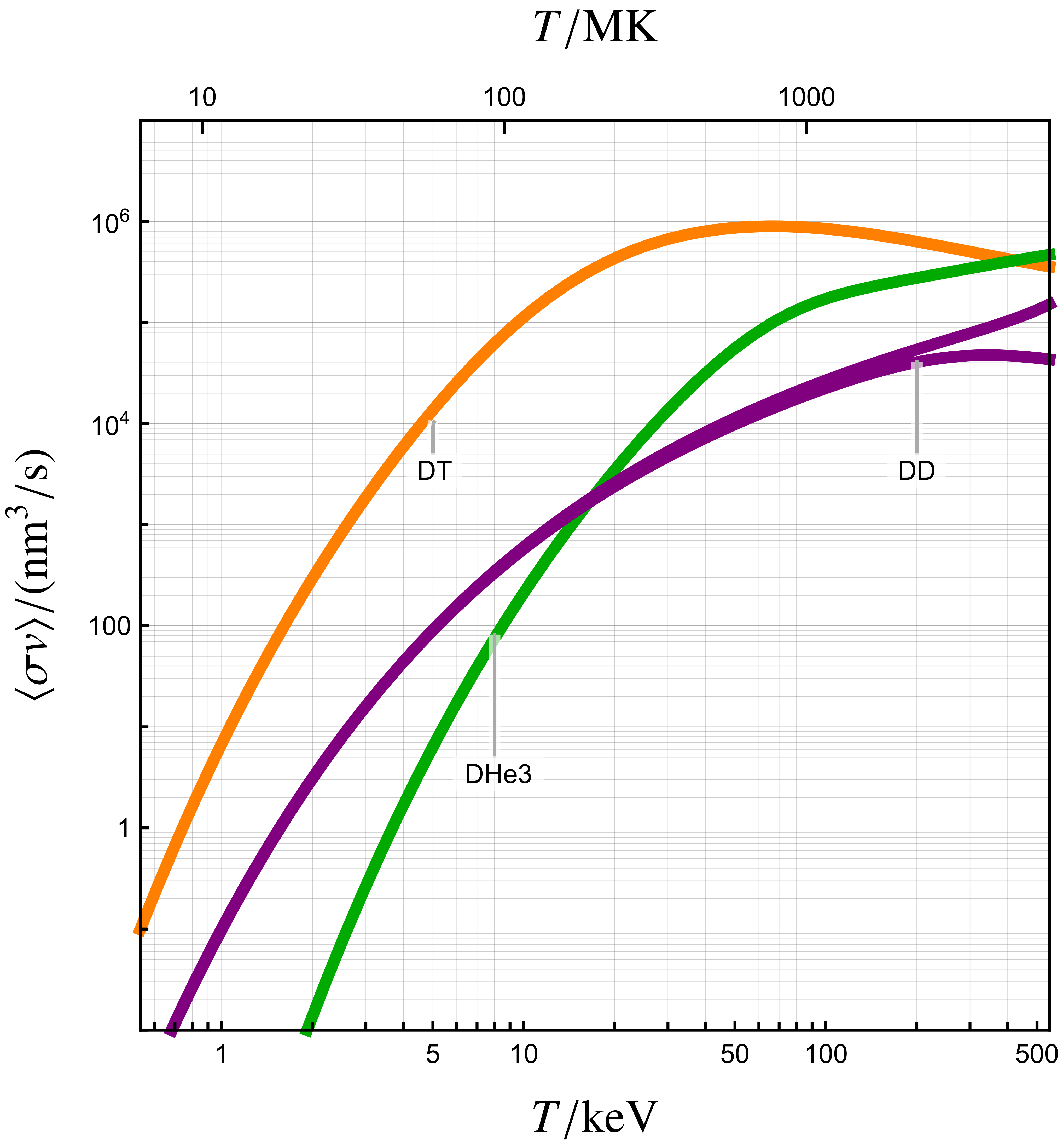
REACTION RATES

Reactants do not have fixed momentum

- Maxwell-averaged cross section $\langle \sigma v \rangle$

  ➡ reaction rate $n_1 n_2 \langle \sigma v \rangle$
- R-matrix parametrisation[1] (see also NCSM[2], NLEFT[3]…)

1. deuterium-deuterium $^2_1\mathrm{H} + ^2_1\mathrm{H} \rightarrow ^3_2\mathrm{He} + ^1\mathrm{n} + 3.3\ \mathrm{MeV}$
2. deuterium-deuterium $^2_1\mathrm{H} + ^2_1\mathrm{H} \rightarrow ^3_1\mathrm{H} + ^1_1\mathrm{H} + 4.0\ \mathrm{MeV}$
3. deuterium-tritium $^2_1\mathrm{H} + ^3_1\mathrm{H} \rightarrow ^4_2\mathrm{He} + ^1\mathrm{n} + 17.6\ \mathrm{MeV}$
4. deuterium-helium3 $^2_1\mathrm{H} + ^3_2\mathrm{He} \rightarrow ^4_2\mathrm{He} + ^1_1\mathrm{H} + 18.3\ \mathrm{MeV}$



---

[1] Bosch/Hale 1992 Nucl. Fusion 32 611

[2] Navrátil/Quaglioni *Phys.Rev.Lett.* 108 (2012) 042503; Hupin/Quaglioni/Navrátil Nature Comm. 10 351 (2019)

[3] PRL 115, 122301 (2015); DD scattering in progress.. Meyer et al.

REACTION RATES

Reactants do not have fixed momentum

- Maxwell-averaged cross section $\langle \sigma v \rangle$

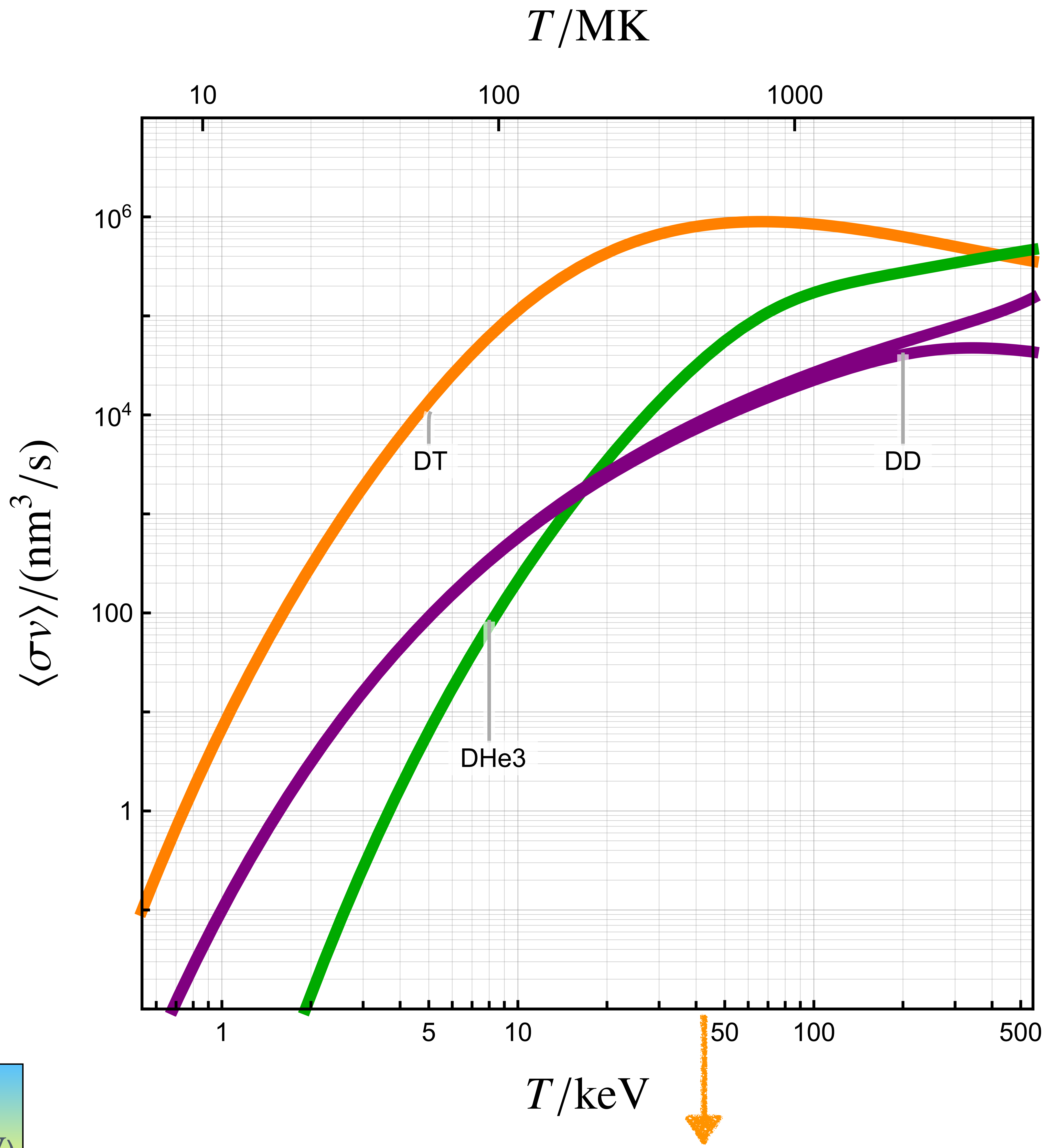
  ➡ reaction rate $n_1 n_2 \langle \sigma v \rangle$

- R-matrix parametrisation[1] (see also NCSM[2], NLEFT[3]…)

1. deuterium-deuterium ${}^2_1\mathrm{H} + {}^2_1\mathrm{H} \to {}^3_2\mathrm{He} + {}^1\mathrm{n} + 3.3\ \mathrm{MeV}$
2. deuterium-deuterium ${}^2_1\mathrm{H} + {}^2_1\mathrm{H} \to {}^3_1\mathrm{H} + {}^1_1\mathrm{H} + 4.0\ \mathrm{MeV}$
3. deuterium-tritium ${}^2_1\mathrm{H} + {}^3_1\mathrm{H} \to {}^4_2\mathrm{He} + {}^1\mathrm{n} + 17.6\ \mathrm{MeV}$
4. deuterium-helium3 ${}^2_1\mathrm{H} + {}^3_2\mathrm{He} \to {}^4_2\mathrm{He} + {}^1_1\mathrm{H} + 18.3\ \mathrm{MeV}$

➡ most favourable* reaction deuterium-tritium (DT)

energy density ($\sim 4 \cdot 10^8\ \mathrm{MJ/kg}$)
orders of magnitude higher than chemical (eV)



$E_T \sim 100\,\mathrm{keV} \gg E_{ionization} \sim 10\,\mathrm{eV}$
Fusion fuel is completely ionized: **Plasma**

[1] Bosch/Hale 1992 Nucl. Fusion 32 611
[2] Navrátil/Quaglioni *Phys.Rev.Lett.* 108 (2012) 042503; Hupin/Quaglioni/Navrátil Nature Comm. 10 351 (2019)
[3] PRL 115, 122301 (2015); DD scattering in progress.. Meyer et al.

III.TECHNICAL REALISATION

Confinement

Stability

Self-efficiency

Tokamak/Stellarator

…

“black board with many formulas blending into a nuclear fusion reactor”
MM@ChatGPT4.0

PLASMA EQUILIBRIUM

SOURCES OF ENERGY

- **External heating** (ohmic, microwaves):

$$S_h$$

- **Fusion reaction** $\mathrm{DT} \rightarrow \alpha(3.5\,\mathrm{MeV}) + \mathrm{n}(14.1\,\mathrm{MeV})$

neutrons: large mean free path ➔ reactor output

alpha: small mean free path $\mathcal{O}(1\,\mathrm{mm})$ ➔ plasma reheating

$$S_\alpha = E_\alpha \frac{n_\alpha^2}{4}\langle\sigma v\rangle = E_\alpha \frac{\langle\sigma v\rangle}{16}\frac{p^2}{T^2}$$

SINKS OF ENERGY

- **Bremsstrahlung** (dominantly electrons)

$$S_B = C_B n^2 T^{1/2} = \frac{C_B}{4}\frac{p^2}{T^{3/2}}$$

- Fluid dynamics of plasma (steady state):
  - ➡ **Heat conduction**: $S_d = \nabla(\kappa \nabla T)$. Empirically,

$$S_d = \frac{2}{3}\frac{p}{\tau_E}$$

… relaxation time τ_E.

- ➡ Corrections: volume expansion/convection/micro-turbulences/temperature profile/…

POWER BALANCE RELATION[1]

$$\frac{E_\alpha\langle\sigma v\rangle}{16}\frac{p^2}{T^2} + S_h = \frac{C_B}{4}\frac{p^2}{T^{3/2}} + \frac{3}{2}\frac{p}{\tau_E} \qquad \left[\frac{W}{m^3}\right]$$

---

[1] Lawson (1957). *Proceedings of the Physical Society* B70, 6

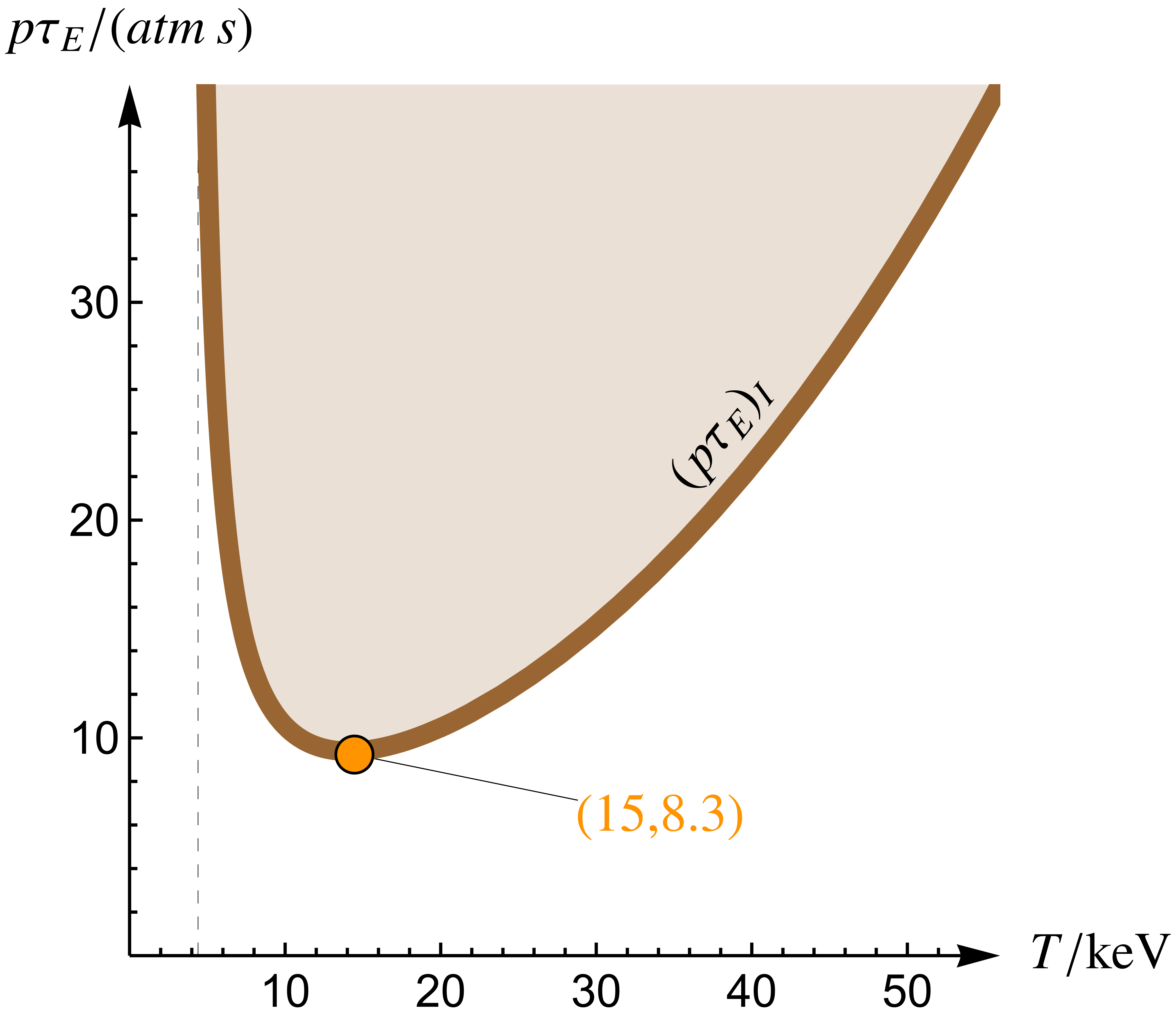
IGNITION

Ignition $(S_\alpha = S_B + S_\kappa)$

- Lawson parameter $(p\tau_E)_I = \dfrac{K_\kappa T^2}{K_\alpha \langle \sigma v \rangle - K_B T^{1/2}}$

… high-T solution — stable equilibrium

… low-T solution — unstable equilibrium → burn control

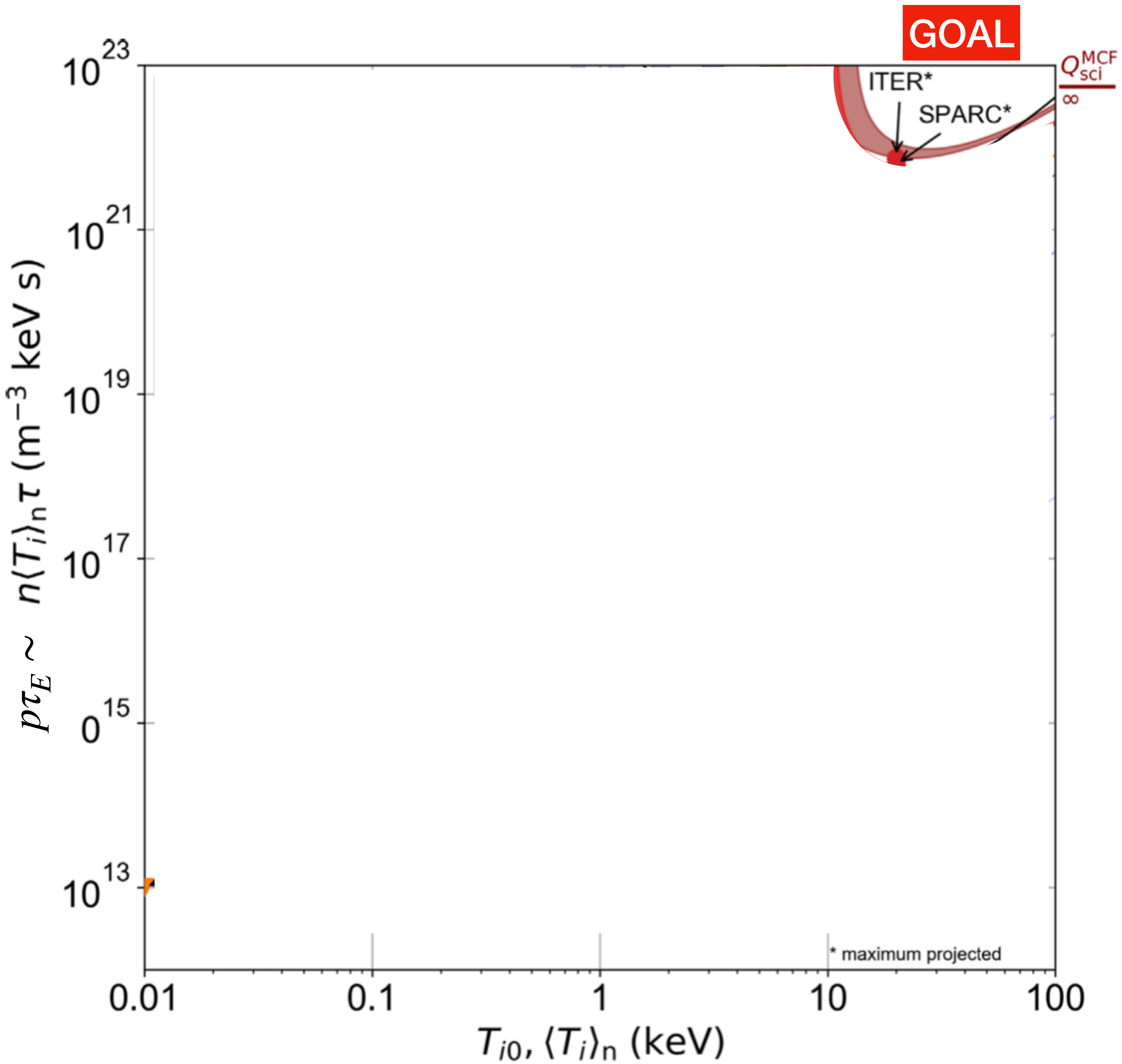


[1] Lawson (1957). *Proceedings of the Physical Society* B70, 6

IGNITION

Ignition $(S_\alpha = S_B + S_\kappa)$

- Lawson parameter $(p\tau_E)_I = \dfrac{K_\kappa T^2}{K_\alpha \langle \sigma v \rangle - K_B T^{1/2}}$



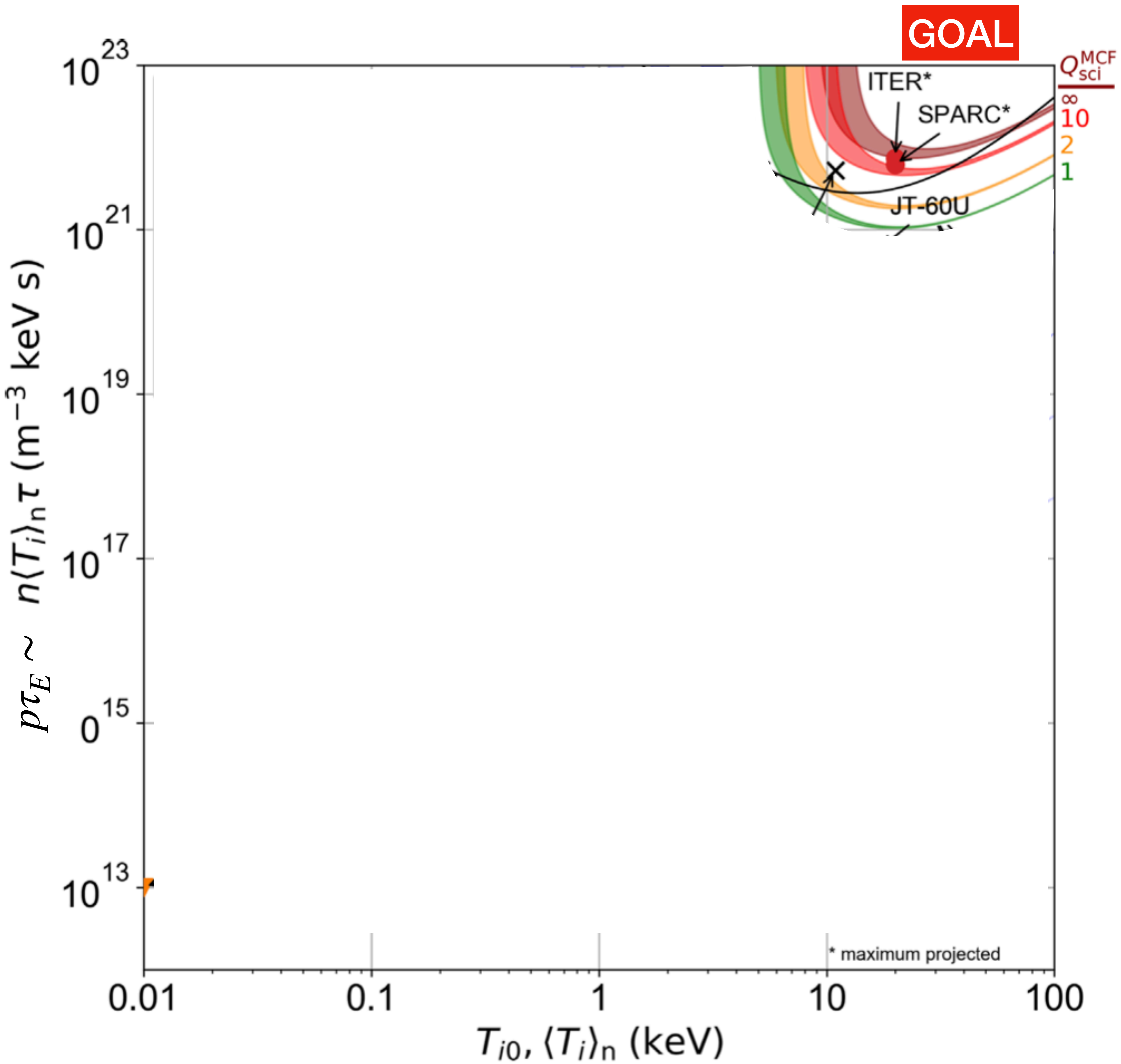
[FIG] Wurzel et al. Phys. Plasmas 29, 062103 (2022)

[1] Lawson (1957). *Proceedings of the Physical Society* B70, 6

IGNITION

Ignition $(S_\alpha = S_B + S_\kappa)$

- Lawson parameter $(p\tau_E)_I = \dfrac{K_\kappa T^2}{K_\alpha \langle \sigma v \rangle - K_B T^{1/2}}$
- External heating $S_h > 0$ can reduce $(p\tau_E)_{min}$ (reducing gain)



[FIG] Wurzel et al. Phys. Plasmas 29, 062103 (2022)

[1] Lawson (1957). *Proceedings of the Physical Society* B70, 6

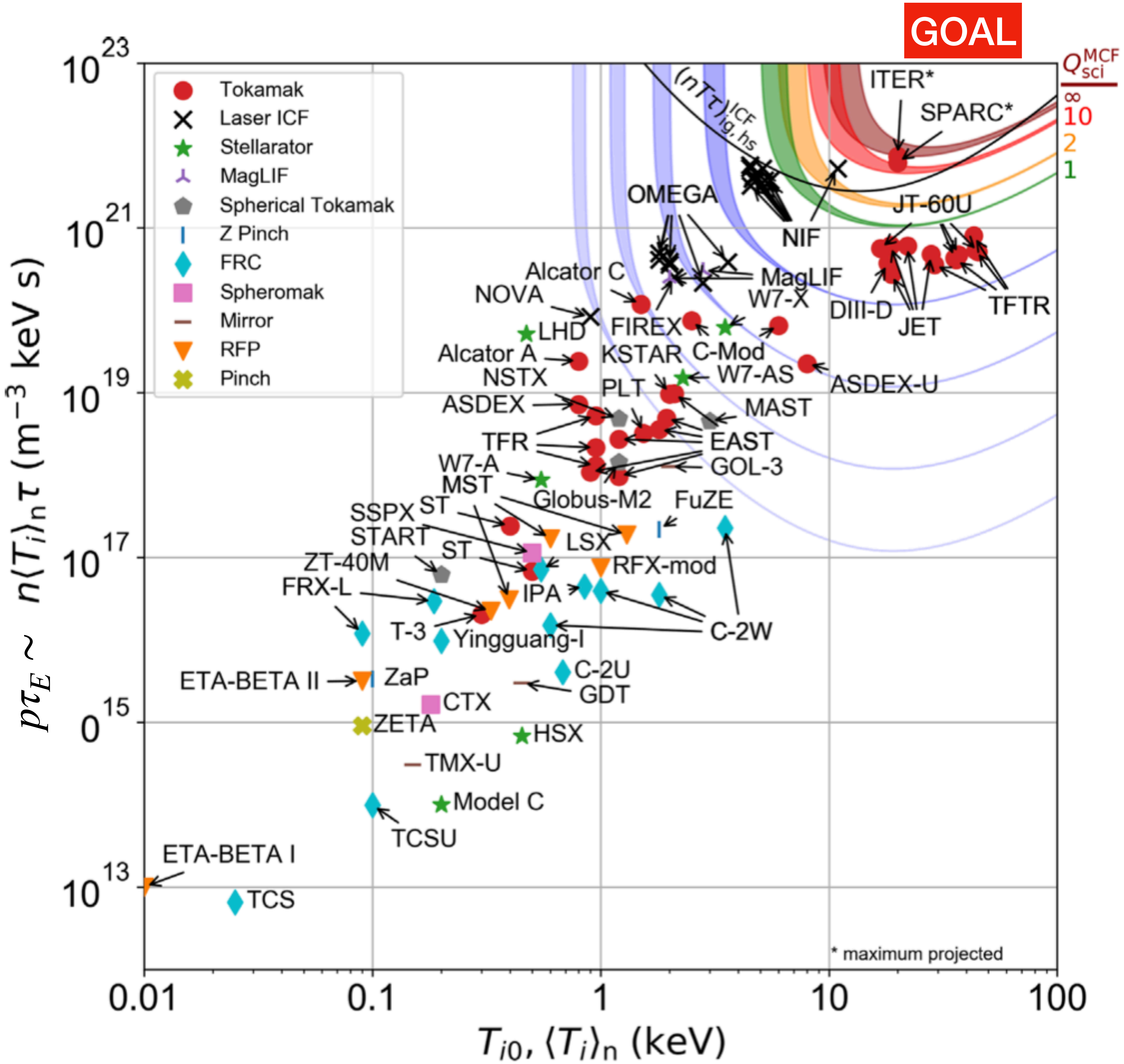
IGNITION

Ignition $(S_\alpha = S_B + S_\kappa)$

- Lawson parameter $(p\tau_E)_I = \dfrac{K_\kappa T^2}{K_\alpha\langle\sigma v\rangle - K_B T^{1/2}}$
- External heating $S_h > 0$ can reduce $(p\tau_E)_{min}$ (reducing gain)

World progress

- Operating and proposed (*) facilities



[FIG] Wurzel et al. Phys. Plasmas 29, 062103 (2022)

---

[1] Lawson (1957). *Proceedings of the Physical Society* B70, 6

PLASMA CONFINEMENT

Overcoming Coulomb-barrier requires $T \sim 10^8 K$

No materials can withstand such temperatures

Fuel-Plasma needs to be confined/controlled[1]

Prospective ansätze:

1. **ICF Inertial Confinement Fusion (Laser, Ion-beam, …)**

Laser induced shock waves in small pellets $\mathcal{O}(1\,\mathrm{mm})$*. Experiments since 1970s.*

[+] Only a small portion needs to be heated up.

[+] Plasma self-heating, mean free path of ${}^4_2\mathrm{He}$: $\mathcal{O}(0.01\,\mathrm{mm})$

[+] Gain: 3.15/2.05 (2022NIF[2])

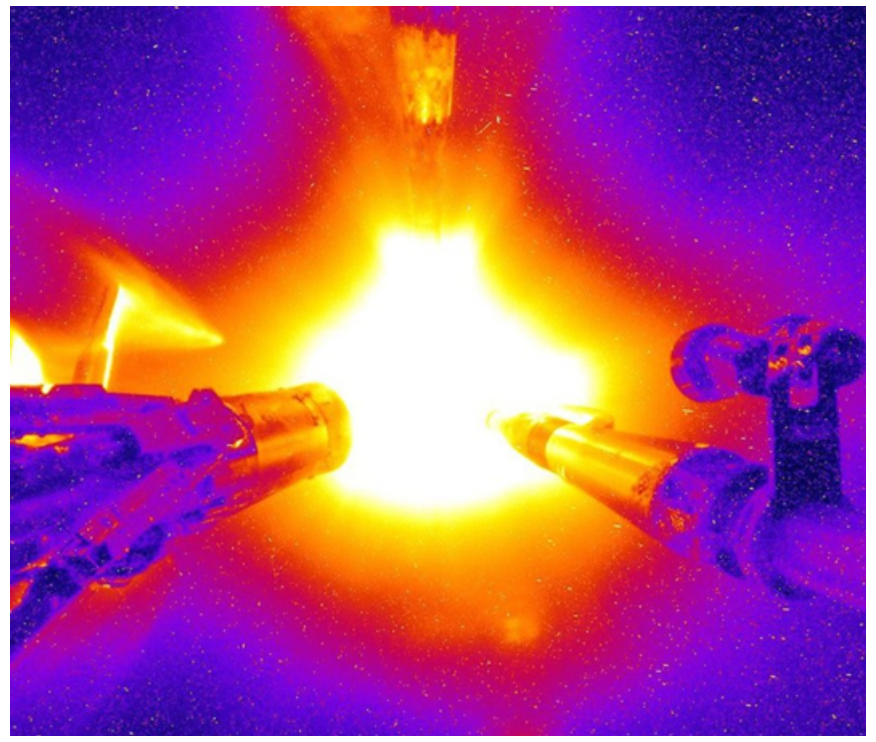
[–] Confinement time $\mathcal{O}(20\,\mathrm{ns})$

[–] Blows apart in the process

… **How to make a continuous process and collect energy?**

Nova Laser Bay

Target



NIF "Big Foot" deuterium-tritium (DT) implosion

---

[1] Lawson 1957 Proc. Phys. Soc. B 70 6

[2] Abu-Shawareb Phys. Rev. Lett. 129, 075001 (2022);

PLASMA CONFINEMENT

Overcoming Coulomb-barrier requires $T \sim 10^8 K$

No materials can withstand such temperatures

Fuel-Plasma needs to be confined/controlled[1]

Prospective ansätze:

2. MCF Magnetic Confinement Fusion (Tokamak, Stellarator, …)

Locally quasi-neutral, high-conductivity (~40x copper) Plasma. Charged particles in the Plasma move along the B-field lines

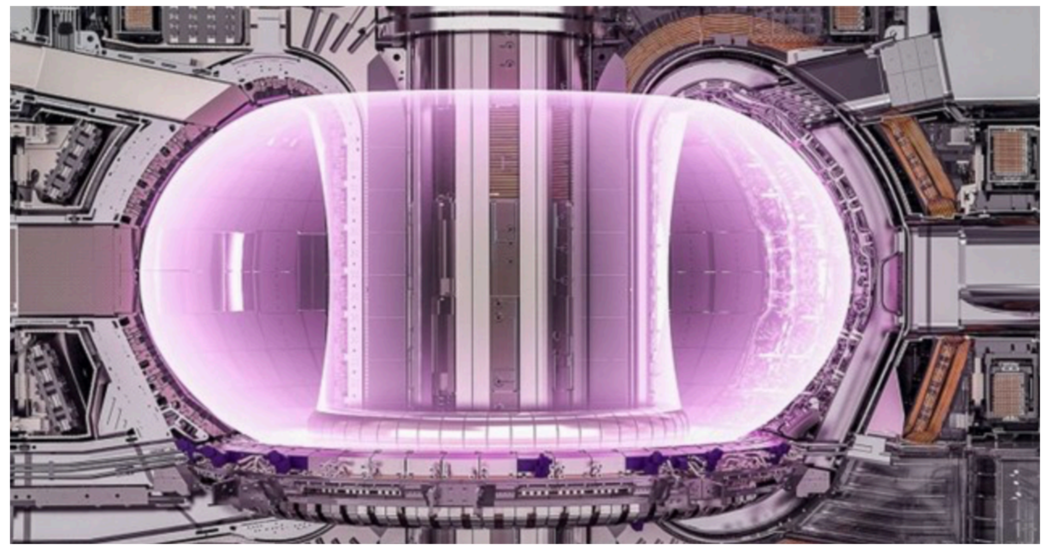
[+] Sustained self-heated plasma conditions $\mathcal{O}(10\,\mathrm{min})$[2]

[+] Semi-realistic reactor designs exist (Energy conversion..)

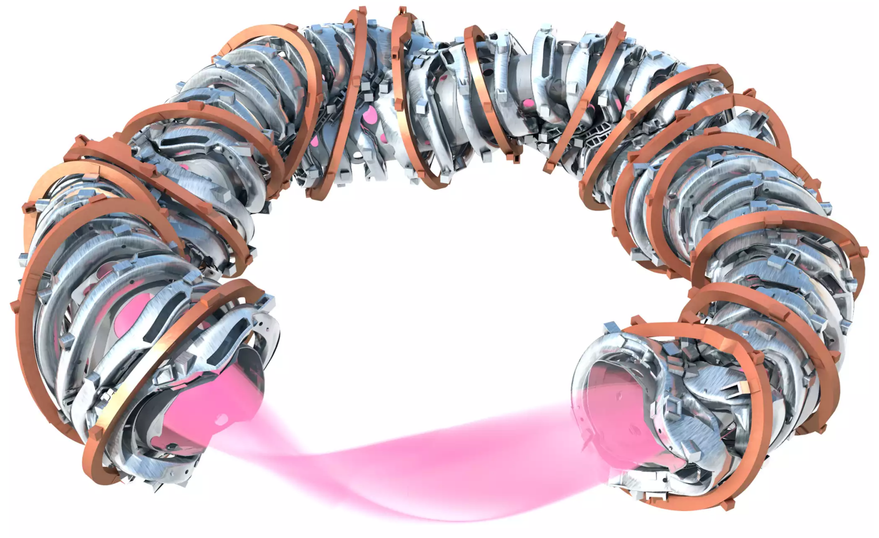
[−] Complex configuration (shielding, …)

[−] Stability, Plasma discharges, Turbulences

[−] Fuel self-sufficiency



ITER (Tokamak)



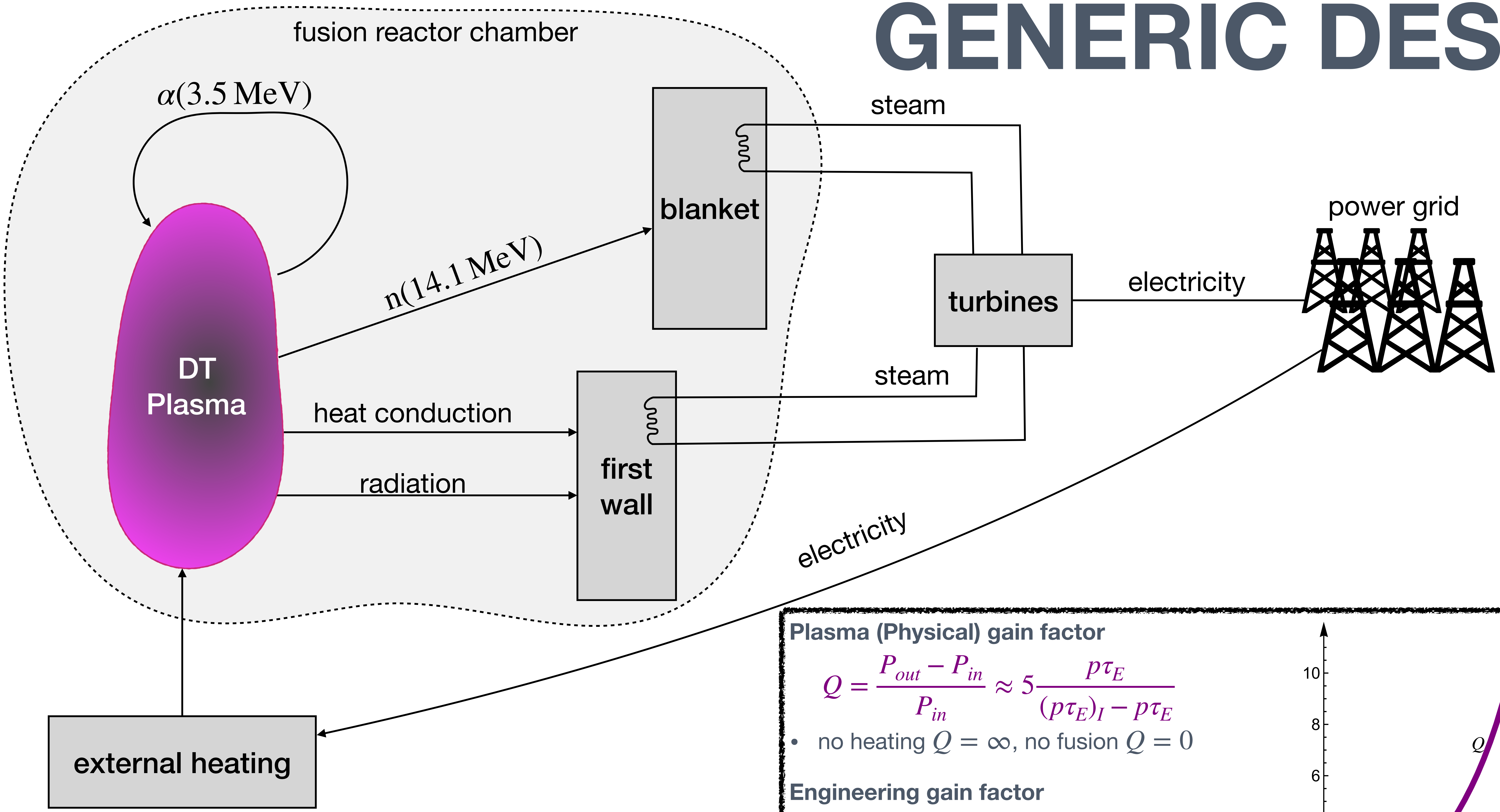
Wendelstein7-X (Stellarator)

---

[1] Lawson 1957 Proc. Phys. Soc. B 70 6

[2] http://east.ipp.ac.cn/; https://www.ipp.mpg.de/5322229/01_23

GENERIC DESIGN


fusion reactor chamber

$\alpha(3.5\,\mathrm{MeV})$

DT Plasma

$\mathrm{n}(14.1\,\mathrm{MeV})$

heat conduction

radiation

blanket

first wall

steam

steam

turbines

electricity

power grid

electricity

external heating

Plasma (Physical) gain factor

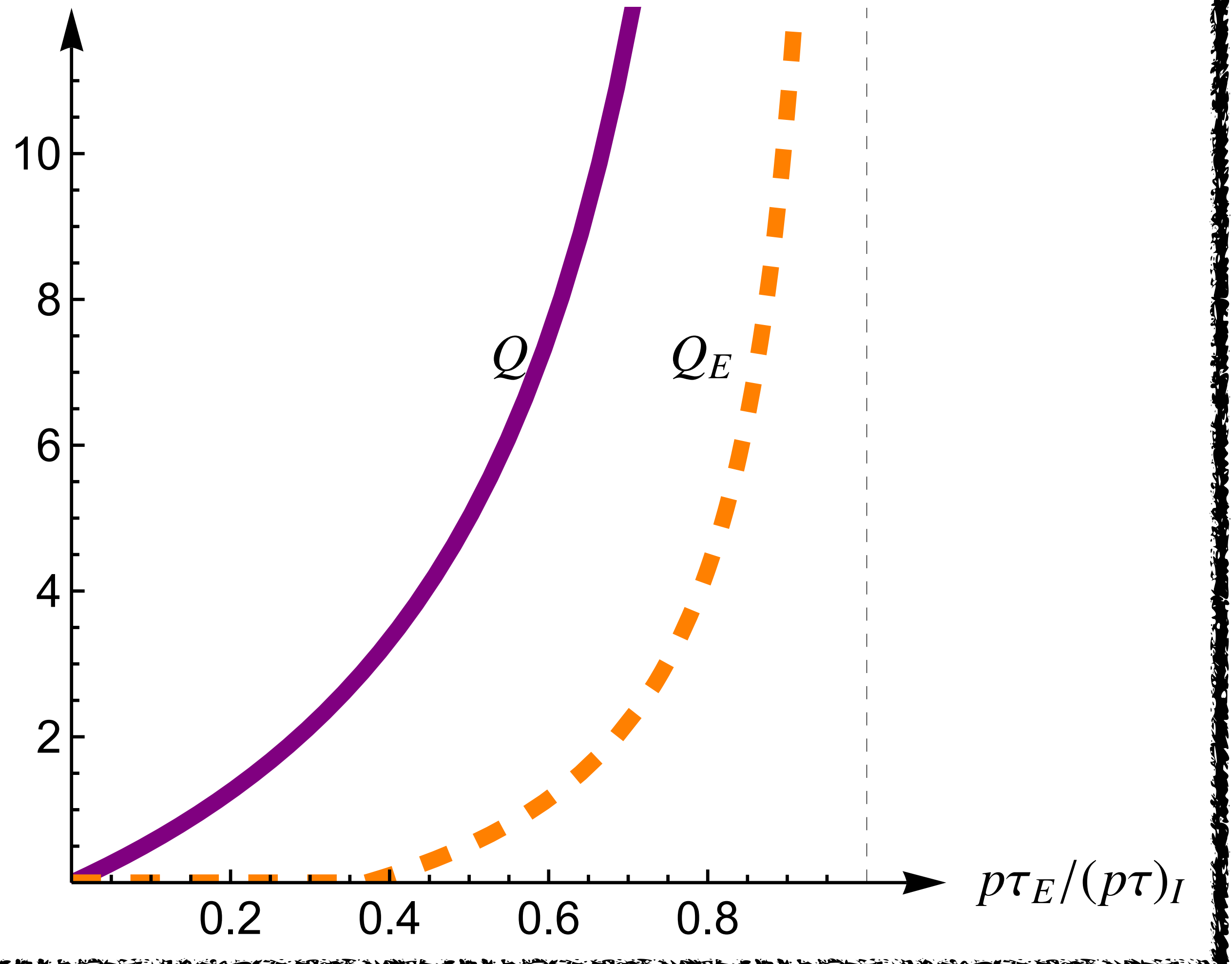
$$Q = \frac{P_{out} - P_{in}}{P_{in}} \approx 5 \frac{p\tau_E}{(p\tau_E)_I - p\tau_E}$$

- no heating $Q = \infty$, no fusion $Q = 0$

Engineering gain factor

$$Q_E = \frac{P^{el}_{out} - P^{el}_{in}}{P^{el}_{in}} \approx 2 \frac{p\tau_E - 0.37(p\tau_E)_I}{(p\tau_E)_I - p\tau_E}$$

- Depends on efficiency (absorption, heating, …)
- “Break-even” Q=0

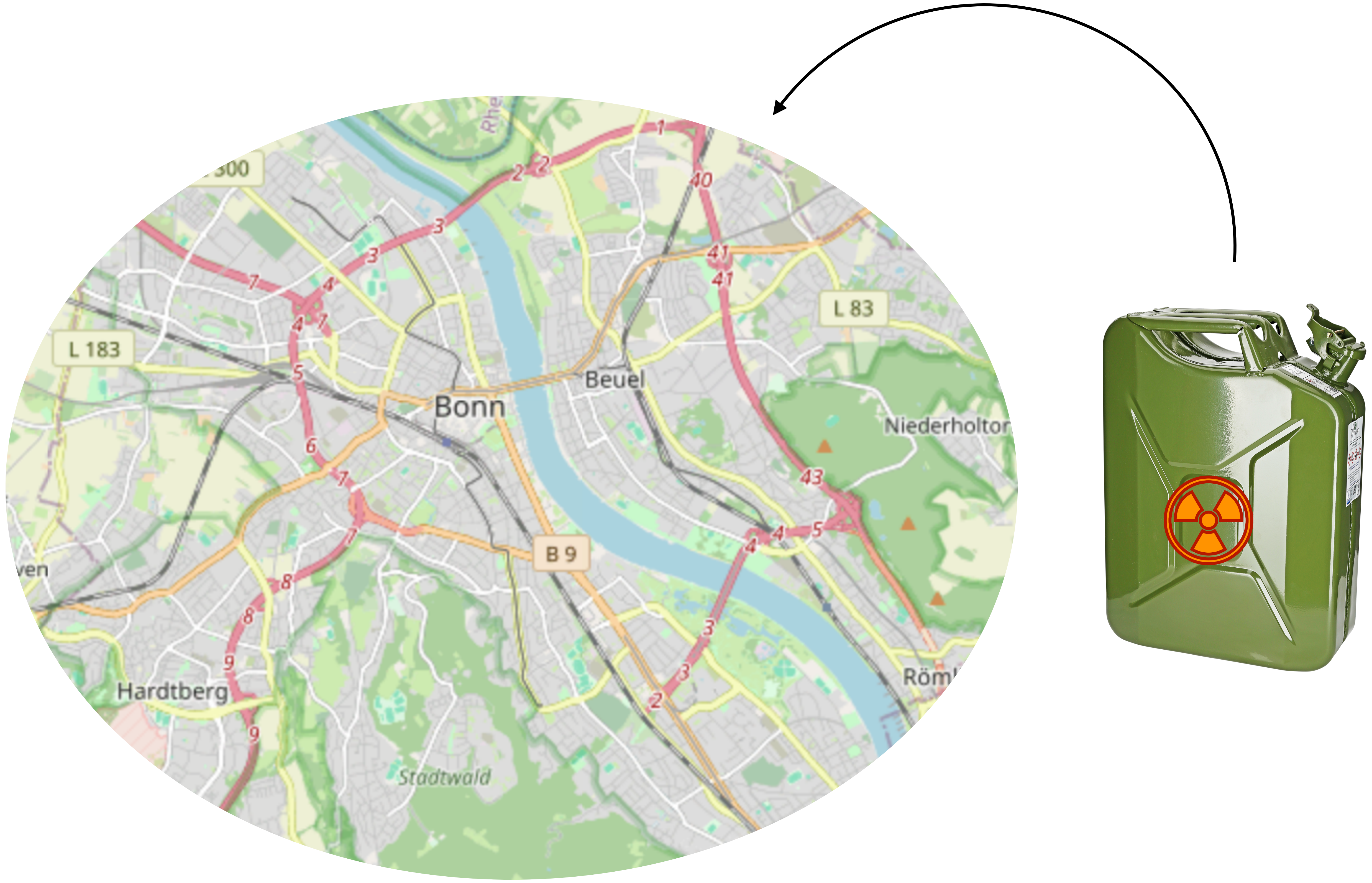


FUEL SELF-SUFFICIENCY

- Good news:

DT fusion has large energy density ($\approx 4 \cdot 10^8\,\mathrm{MJ/kg}$)

$$\frac{\text{Consumption(Bonn)}}{\text{year}} \sim 16\,\mathrm{kg} \quad \text{Tritium}$$



FUEL SELF-SUFFICIENCY

- Good news:

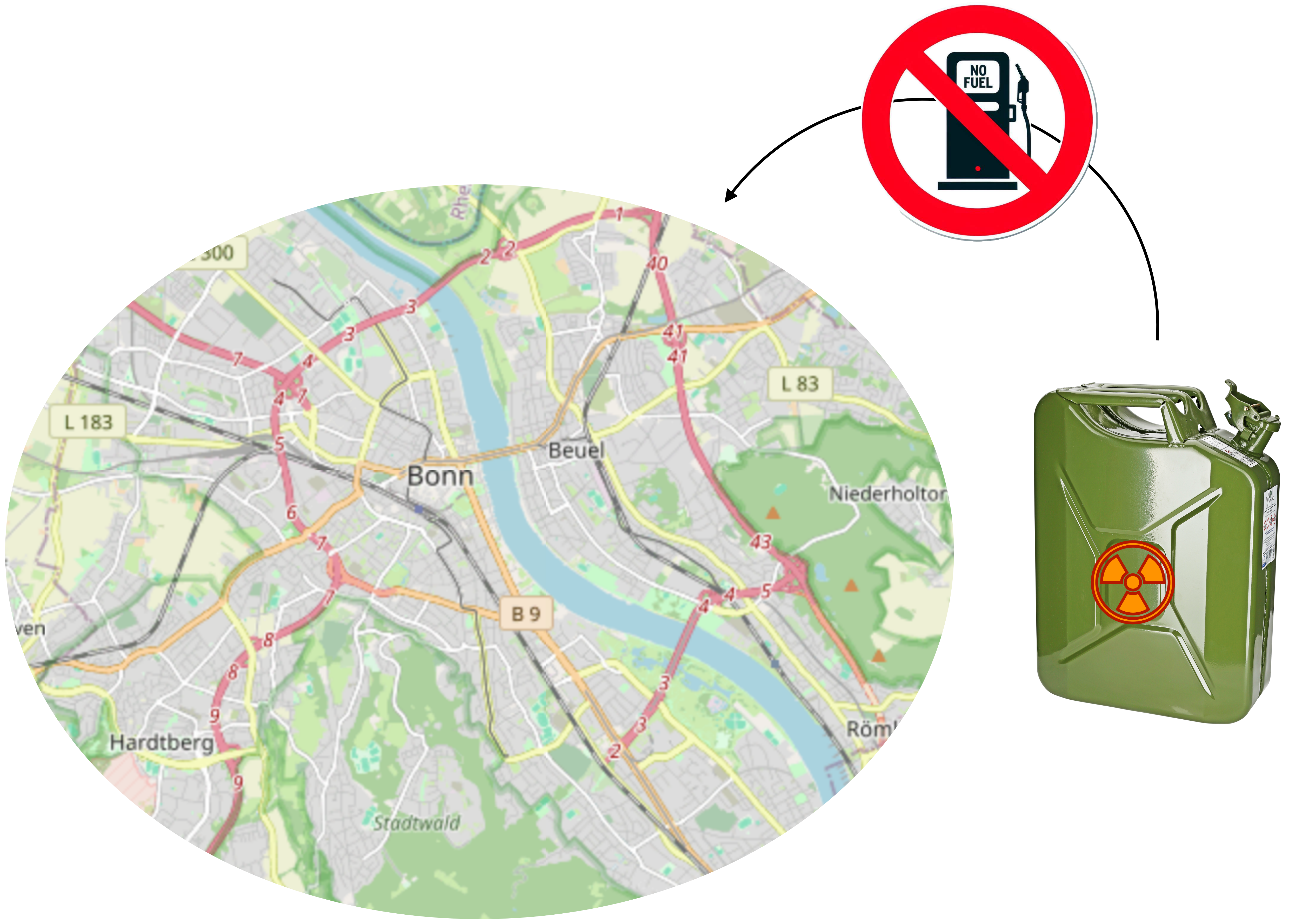
DT fusion has large energy density ($\approx 4 \cdot 10^8\,\mathrm{MJ/kg}$)

$$\frac{\text{Consumption(Bonn)}}{\text{year}} \sim 16\,\mathrm{kg} \quad \text{Tritium}$$

- Bad news:

$$\frac{\text{Tritium}}{\text{Earth}} \sim 20\,\mathrm{kg}$$

➡ Concept for Tritium breeding is needed …



TRITIUM BREEDING

Typical concept: **breeding in a lithium blanket**

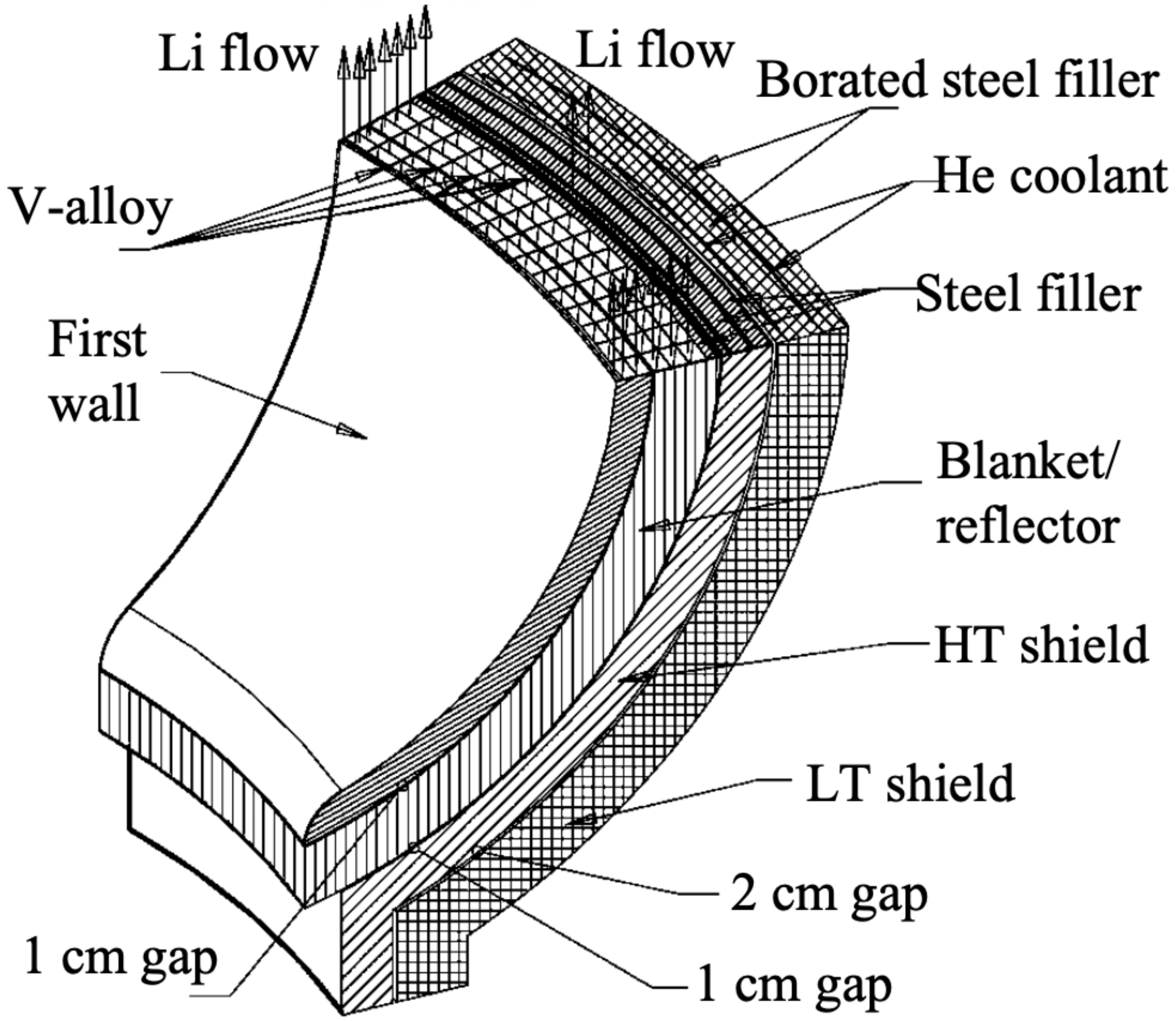
$$^{6}_{3}\mathrm{Li} + \mathrm{n} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H} + 4.78\,\mathrm{MeV}$$
$$^{7}_{3}\mathrm{Li} + \mathrm{n} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{3}_{1}\mathrm{H} + \mathrm{n} - 2.7\,\mathrm{MeV}$$

Technological challenges[1]

- $1\,\mathrm{m}$ thick layer around $1000\,\mathrm{m}^3$ volume
- Neutrons need to be slowed down for efficiency
- complex configuration
- high demand on materials

…

- Essential experiments @ITER (20??), @DEMO (2???)



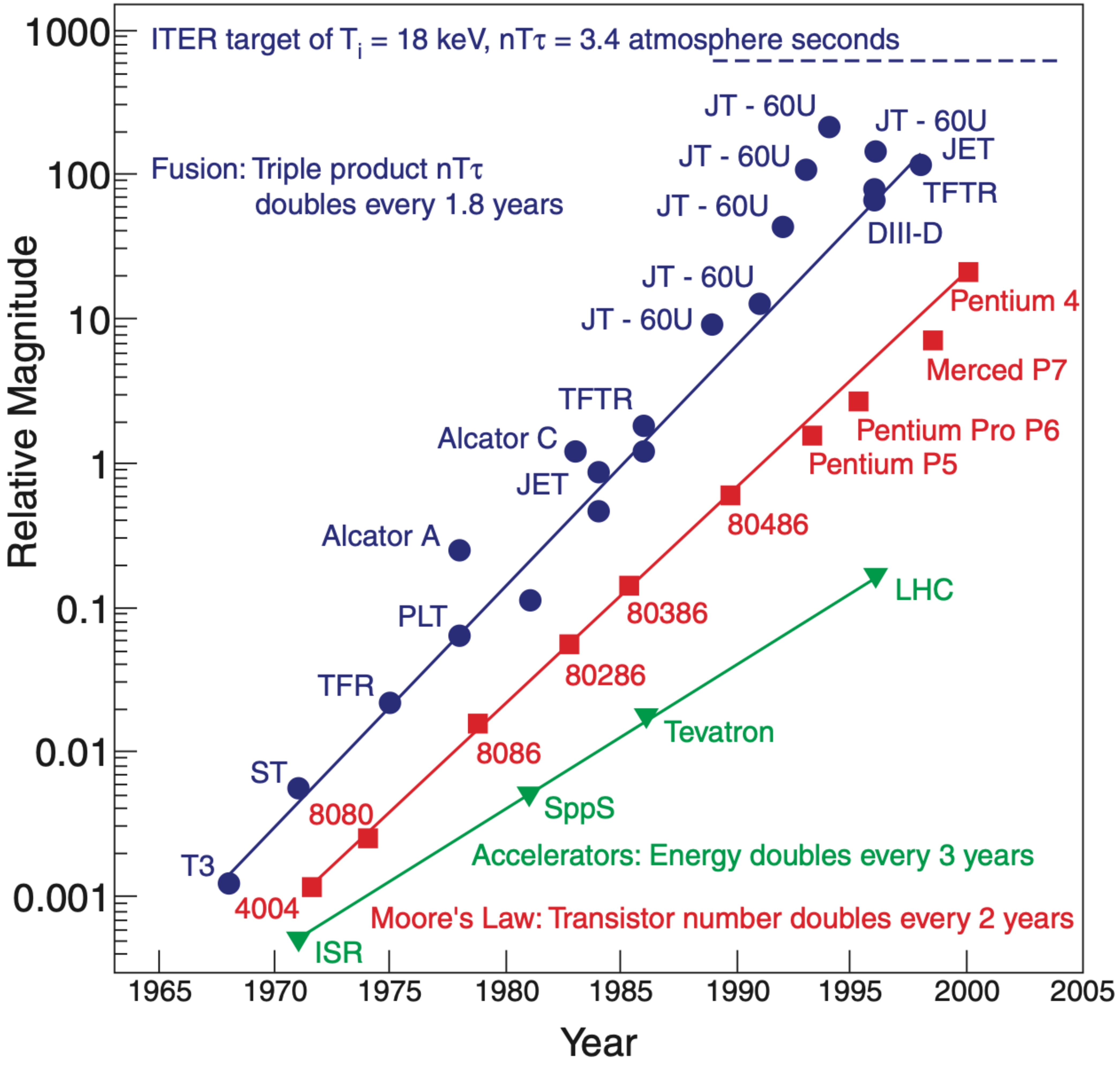
[FIG] Raffray et al. / Journal of Nuclear Materials 307–311 (2002) 21–30
[1] Rubel, Journal of Fusion Energy (2019) 38:315–329; Dittmar, Energy 37 (2012) 35-40

SUMMARY

NUCLEAR FUSION — ASTONISHINGLY SIMPLE & ASTONISHINGLY COMPLEX

Progress

- Basic design well understood
- Many test reactor facilities $\mathcal{O}(100)$
- Confinement time records, e.g., $t_C = 480\,\mathrm{s}$ @ Wendelstein7-X
- Net (plasma) energy gain, e.g., $Q \sim 3/2$ @NIF



[FIG] Webster, "Fusion: Power for the future", Physics Education 38, 135 (2003)

SUMMARY

NUCLEAR FUSION — ASTONISHINGLY SIMPLE & ASTONISHINGLY COMPLEX

Progress

- Basic design well understood
- Many test reactor facilities $\mathcal{O}(100)$
- Confinement time records, e.g., $t_C = 480\,\mathrm{s}$ @ Wendelstein7-X
- Net (plasma) energy gain, e.g., $Q \sim 3/2$ @NIF

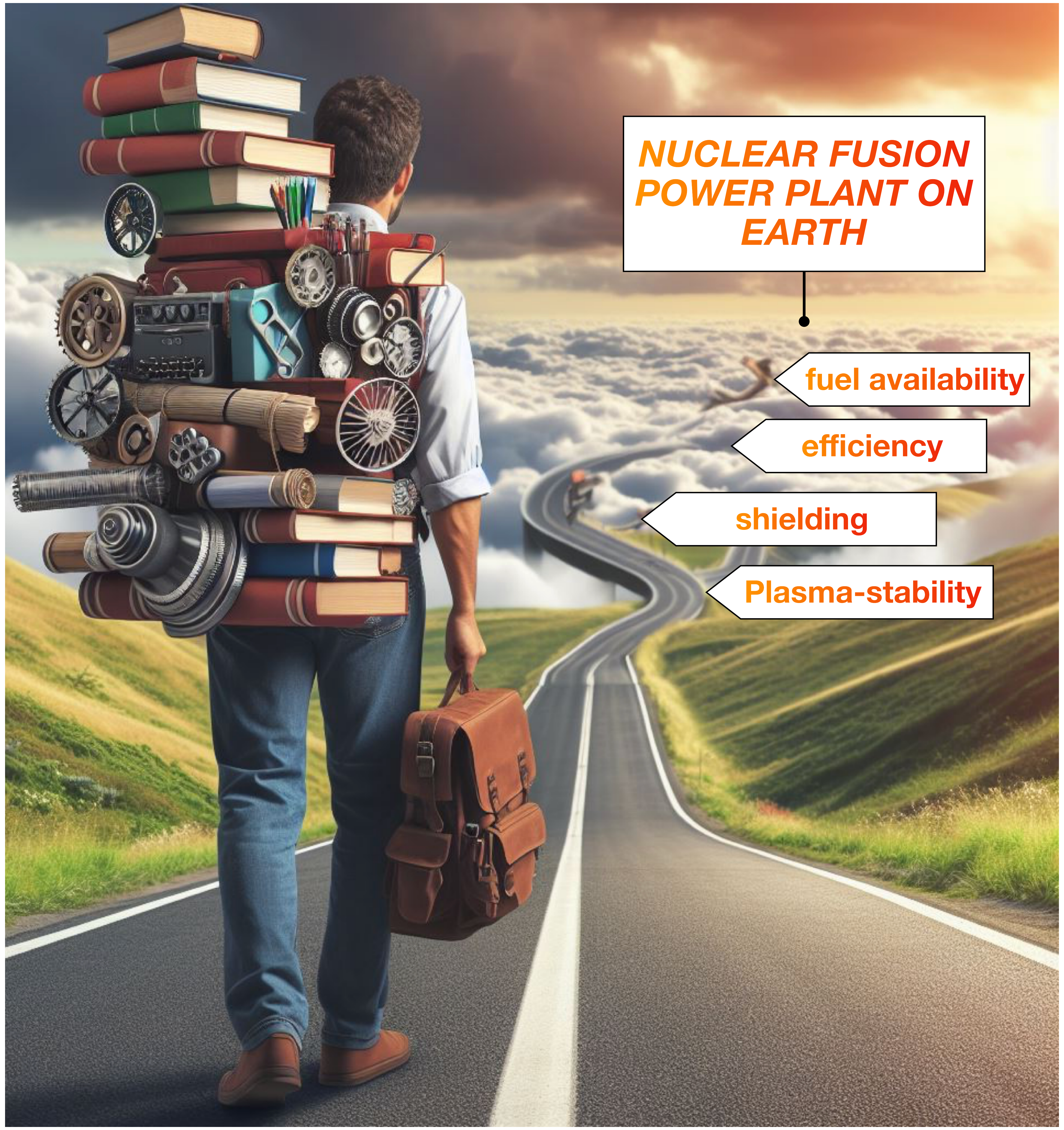
Challenges

- Plasma stability
- Net (engineering) gain $Q_E \sim 0$

  … *efficient thermal-electric conversion*
- Shielding vs. very high energy, flux neutrons

  … *radioactive waste* $T_{1/2} \sim \mathcal{O}(10\mathrm{y})$
- Tritium-breeding concept very theoretical:

  … *experiments/design updates will take decades*



A mechanical engineer with many books in his backpack walking a very windy road
MM@ChatGPT4.0

POWER BALANCE

Physical gain factor

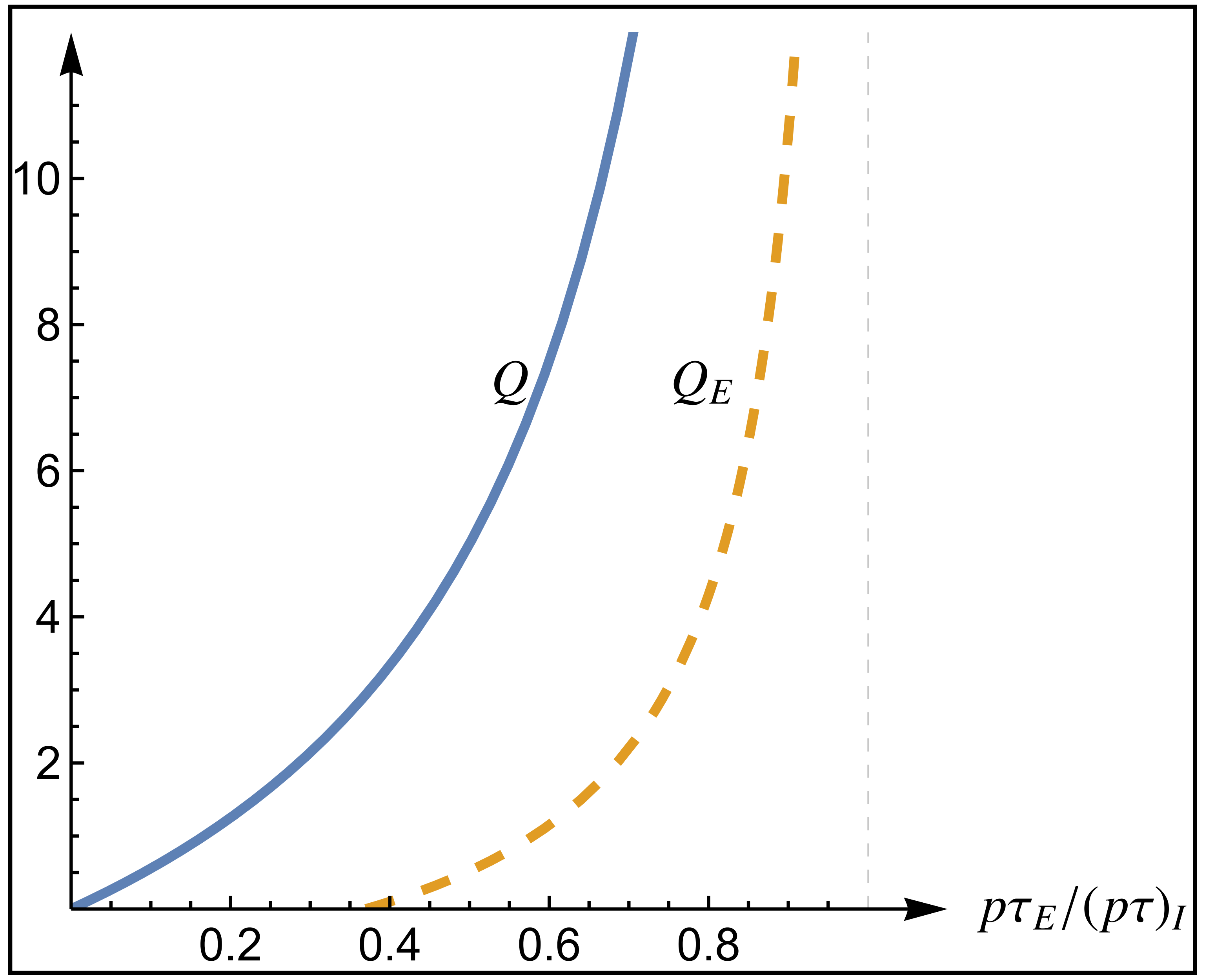
$$Q = \frac{P_{out} - P_{in}}{P_{in}} \approx 5 \frac{p\tau_E}{(p\tau_E)_I - p\tau_E}$$

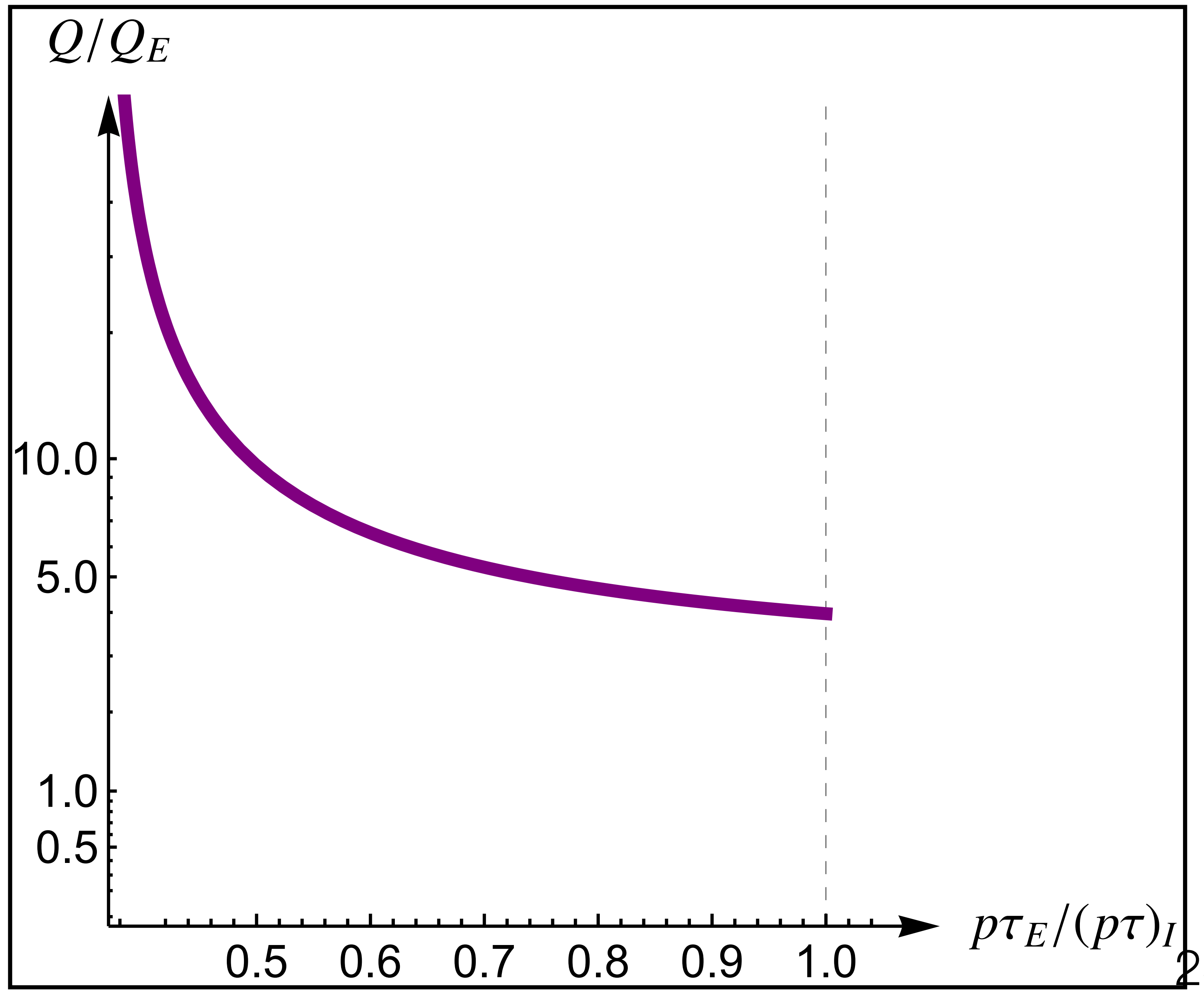
- no heating $Q = \infty$
- no fusion $Q = 0$

Engineering gain factor

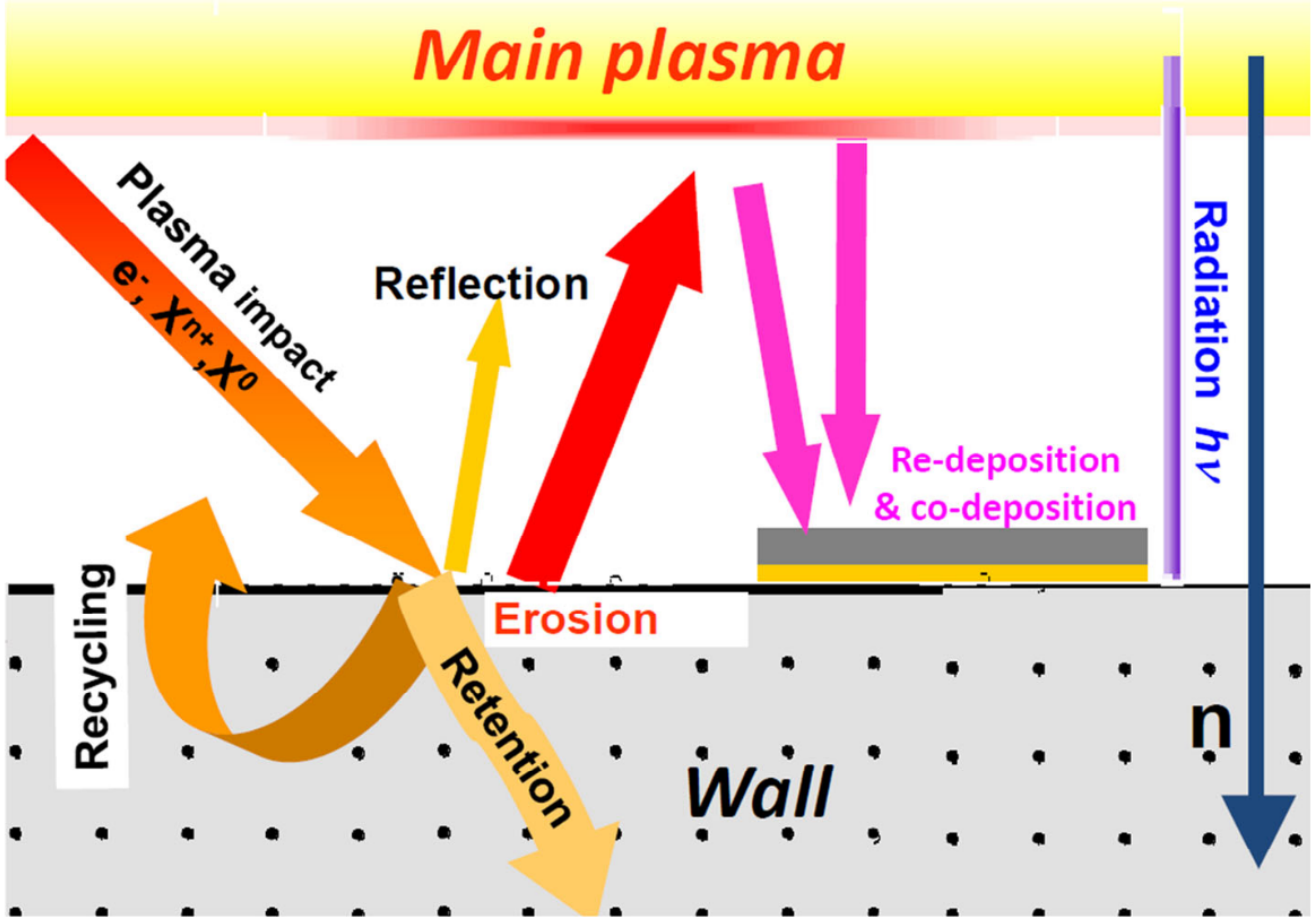
$$Q_E = \frac{P^{el}_{out} - P^{el}_{in}}{P^{el}_{in}} \approx 2 \frac{p\tau_E - 0.37(p\tau_E)_I}{(p\tau_E)_I - p\tau_E}$$

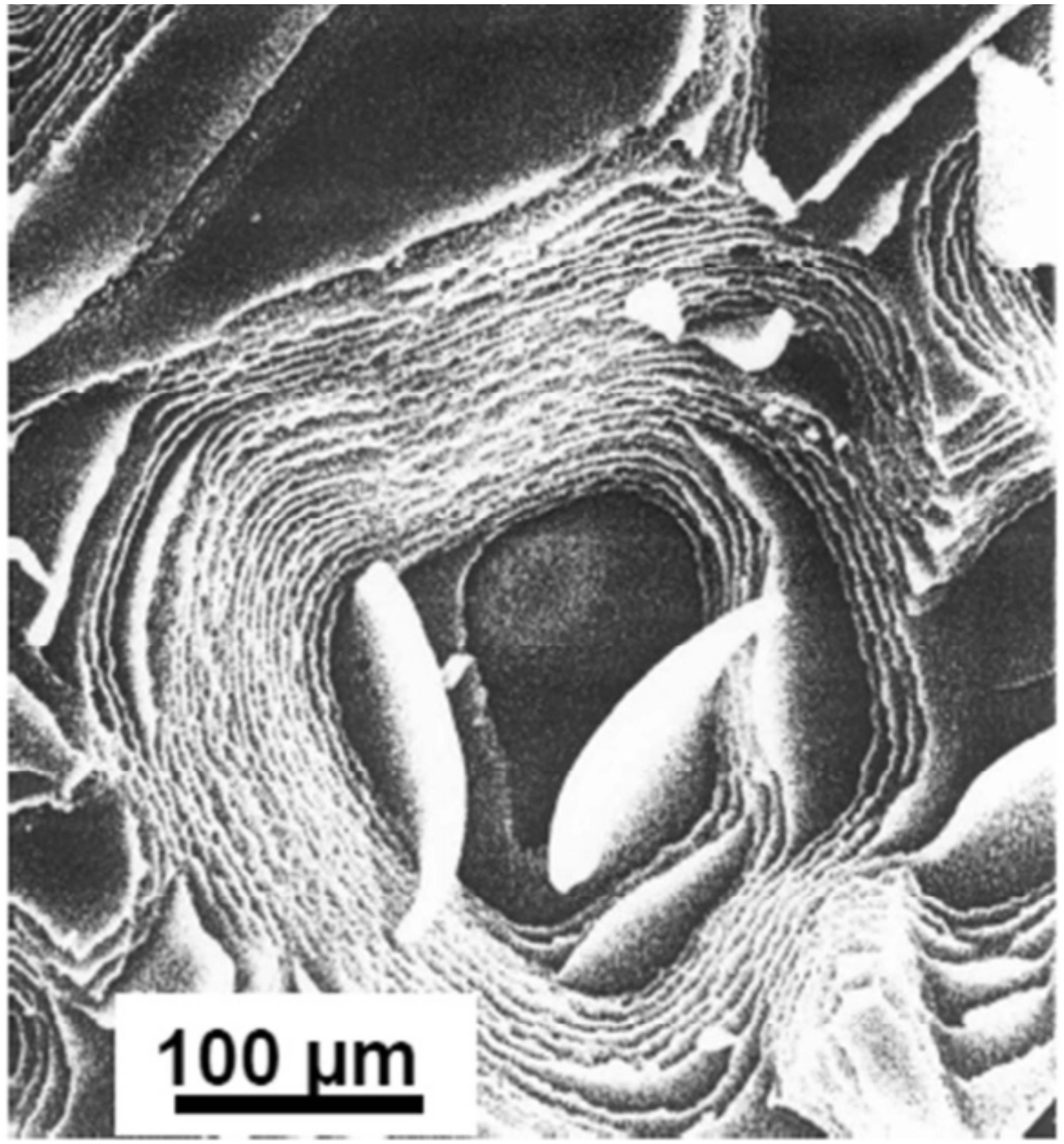
- Depends on efficiency (absorption, heating, Li-breeding)
- “Break-even” Q=0





TRITIUM BREEDING CHALLENGES

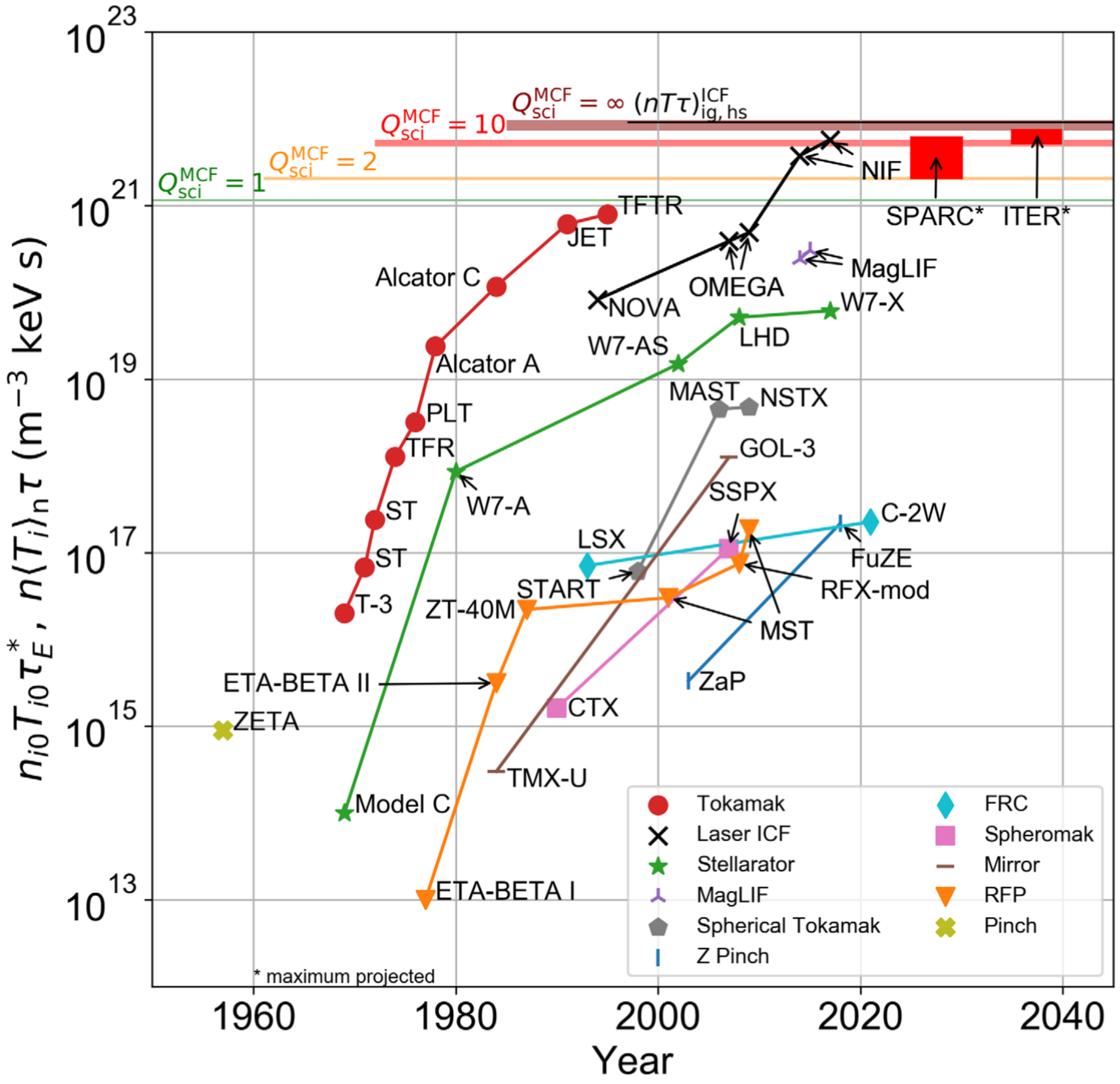




Exfoliation of steel caused by high-dose irradiation

[1] M. Rubel, Journal of Fusion Energy (2019) 38:315–329

TIMELINES



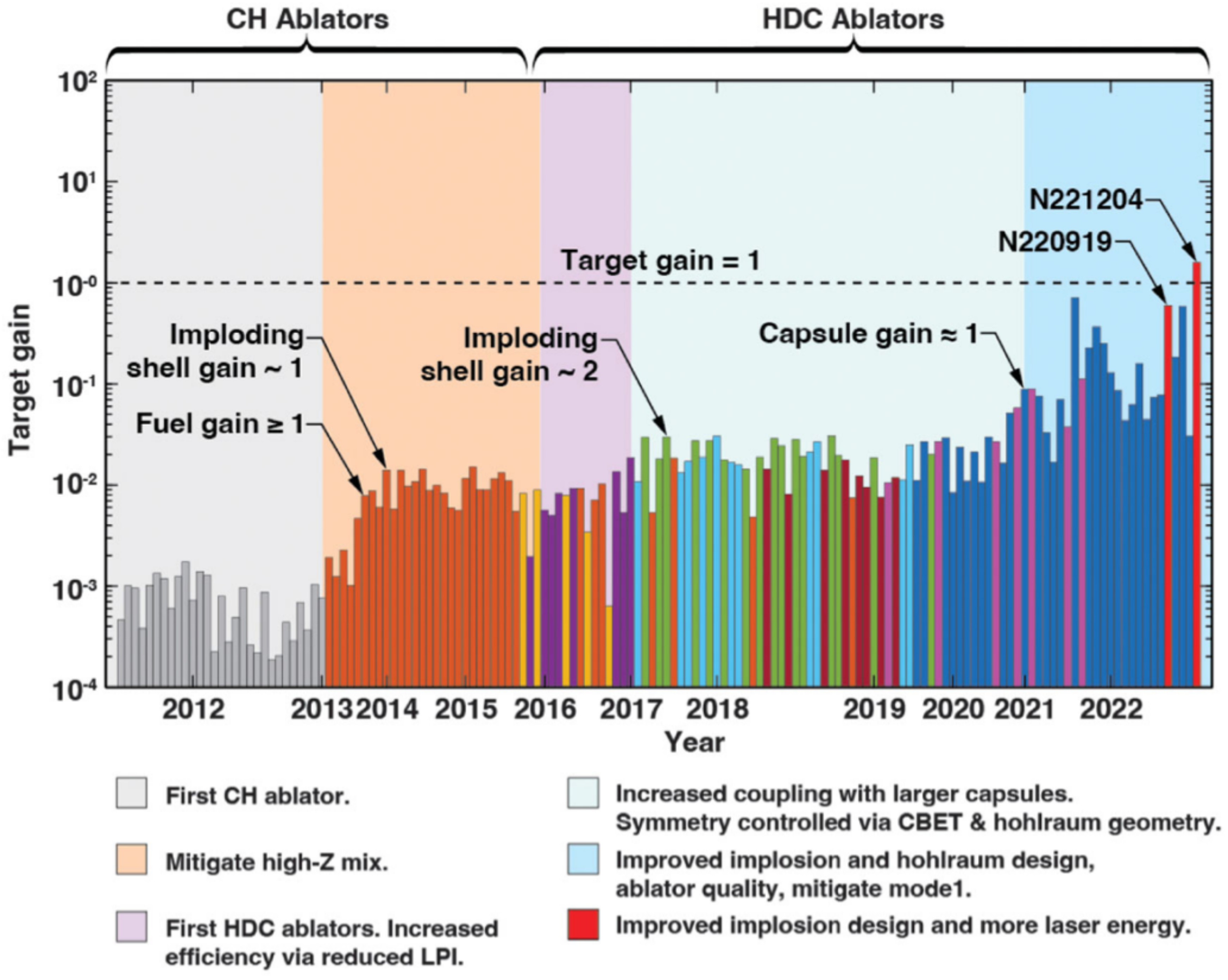
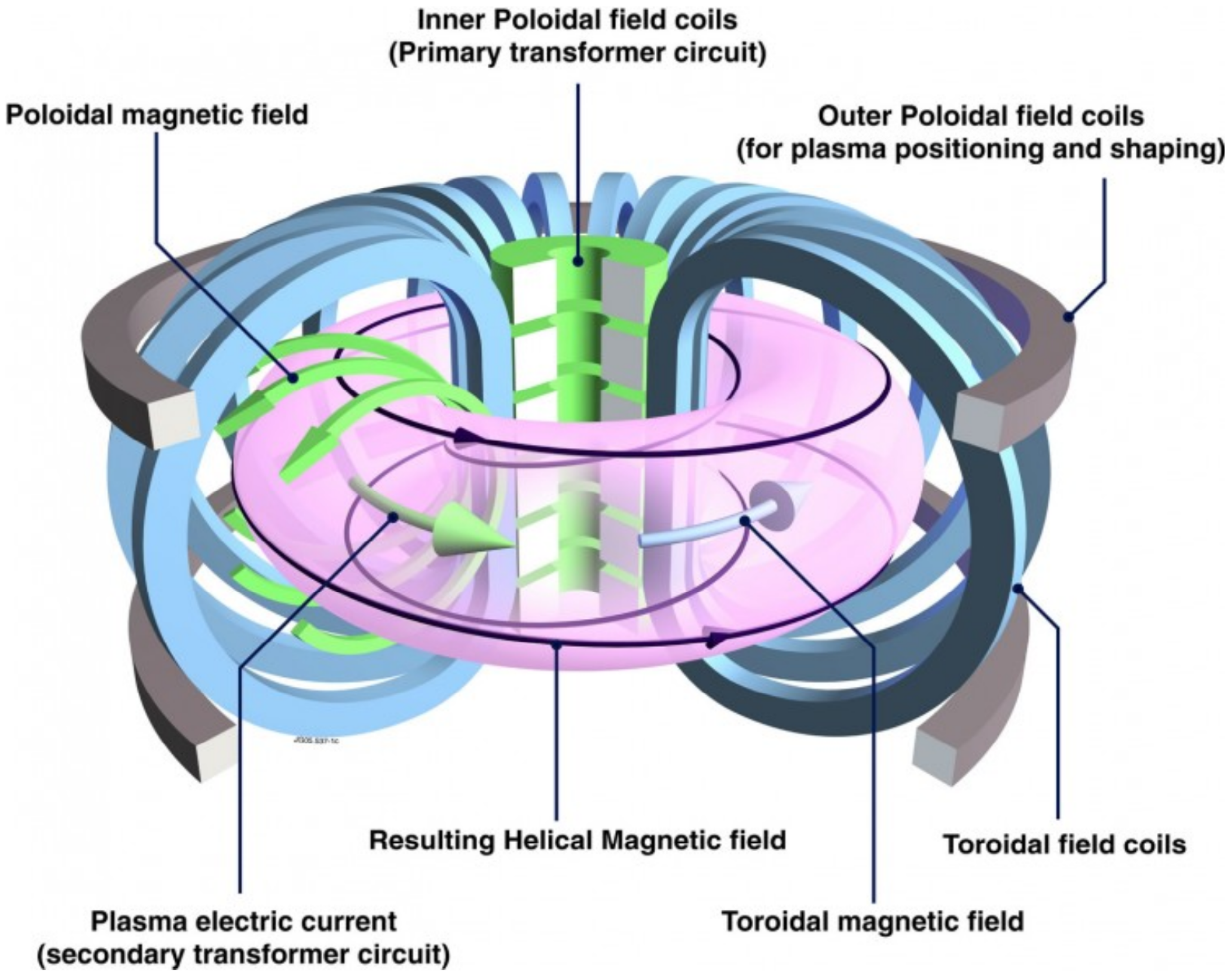
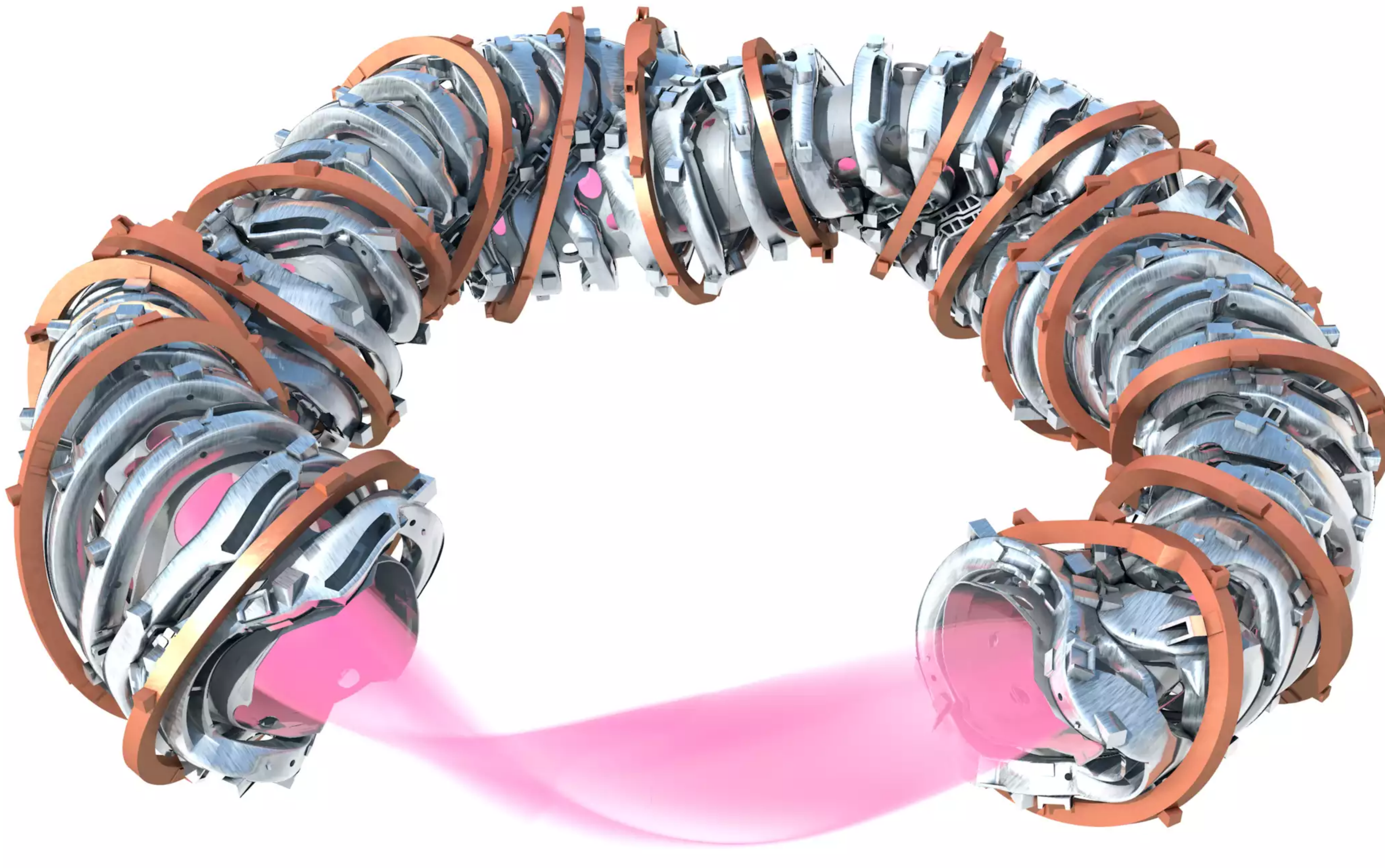
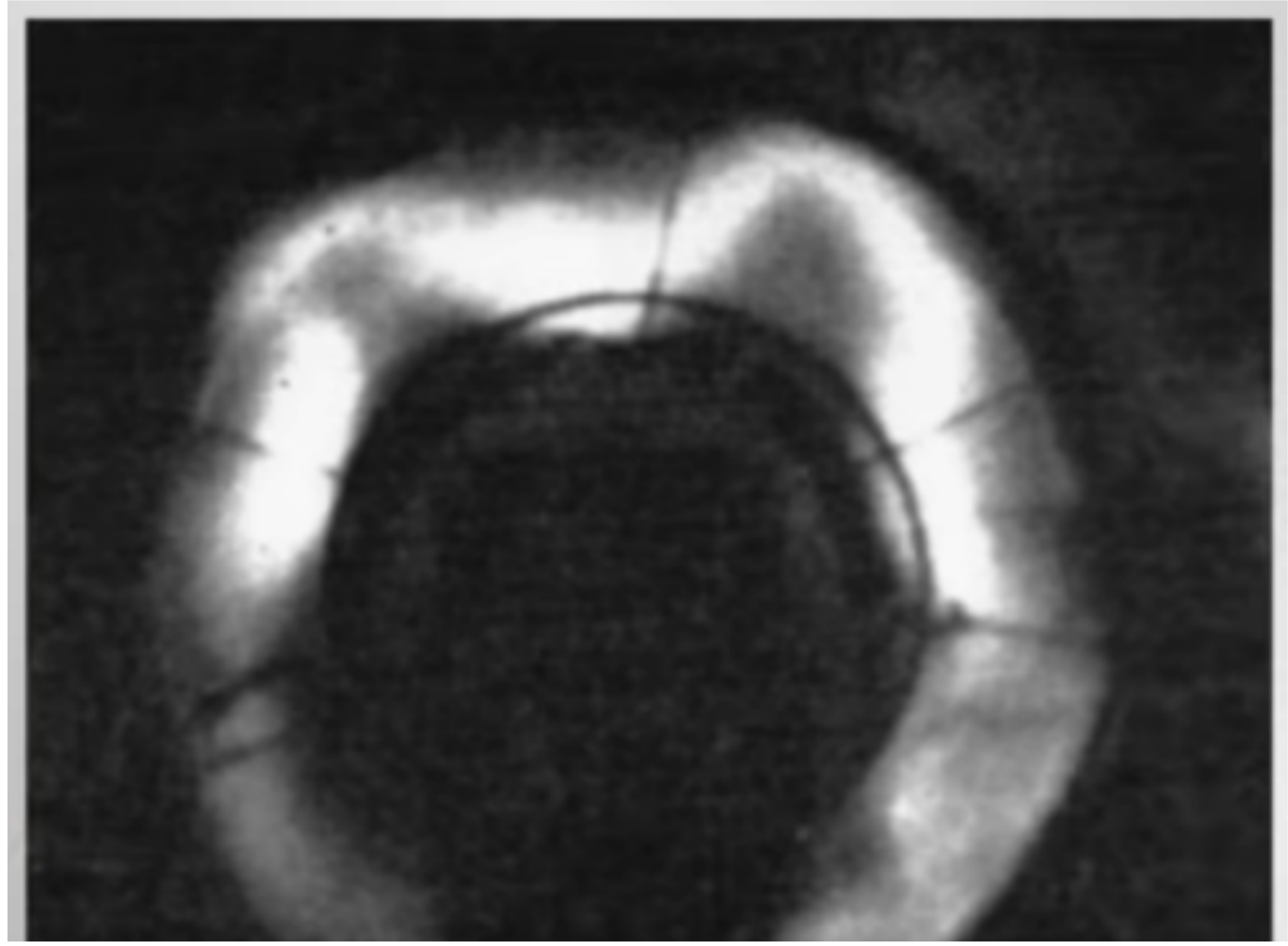


FIG. 2. Target gain vs calendar date. The horizontal labels mark the beginning of each year. The color of the narrow target gain bars represents different implosion designs, and the dashed horizontal line represents the target gain = 1 per the NAS ignition criteria [26].

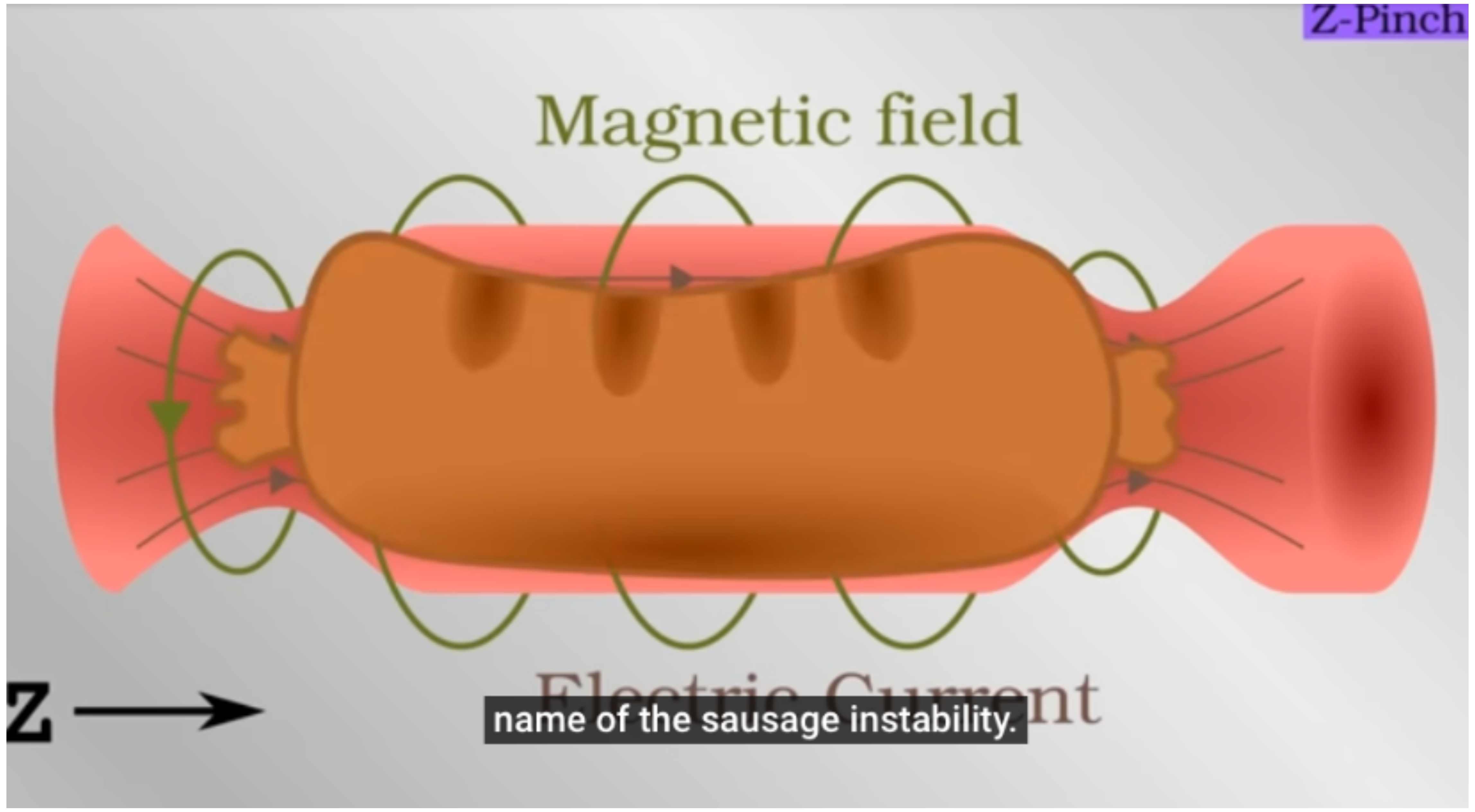




Wendelstein7-X



Plasma Instability: Z-Pinch



Plasma Instability in a mirror device

https://www.youtube.com/watch?v=gwOrbr8KWDs&list=PLbhKQRV6Toq4ocE3C1EwVbeY4ofwrPLn_&index=5

Plasma Physics Reports, 2021, Vol. 47, No. 8, pp. 814–825

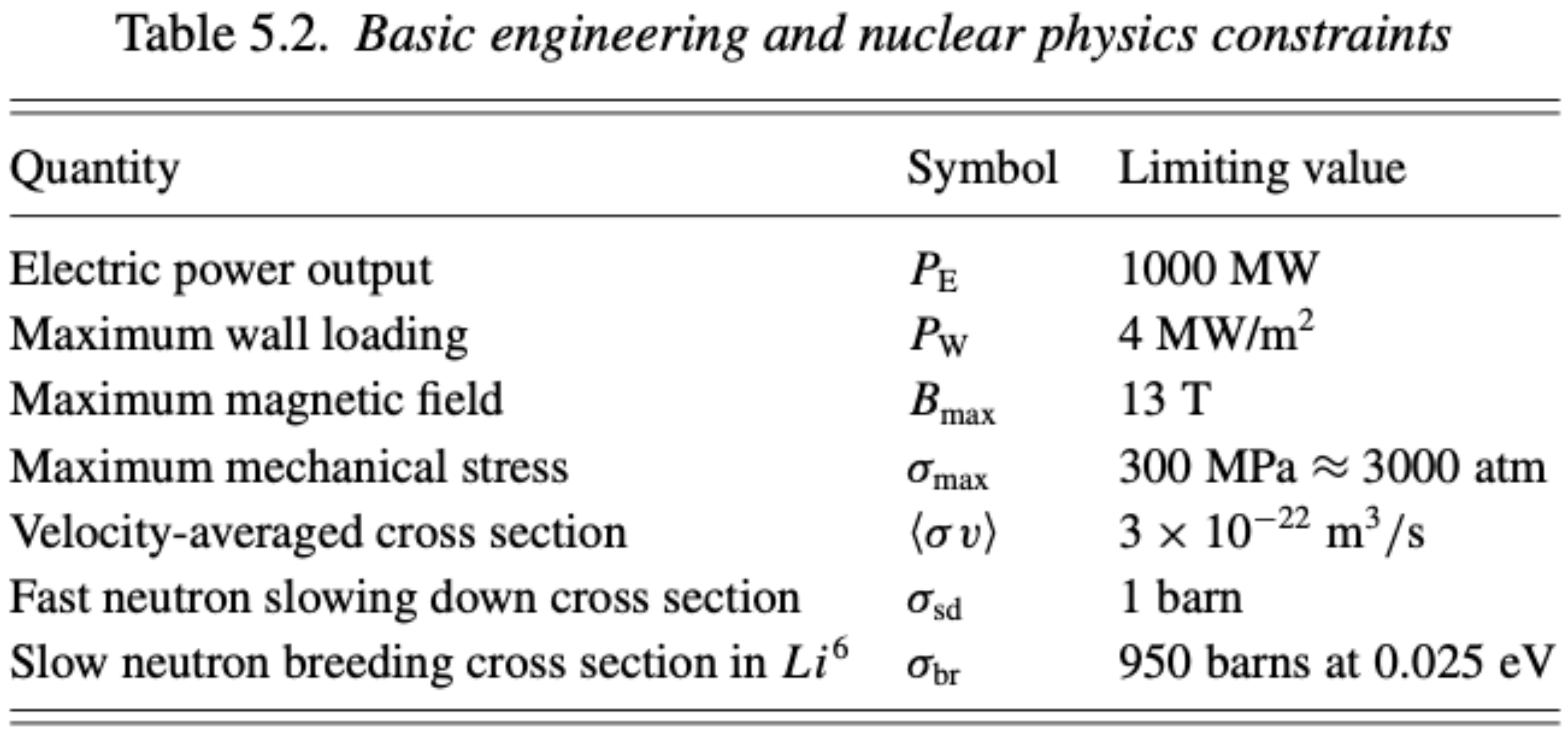


Table 5.2. *Basic engineering and nuclear physics constraints*

Quantity	Symbol	Limiting value
Electric power output	P_E	1000 MW
Maximum wall loading	P_W	4 MW/m^2
Maximum magnetic field	B_{max}	13 T
Maximum mechanical stress	σ_{max}	300 MPa $\approx$ 3000 atm
Velocity-averaged cross section	$\langle \sigma v \rangle$	3×10^{-22} m^3/s
Fast neutron slowing down cross section	σ_{sd}	1 barn
Slow neutron breeding cross section in Li^6	σ_{br}	950 barns at 0.025 eV

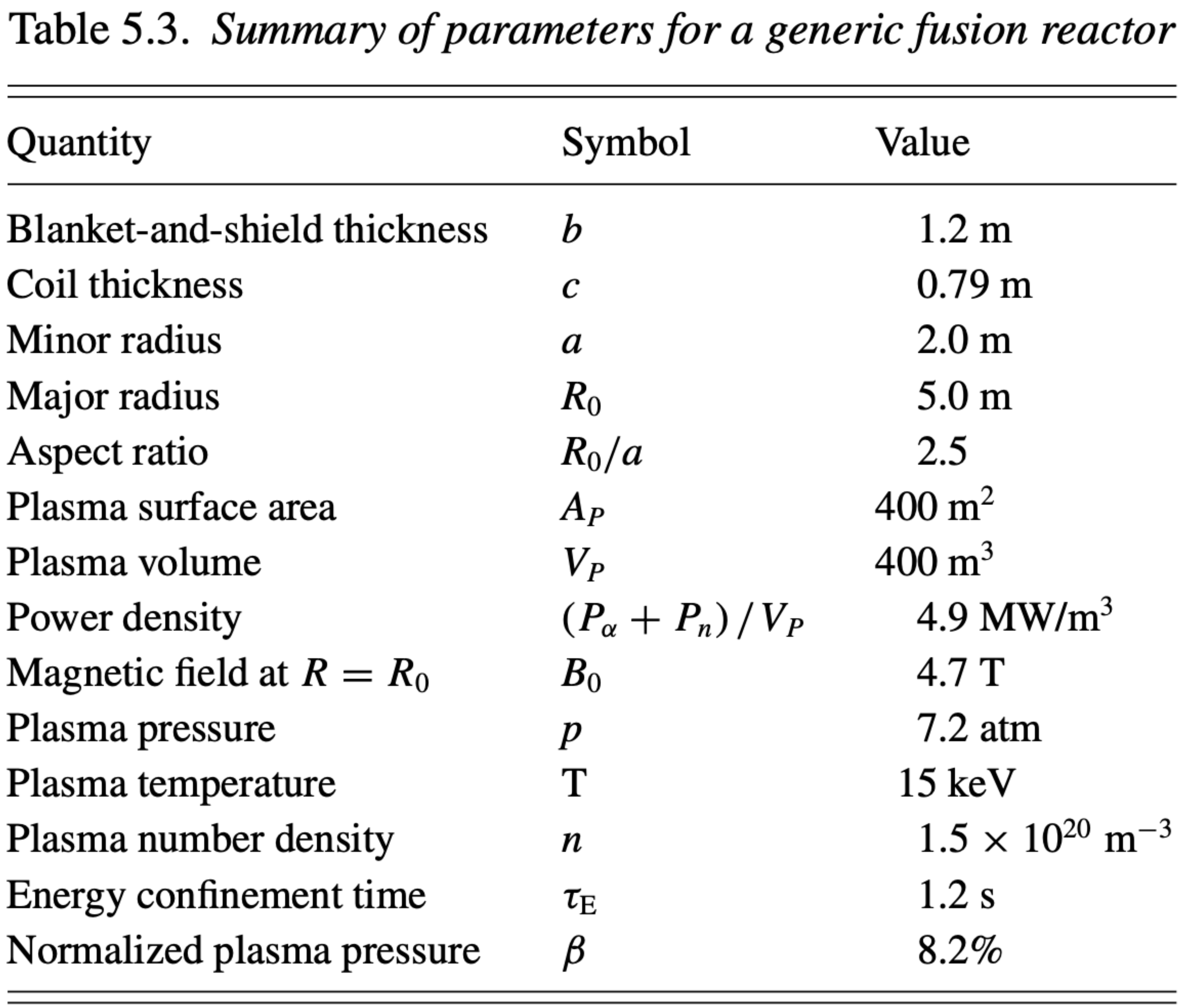


Table 5.3. *Summary of parameters for a generic fusion reactor*

Quantity	Symbol	Value
Blanket-and-shield thickness	b	1.2 m
Coil thickness	c	0.79 m
Minor radius	a	2.0 m
Major radius	R_0	5.0 m
Aspect ratio	R_0/a	2.5
Plasma surface area	A_P	400 m^2
Plasma volume	V_P	400 m^3
Power density	$(P_\alpha + P_n)/V_P$	4.9 MW/m^3
Magnetic field at $R = R_0$	B_0	4.7 T
Plasma pressure	p	7.2 atm
Plasma temperature	T	15 keV
Plasma number density	n	1.5×10^{20} m^{-3}
Energy confinement time	τ_E	1.2 s
Normalized plasma pressure	β	8.2%

[1] Freidberg “PLASMA PHYSICS AND FUSION ENERGY ”, Cambridge University Press 2007