

A photograph of the Golden Gate Bridge in San Francisco, California. The bridge is a suspension bridge with two large towers and numerous cables. The bridge spans across the water, and a small sailboat is visible in the lower part of the image. The sky is blue with some clouds, and the water is a deep blue. The bridge is the central focus of the image, extending from the left side towards the right.

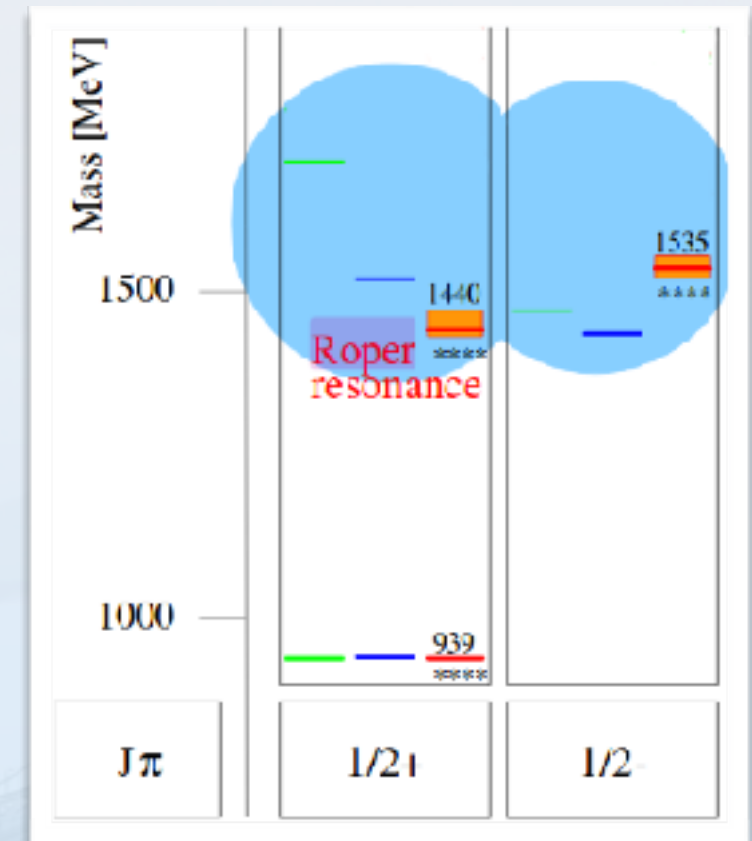
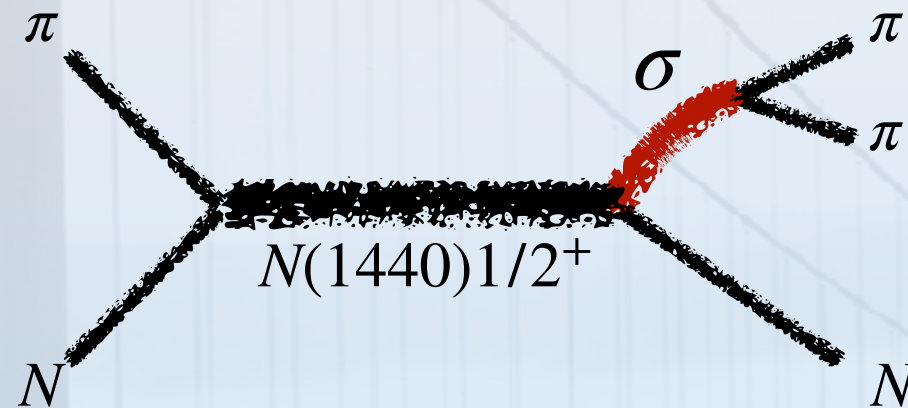
Bridging phenomenology & lattice QCD in the 3-body sector

Maxim Mai

MOTIVATION: QCD PUZZLES

● Roper-puzzle

- reversed mass pattern cf. constituent Quark Model
- large coupling to $\pi\pi N$ channels



Loring et al. EPJA10 (2001)

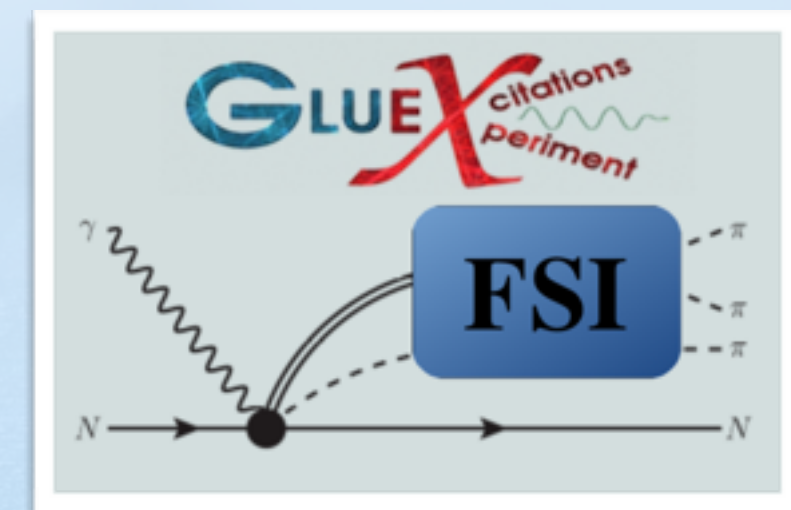
● $a_1(1260)$ & spin-exotics (GlueX, COMPASS, BESIII)

- indicator for the importance of gluonic d.o.f.
- cannot decay into $\pi\pi$ but only $\pi\pi\pi$ channel

● $X(3872)$

- decays dominantly into $D\bar{D}\pi$

● ...



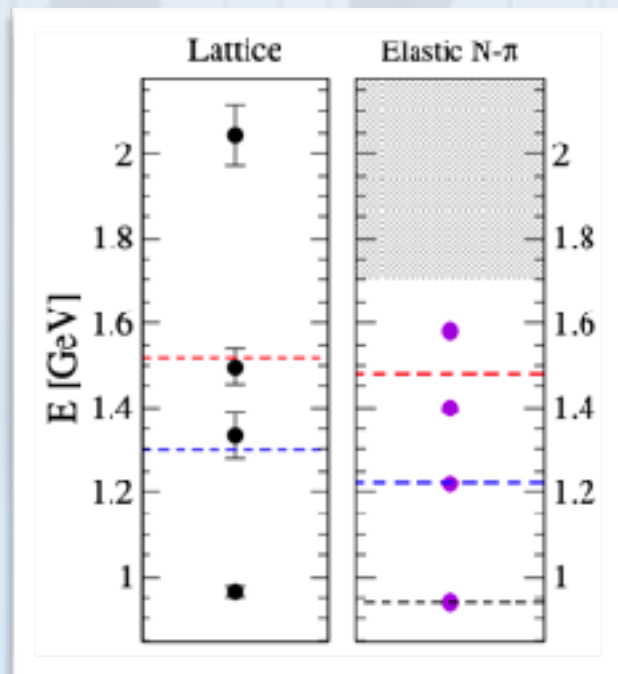
AB-INITIO APPROACH: LATTICE QCD

- first Lattice results are available

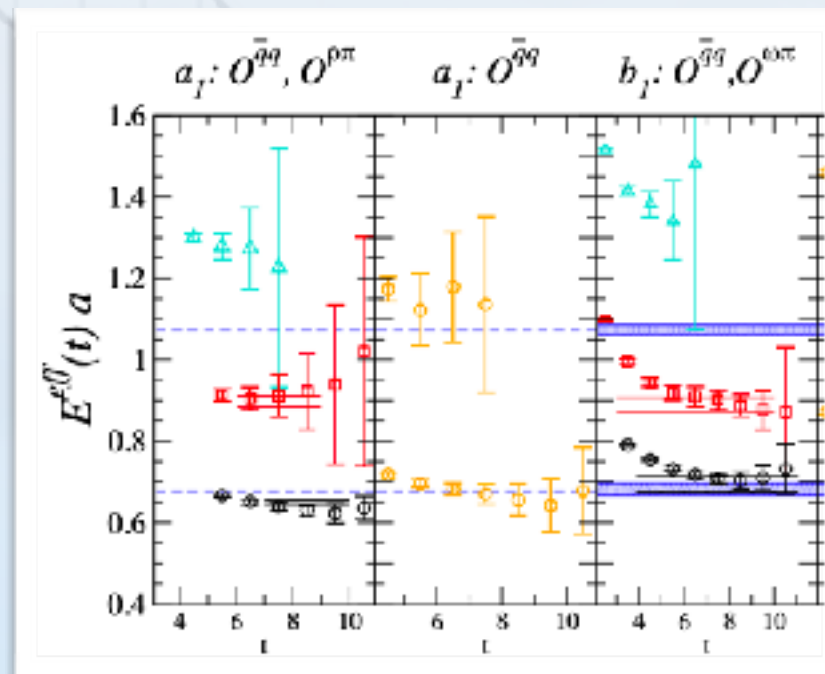
● $N(1440)1/2^+$

● $a_1(1260)$

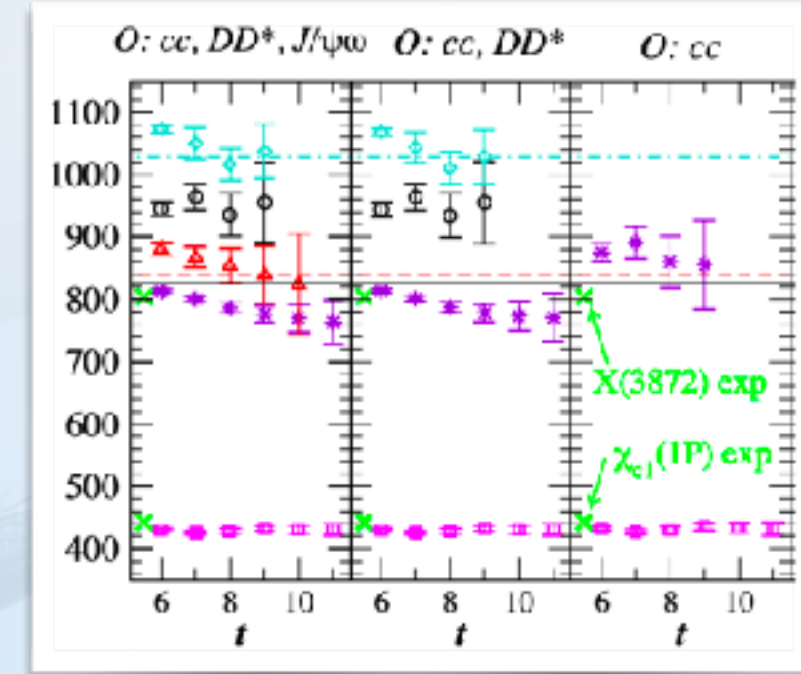
● $X(3872)$



Lang et al. PRD 95(2017)



Lang et al. JHEP 1404



Prelovsek, Leskovec PRL111 (2013)

● $I=2 \pi\rho$

Woss et al. JHEP 1807 [Hadspec]

● $I=3 \pi\pi\pi \rightarrow$ brand new calculations

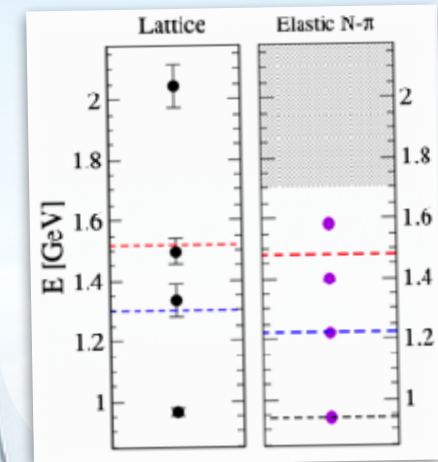
Hoerz/Hanlon (2019)

● ... more to come

Lattice QCD:

Euclidean Space-time in **finite volume**

→ results are **real and discrete!**

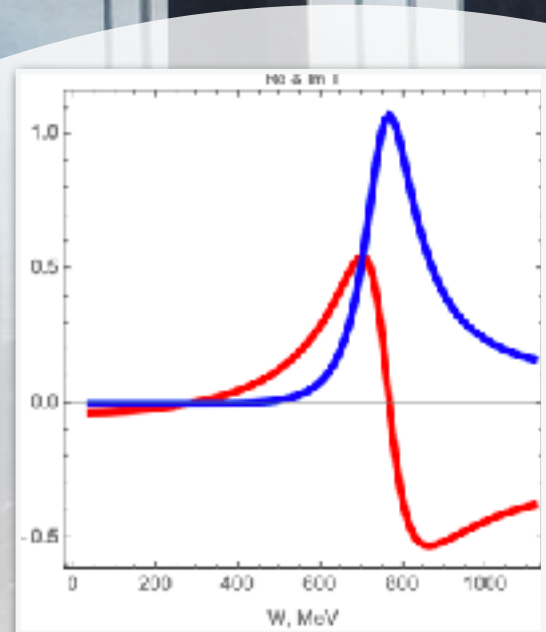


3-BODY QUANTIZATION CONDITION

Infinite volume world = phenomenology:

Universal properties of hadronic systems encoded in the **S-matrix**

→ **complex and continuous**



STATE OF THE ART

© 2-body case – Lüscher’s method

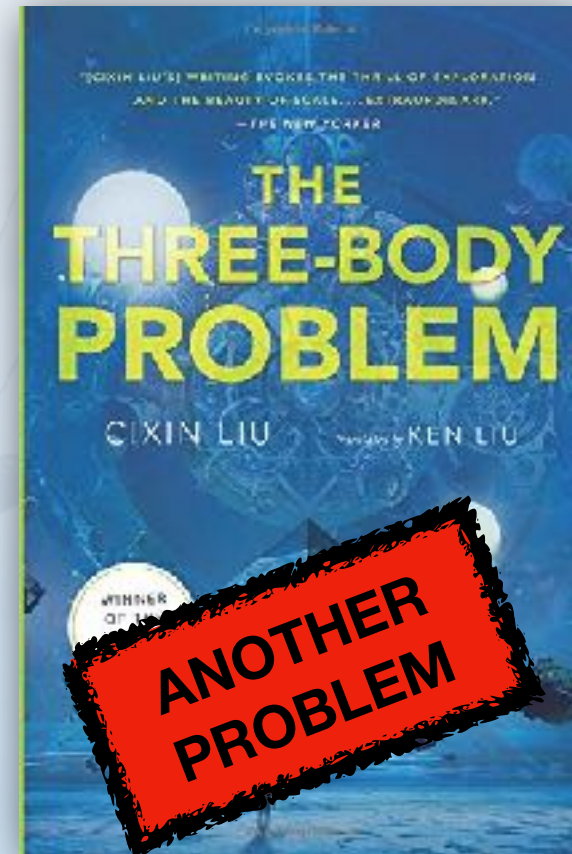
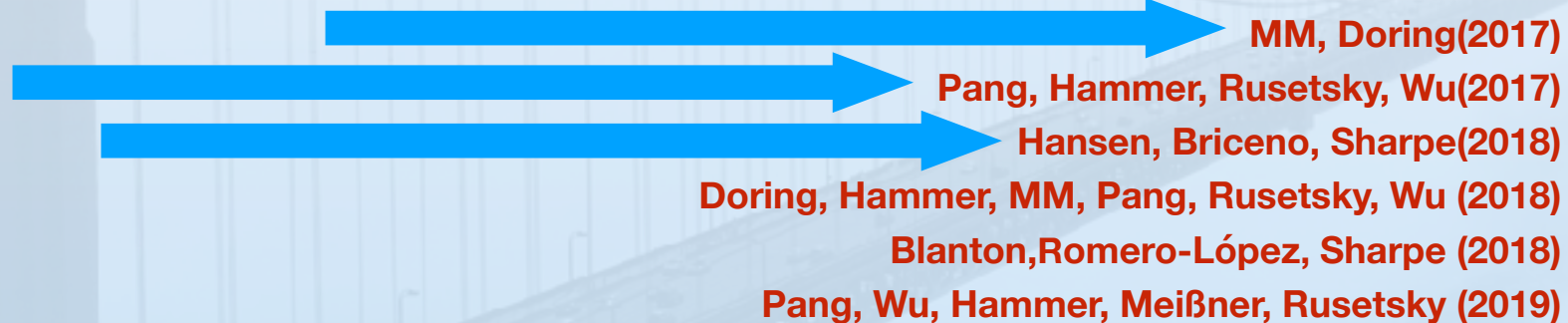
- *one-to-one mapping (also with extensions to multi-channels, spin, ...)*

Gottlieb, Rummukainen, Feng, Meißner, Li, Liu, Doring, Briceno, Rusetsky, Bernard...

© 3-body case

- *presumably no one-to-one mapping:*
complex kinematics (8 variables)
sub-channel dynamics
- *theoretical developments and pilot numerical investigation*

Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo...



- *first application to Lattice QCD results :*

finite volume ($\pi^+\pi^+$) and ($\pi^+\pi^+\pi^+$) spectrum

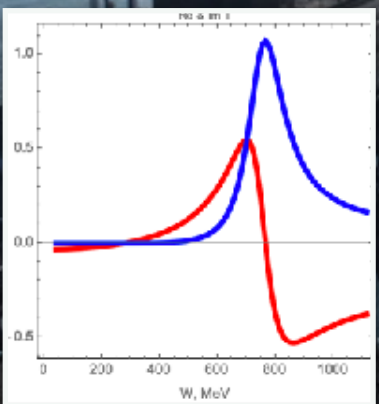
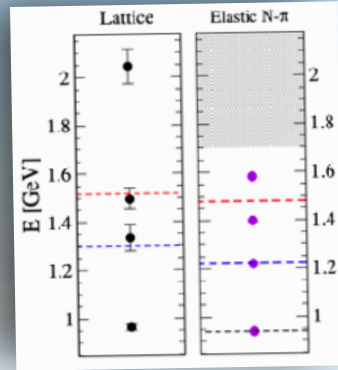
MM, Doering (2018) → PRL122 (2019)

- *analysis of 2019 Hoerz/Hanlon ($\pi^+\pi^+\pi^+$) spectrum (boosted systems/...)*

MM, Alexandru, Culver, Doering [1909.05749] Blanton, Romero-López, Sharpe [1909.02973]

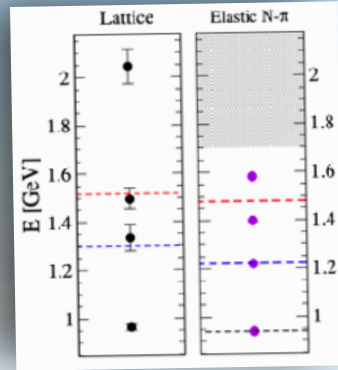
RELATIVISTIC 3B-QC IN A NUTSHELL

3-BODY QUANTIZATION CONDITION

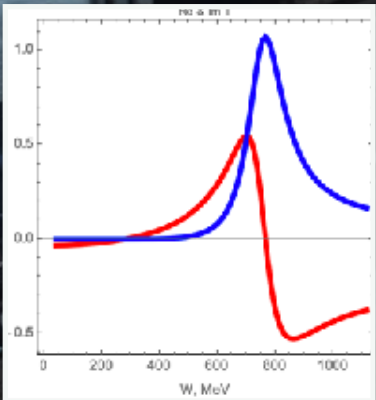


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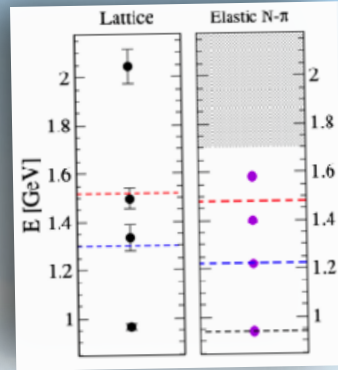
3-BODY QUANTIZATION CONDITION



Unitarity



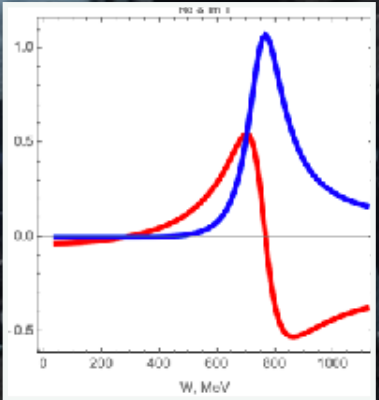
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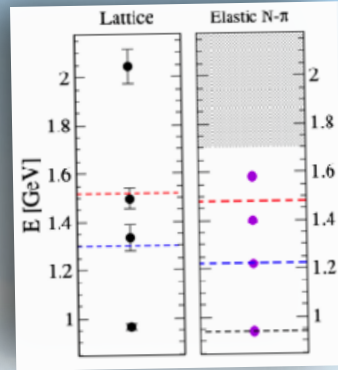
3-BODY QUANTIZATION CONDITION

Im T in infinite volume

Unitarity



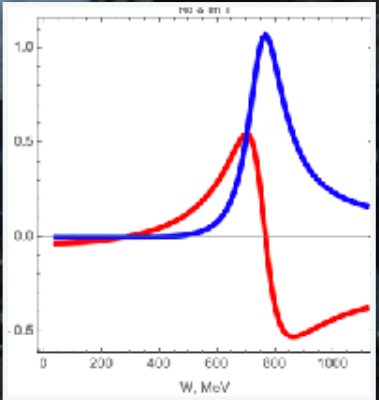
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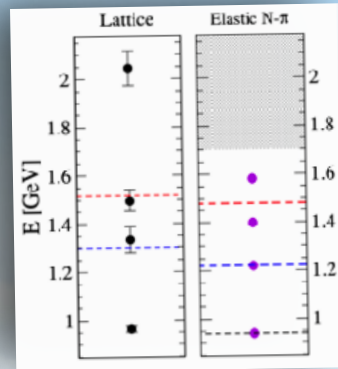
on-shell configurations

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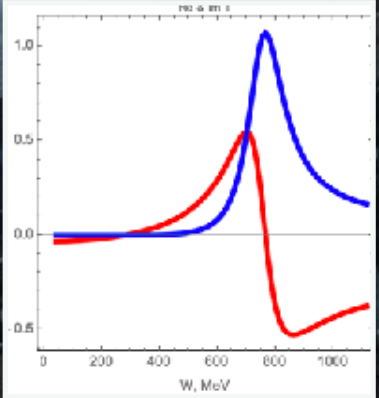


***Power-law
finite-volume effects***

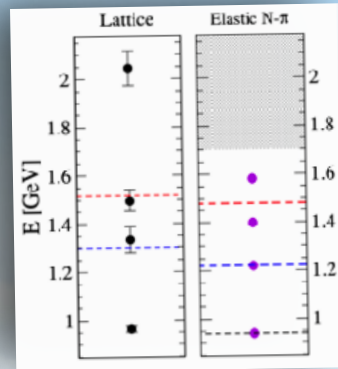
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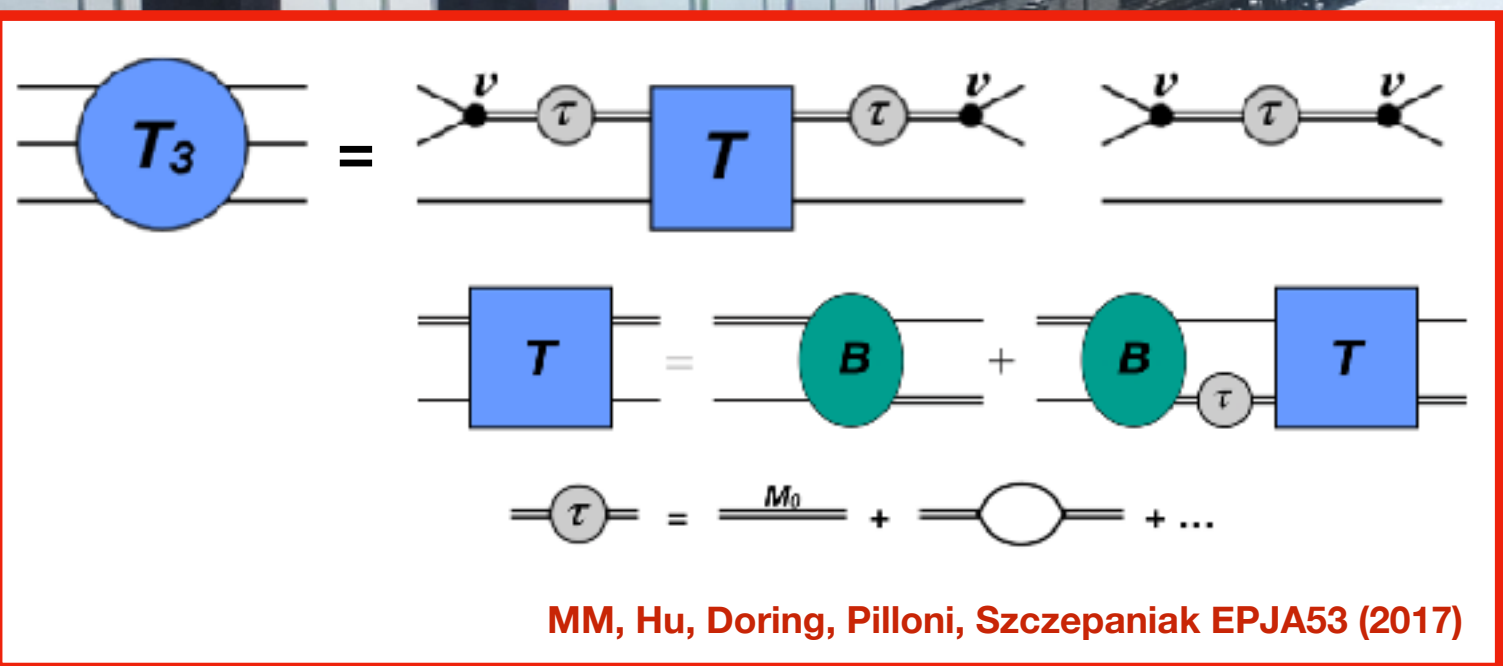


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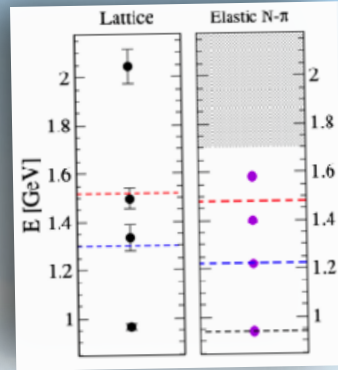
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on-shell configurations



MM, Hu, Doring, Pilloni, Szczepaniak EPJA53 (2017)

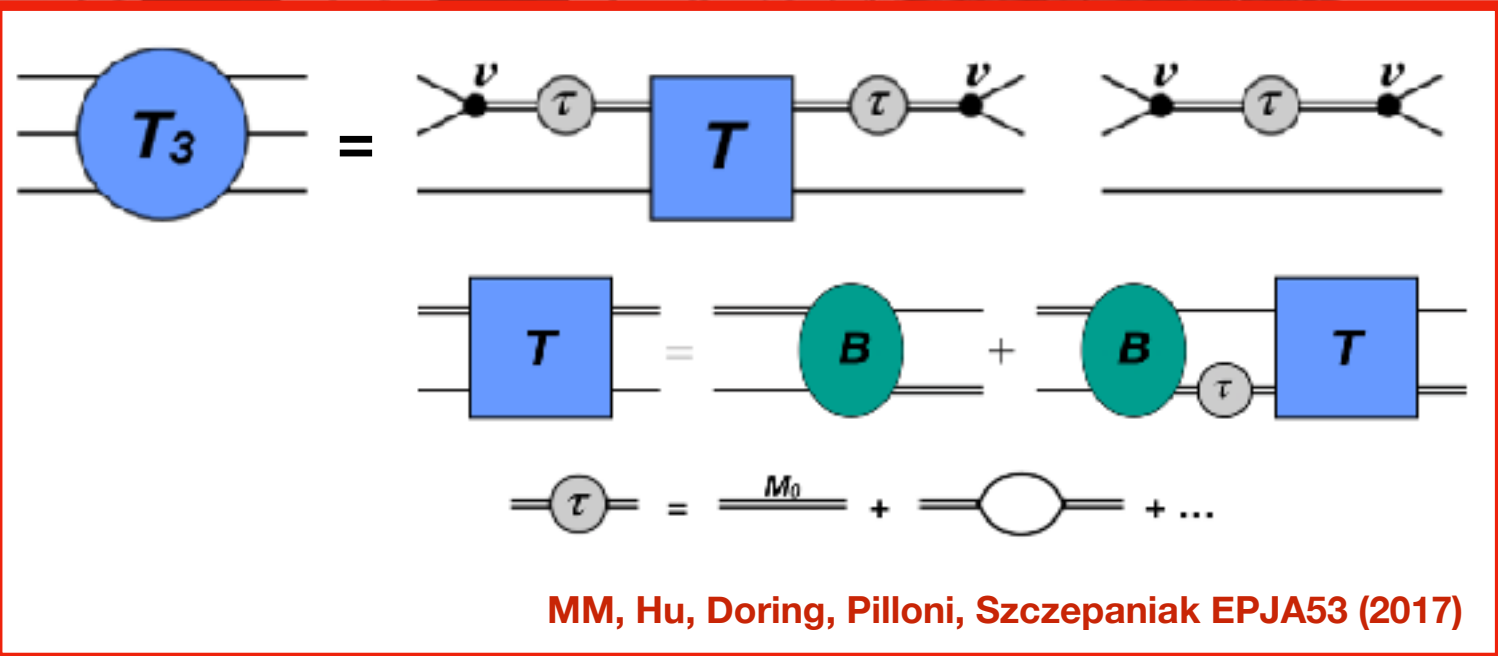
RELATIVISTIC 3B-QC IN A NUTSHELL



3-BODY QUANTIZATION CONDITION

$$\text{Det} \left(B^{ss'}(W^2) + \tau_s^{-1}(W^2, L) \delta_{ss'} \right) = 0$$

MM, Doring EPJA53 (2017)

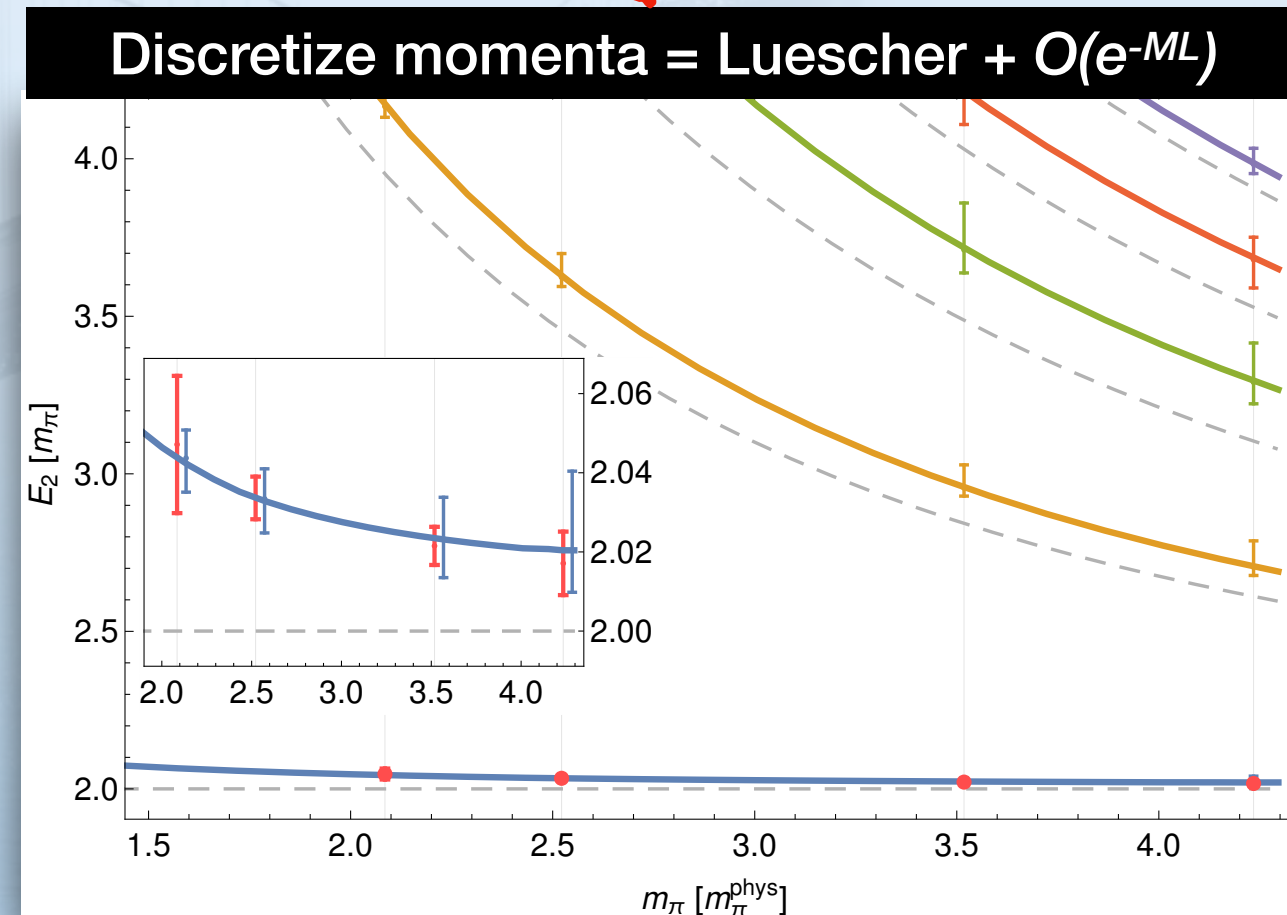
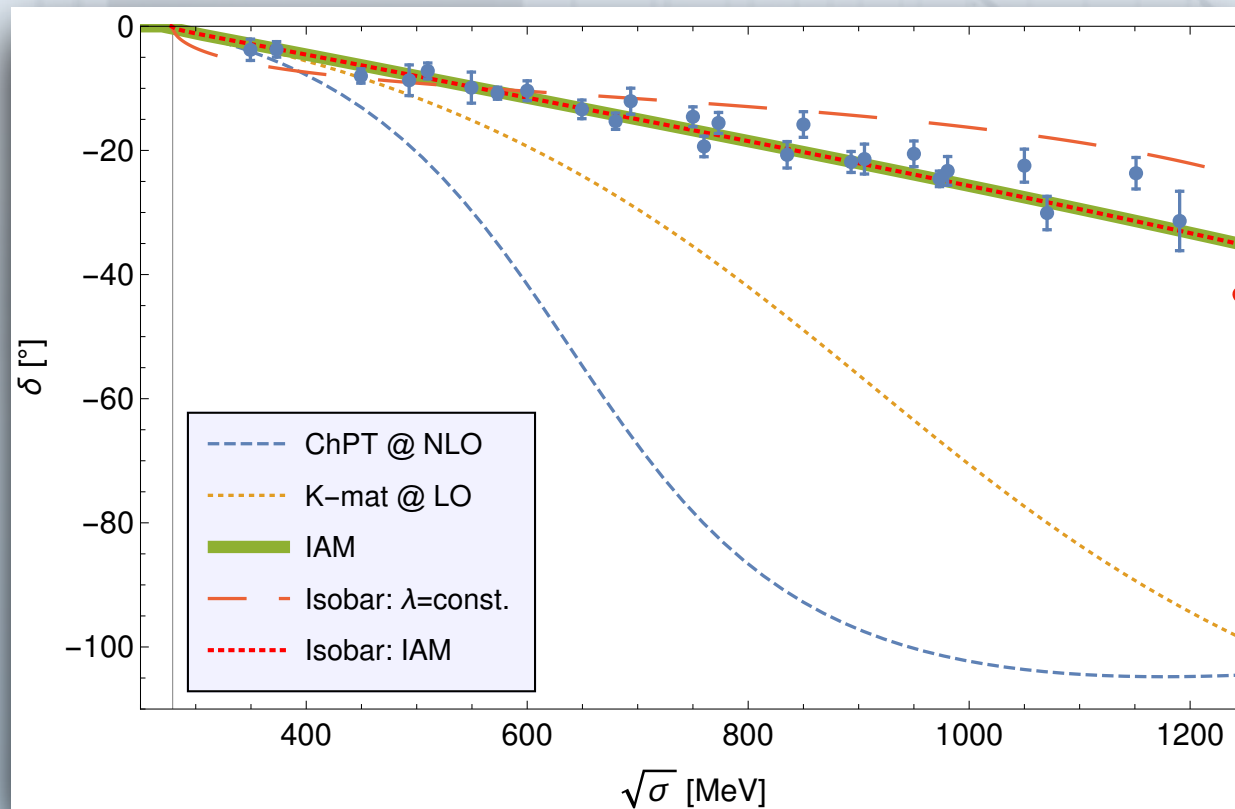


PHYSICAL APPLICATION

MM, Doring (2018) PRL 122

◎ 2-body sub-channel:

- One-channel problem — $\pi^+\pi^+$ system in S -wave
- IAM parametrized scattering amplitude



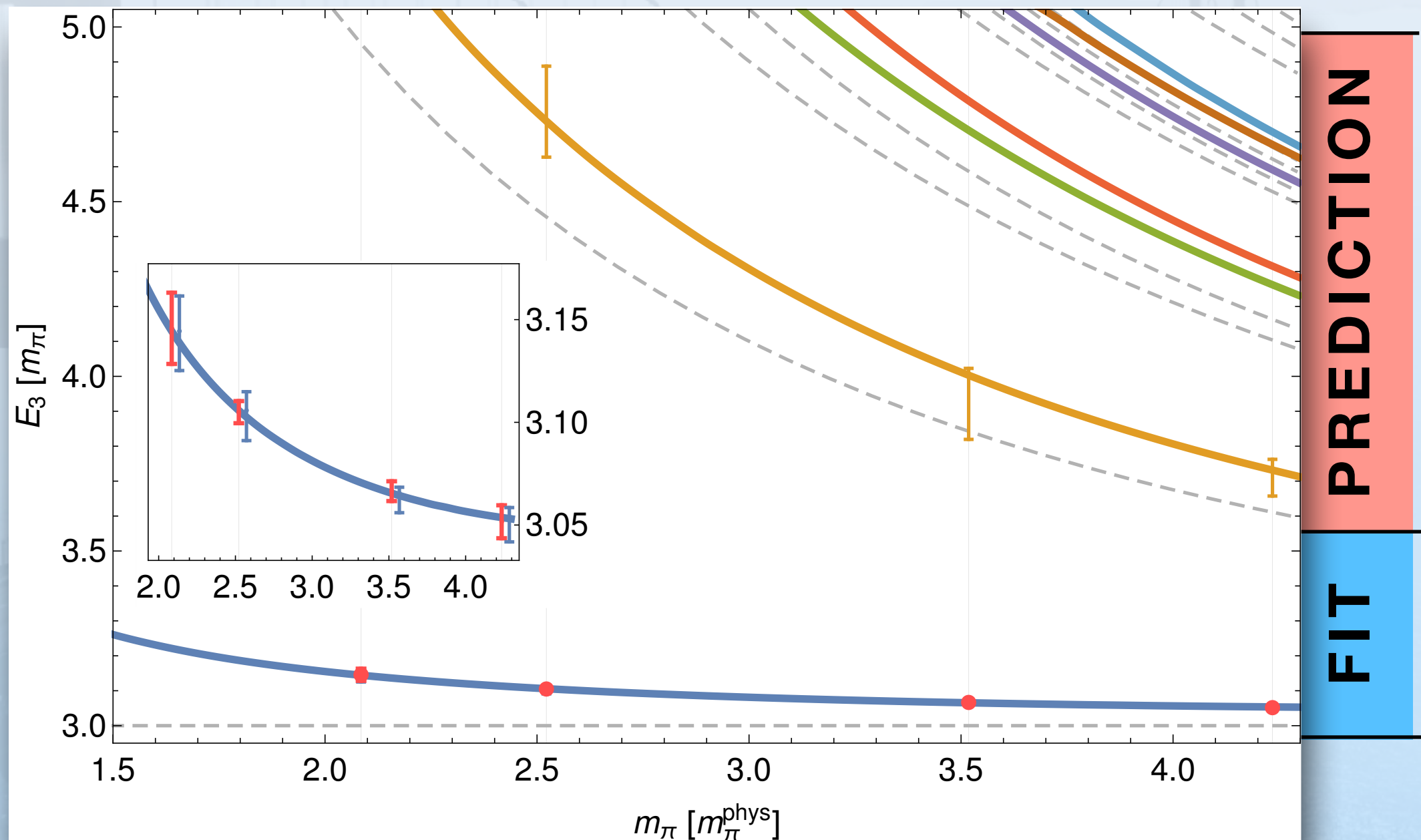
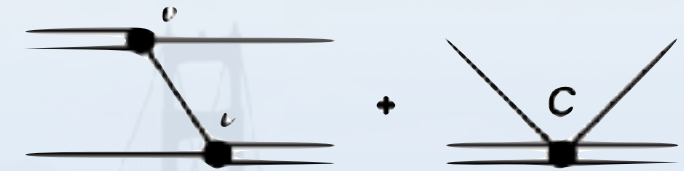
PHYSICAL APPLICATION

MM, Doring (2018) PRL 122

3-body spectrum

- ansatz: $\mathbf{C} = c \delta^{(3)}(\mathbf{p} - \mathbf{q}) \rightarrow c = 0.2 \pm 1.5 \cdot 10^{-10}$

$$\text{Det} \left(B^{ss'}(W^2) + \tau_s^{-1}(W^2, L) \delta_{ss'} \right) = 0$$



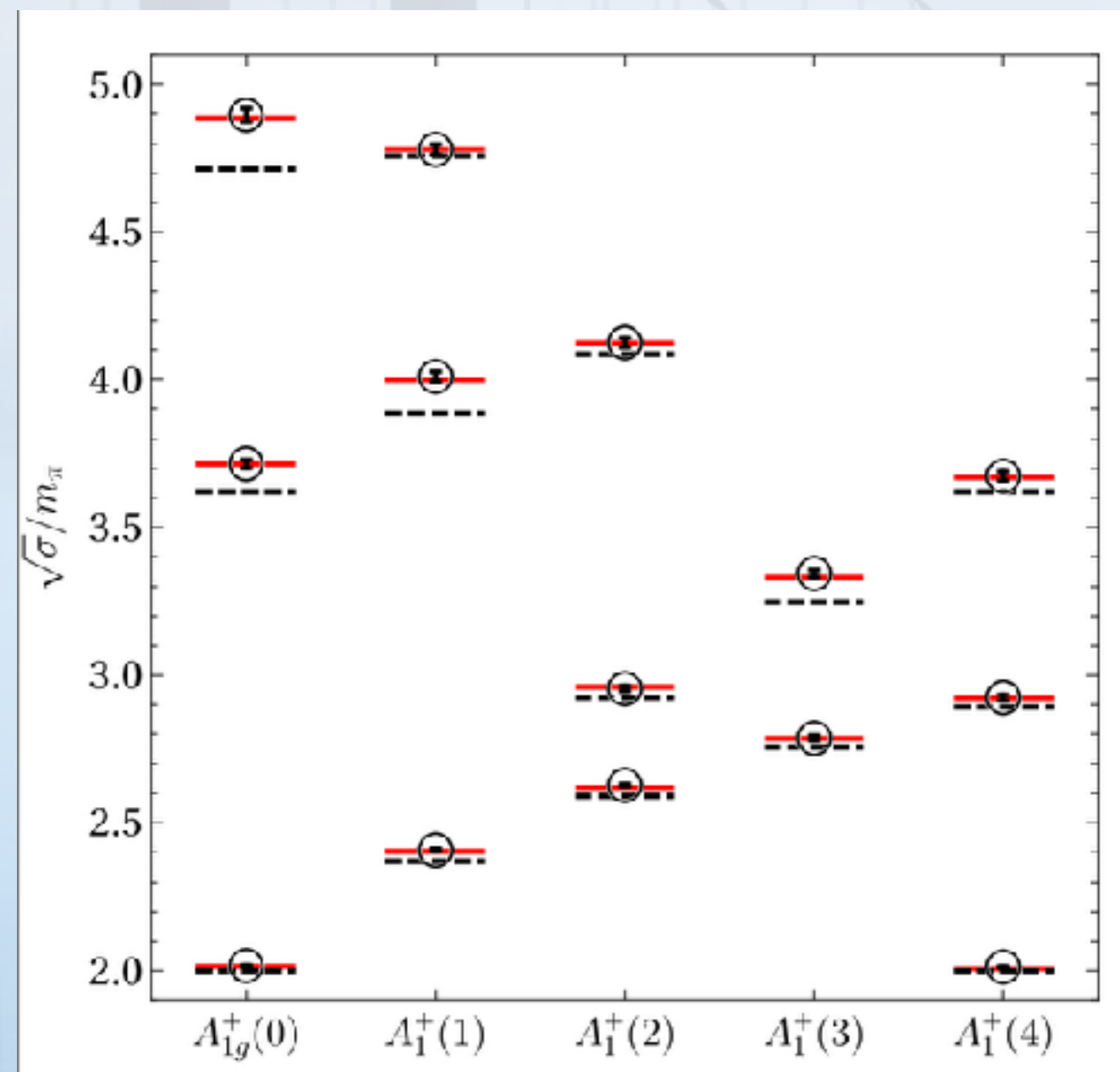
BOOSTED SYSTEMS ETC..

© New data is now available for higher boosts and irreps

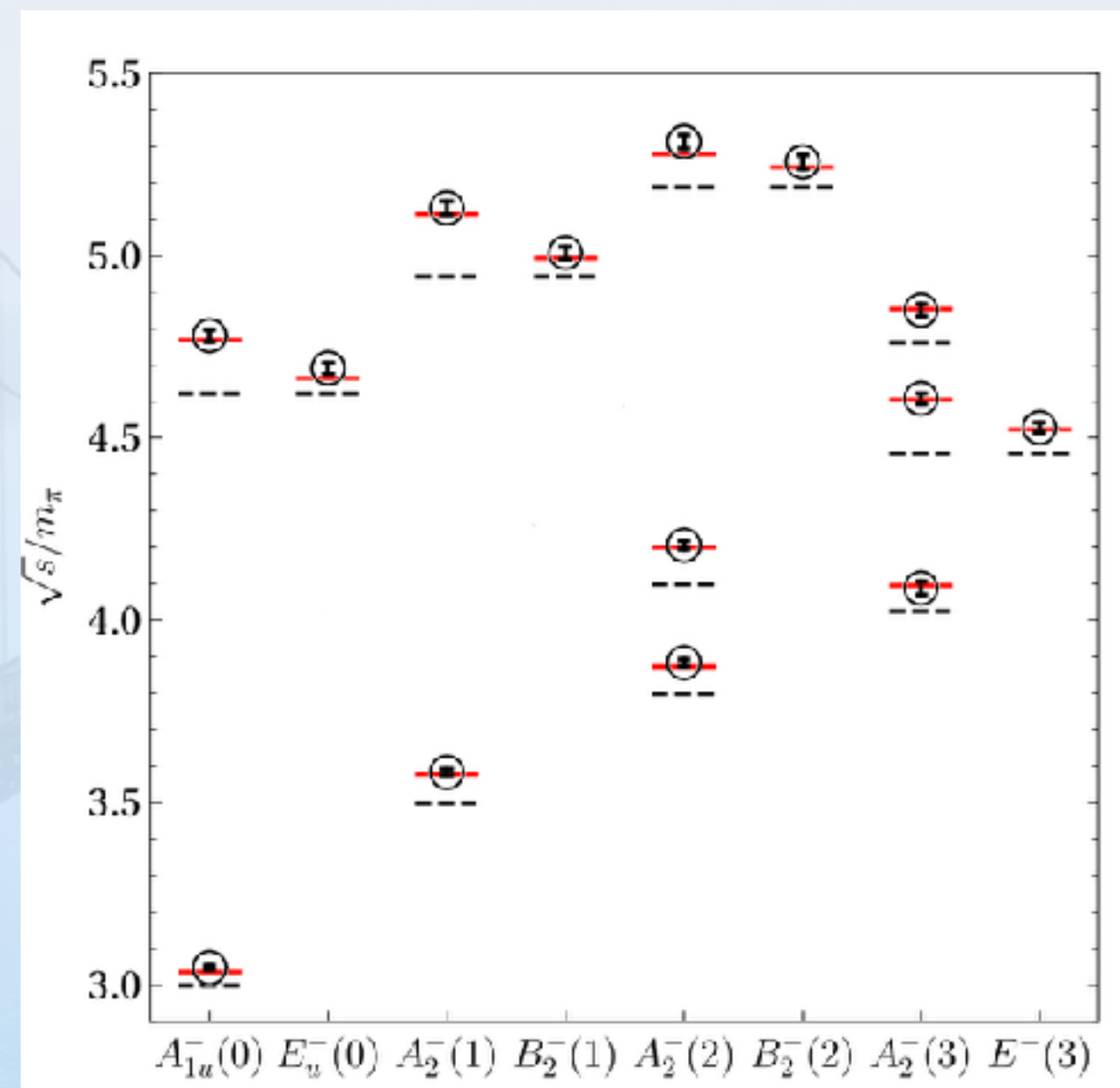
Hoerz Hanlon PRL 123 (2019)

- compare with the 3b-QC predictions:

MM, Alexandru, Culver, Doering [1909.05749]



2-body spectrum



3-body spectrum

SUMMARY

- ★ *Discretization & Projection to irreps of O_h leads to a **relativistic 3body QC***
- ★ *Excited spectrum of $\pi^+\pi^+$ & $\pi^+\pi^+\pi^+$*
- ★ ***NPLQCD(2009)** and **Hoerz/Hanlon(2019)** results ($2\pi^+$ & $3\pi^+$) analyzed*
- ★ *Predictions at physical pion mass*

EPJA53 (2017) PRD97 (2018) PRL 122 (2019) arXiv:1909.05749

- ★ *Parametrization via 2-b. sub-channel amplitudes*
- ★ *Relativistic integral equation **EPJA53 (2017)***

*this and other related talks are on
<https://maxim-mai.github.io/>*

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THANK YOU