Recoil corrections in antikaon-deuteron scattering

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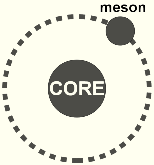
MOTIVATION

- The $\bar{K}N$ system as a testing ground for low energy $SU(3)$ meson-baryon dynamics
- Two fundamental quantities: $a_{I=0}$ and $a_{I=1}$

Two experiment(s):

- $K^- p$: energy shift and width of the $1s$ level for kaonic hydrogen in SIDDHARTA[1]@ DAΦNE
- $K^- d$: X-ray yield of kaonic deuterium derived. Important for:
  - planned upgrade to SIDDHARTA-2
  - Kaon implantation experiment @ J-PARC



[1] **Bazzi et al. (2011)**

EXPERIMENT ↔ THEORY

1. “Unitarized ChPT“ for meson-baryon scattering
   a) Extract a_{K^-p} from modified Deser-type formula[2]
   b) Construct a unitary amplitude from chiral potential[3], adjust free parameters
2. Three-body Faddeev equation:
   a) Assume NN and $\bar{K}N$ potential
   b) Determine the $\bar{K}NN$ amplitude numerically

	$\bar{K}N$ system	$\bar{K}d$ system
1. UChPT	a_0, a_1 calculated ✓	A not addressed X
2. Faddeev	potential assumed X	A predicted[4] ✓

GOAL: Explicit relation btw. a_1, a_0 and A

[2] Meißner, Raha, Rusetsky(2004)
[3] Ikeda, Hyodo, Weise(2012) MM, Meißner(2012) Borasoy, Nißler, Weise(2006)...
[4] Shevchenko (2012)

MOTIVATION

- Multiple-scattering series $\to$ poor convergence
- Resummation of the series $\hat{=}$ static approximation ($m_N \to \infty$) to Faddeev Equations: *Brueckner-type formula*[5]

$$A_{st} = \langle |\Psi(r)|^2 \frac{4r\tilde{a}_0\tilde{a}_1 + r^2(\tilde{a}_0 + 3\tilde{a}_1)}{2r^2 + r(\tilde{a}_0 - \tilde{a}_1) - 2\tilde{a}_1\tilde{a}_0} \rangle_r$$

$\tilde{a} = (1+\xi)a,\ \xi = \frac{M_K}{m_N}$

! Nucleon recoil corrections start with $\sqrt{\xi} \sim 0.7$

! But quantitatively:

- $\to$ at NLO: 15% effect in double scattering[6]
- $\to$ numerical solutions of Faddeev eqn. suggest $\sim$ 15% effect[7]

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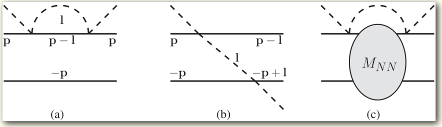
[5] **Kamalov, Oset, Ramos (2001)**
[6] **Baru, Epelbaum, Rusetsky (2009)**
[7] **Gal (2008)**

RECOIL CORRECTIONS - IDEA

- $\bar{K}N$ scale is large ($\sim M_\rho$) $\rightarrow$ amplitude is parameterized by scattering lengths, effective radii, etc..
- NN potential is characterized by a soft scale ($\sim M_\pi$) $\rightarrow$ take explicitly.

$\rightarrow$ Three types of interactions:



$\rightarrow$ In the static limit only (b) contributes $\rightarrow$ rewrite the three-particle $\bar{K}NN$ propagator: $g = (g - g_{st}) + g_{st} := \Delta g + g_{st}$

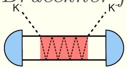
$$\begin{aligned} A = &\ \tilde{a} + \tilde{a}^2 g + \tilde{a}^3 g^2 + \cdots \\ = &\ \left\{\tilde{a} + \tilde{a}^2 g_{\text{st}} + \cdots\right\} + \left\{\tilde{a} + \tilde{a}^2 g_{\text{st}} + \cdots\right\} (\Delta g) \left\{\tilde{a} + \tilde{a}^2 g_{\text{st}} + \cdots\right\} + \cdots \\ = &\ A_{\text{st}} + A^{(1)} + A^{(2)} + \ldots \end{aligned}$$

$\rightarrow$ include interactions of type (a) and (c)

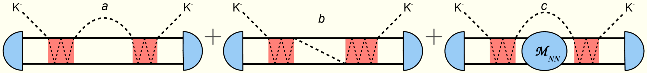
RECOIL CORRECTIONS - IDEA

Full set of Feynman diagrams:

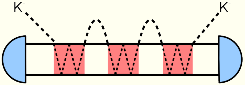
- Static part (*Brueckner formula*) $\Rightarrow$ one diagram

$A_{st} =$ 

- One recoil insertions $\Rightarrow$ three diagrams, 3×3 integrations

$A^{(1)} =$ 

- Two recoil insertions $\Rightarrow$ six diagrams, 6×3 integrations

$A^{(2)} =$  $+ \ldots$

- Higher order corrections can be calculated analogously

➡ computational challenge: rising number of integrals

➡ ***one*** *insertion is done,* ***two*** *is in preparation*

IS THIS A GOOD APPROACH?

IF “YES“, WHAT DOES IT PREDICT?

TEST OF THE FRAMEWORK

- Convergence of $A^{(n)}$ series in $\sqrt{\xi}$

  $\hookrightarrow$ Sign for a good counting scheme of $A = A_{\mathsf{st}} + A^{(1)} + A^{(2)} + \ldots$

- Uniform expansion[8] of $A^{(n)}$:

  - Independent of the regularization procedure
  - Applicable to any Feynman diagram

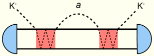
  Recipe:

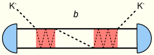
  a) Identify the momentum scales, e.g. small scale λ, large scale Λ.
  b) Expand the integrand $f(\lambda, q, \Lambda)$ in the low-, high- and intermediate momentum regime, i.e $\lambda \sim q \ll \Lambda$, $\lambda \ll q \sim \Lambda$ and $\lambda \ll q \ll \Lambda$.
  c) $\int_q f(\lambda, q, \Lambda) = \int_q f_l(\lambda, q, \Lambda) - \int_q f_i(\lambda, q, \Lambda) + \int_q f_h(\lambda, q, \Lambda)$.

[8] **Beneke, Smirnov (1998) Baru, Epelbaum, Rusetsky (2009)**

Expansion in powers of $\sqrt{\xi}$

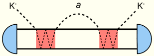
- Isospin (NN interm. state) decomposition reveals cancellation pattern:

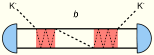
  1) I=1 (S=1,L=1): 

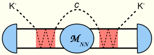
 − 

  ↪ cancels exactly at $\mathcal{O}(\sqrt{\xi})$

  2) I=0 (S=1,L=0):

 + 

 + 

  ↪ cancels exactly at $\mathcal{O}(\sqrt{\xi})$, if X=const.

  orthogonality of bound state and continuum w.f.

  ↪ in general screened by 

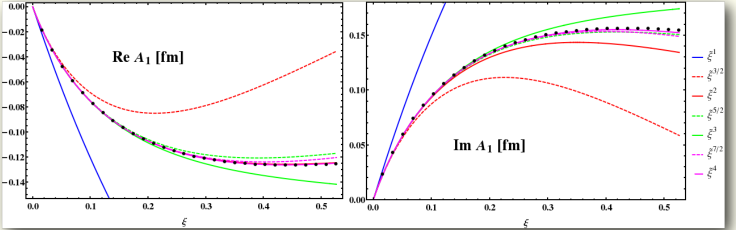
- NN-interaction parametrized (*for convergence test!*) by Hulthén potential ($\beta = 1.4 fm^{-1}$):

$$V_{NN}(p,q) = \lambda g(p) g(q)\,, \quad g(p) = \frac{1}{\beta^2 + p^2}\,, \quad \lambda = 32\pi m_N \beta\,(\beta+\gamma)^2$$

Results of the expansion, I=1

- Expansion in $\tilde{\xi} := \xi/(1+\xi/2)$ yields:

$$A_1 = \frac{8\pi}{(1+\xi/2)^2}\left(c_{1,1}\tilde{\xi} + c_{1,1}\tilde{\xi}^2 + \ldots + b_{1,1}\tilde{\xi}^{3/2} + b_{1,2}\tilde{\xi}^{5/2} + \ldots\right)$$



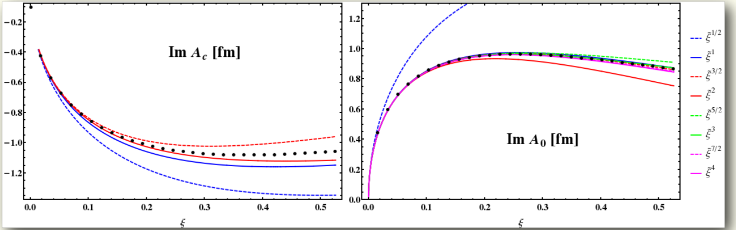
$\rightsquigarrow$ Convergence after a few orders in $\tilde{\xi}^{1/2}$

Results of the expansion, I=0

- Expansion in $\tilde{\xi} := \xi/(1+\xi/2)$ yields:

$$A_0 = \frac{8\pi}{(1+\xi/2)^2}\left(c_{0,1}\tilde{\xi} + c_{0,1}\tilde{\xi}^2 + \ldots + b_{0,0}\tilde{\xi}^{1/2} + b_{0,1}\tilde{\xi}^{3/2} + \ldots\right)$$

$$A_c = \frac{8\pi}{(1+\xi/2)^2}\left(C_1\tilde{\xi} + C_2\tilde{\xi}^2 + \ldots. + B_1\tilde{\xi}^{1/2} + B_2\tilde{\xi}^{3/2} + \ldots\right)$$



⇝ Convergence after a few orders in $\tilde{\xi}^{1/2}$

⇝ LO sizable cancellations: $b_{0,0} = -0.047 + i0.154 \leftrightarrow B_1 = +0.047 - i0.132$

IS THIS A GOOD APPROACH? ✓

WHAT DOES IT PREDICT?

A. CHOICE OF NN POTENTIAL

- NN: Hulthén and PEST[9] potential (short range physics)
- $\bar{K}N$: ($a_1 = -1.62 + i0.78$ fm, $a_0 = +0.18 + i0.68$ fm)[10]

Hulthén

A_{st}		$-1.492 + i1.187$
$A^{(1)}$	A_1	$-0.004 - i0.045$
	A_0	$-0.380 + i1.192$
	A_c	$+0.352 - i1.058$
	Sum:	$-0.031 + i0.090$
$A_{st} + A^{(1)}$		**-1.523+i1.277**

PEST

A_{st}		$-1.549 + i1.245$
$A^{(1)}$	A_1	$+0.002 - i0.039$
	A_0	$-0.401 + i1.309$
	A_c	$+0.356 - i1.193$
	Sum:	$-0.043 + i0.076$
$A_{st} + A^{(1)}$		**-1.593+i1.322**

➠ one insertion corrections are moderate for both NN potentials

[9] **Zankel et al. (1983)**
[10] **Shevchenko (2012)**

B. HIGHER ORDERS? *(preliminary)*

- Do we need the two, three, ... recoil insertions corrections?
- Formally they start at $(\xi^{1/2})^2$, $(\xi^{1/2})^3$, ...
- First estimation of two insertion corrections (Hulthén):

A_{st} [fm]		$A^{(1)}$ [fm]		$A^{(2)}$ [fm]	
		I		I	
		1	$-0.00 - i0.04$	11	$+0.01 - i0.01$
		0	$-0.03 + i0.13$	00	$+0.04 + i0.09$
				10	$+0.01 - i0.00$
$\sum$	$-1.49 + i1.19$	$\sum$	$-0.03 + i0.09$	$\sum$	$+0.06 + 0.07$

⇝ Estimate two recoil corrections:

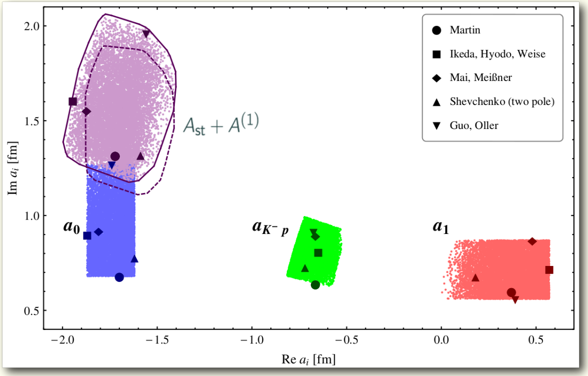
$$\xi^{1/2} Im(A_1^{(1)}) = -0.03 \text{ fm}, \; \xi^{1/2} Im(A_0^{(1)}) = 0.09 \text{ fm}$$

⇝ Estimate three recoil corrections: $\xi Im(A_0^{(1)}) = 0.07 \text{ fm} \approx 6\% A_{\mathrm{st}}$

⇝ Further cancellations might reduce the size of recoil corrections

C. PREDICTION

- NN: PEST potential
- $\bar{K}N$: syntetic data around literature values, restricted by SIDDHARTA



⇝ $(A_{st} + A^{(1)})$ depends strongly on the choice of $\bar{K}N$ s.l.

➠ precise exp. data on $\bar{K}d$ system can restrict a_0 and a_1 significantly!

Conclusion

- ✓ Analytic formulas for multiple insertion corrections
- ✓ Expansion of $A^{(1)}$ in powers of ξ converges
- ✓ Large cancellations at LO in one insertion corr.
- ✓ One insertion corr.: $7 - 8\%$ of the static result $\Rightarrow$ Good news!
- ✓ A is sensitive to a_0 and a_1 $\Rightarrow$ Good news for future experiment on kaonic deuterium

- ! Finite range/relativistic corrections
- ! Investigation of results for two insertion correction

... *in progress*

SPARES: two insertions

$$A^a = \langle f(p,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}}X_a(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}'}\rangle_{p,l,r,r'}$$

$$A^b = \langle f(p,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}}X_b(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{p}-\vec{l}/2)\cdot\vec{r}'}\rangle_{p,l,r,r'}$$

$$A^c = \langle d(p,l)d(q,l)M_{NN}(p,q,l)\Psi(r)e^{-i(\vec{p}+\vec{l}/2)\cdot\vec{r}}X_c(r,r',a_1,a_0)\Psi(r')e^{-i(\vec{q}+\vec{l}/2)\cdot\vec{r}'}\rangle_{p,q,l,r,r'}$$

$$d(p,l) := \frac{1}{l^2(1+\xi/2)+2\xi(p^2+m_N\epsilon_d)^2}\,, \quad f(p,l) := d(p,l) - \frac{1}{(1+\xi)l^2}$$