GOING FULL CIRCLE: QCD TO EFT TO RESONANCES AND BACK TO (LATTICE) QCD

MAXIM MAI

ALBERT EINSTEIN CENTER

UNIVERSITY OF BERN

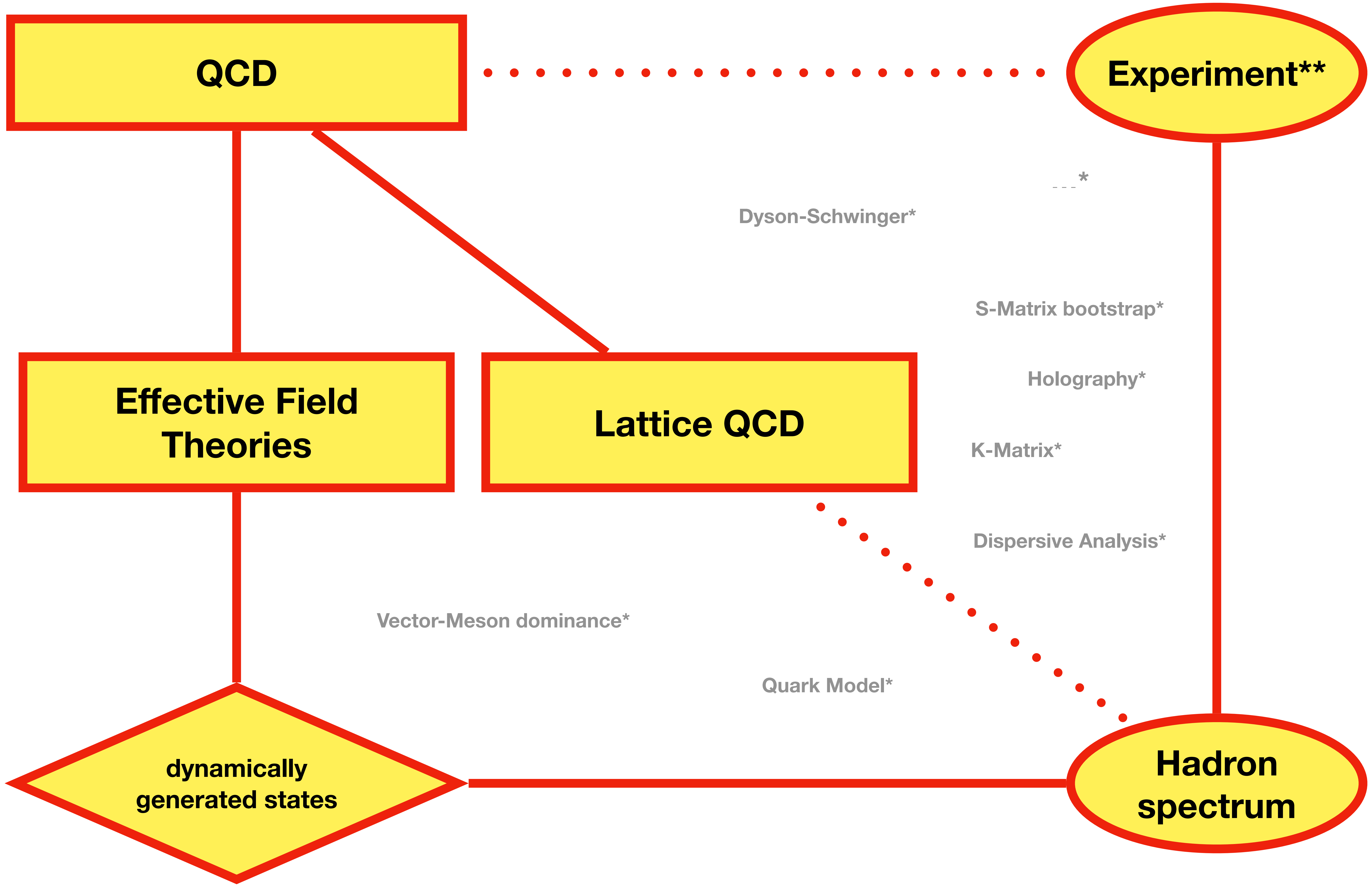
Colloquium @ Peking University

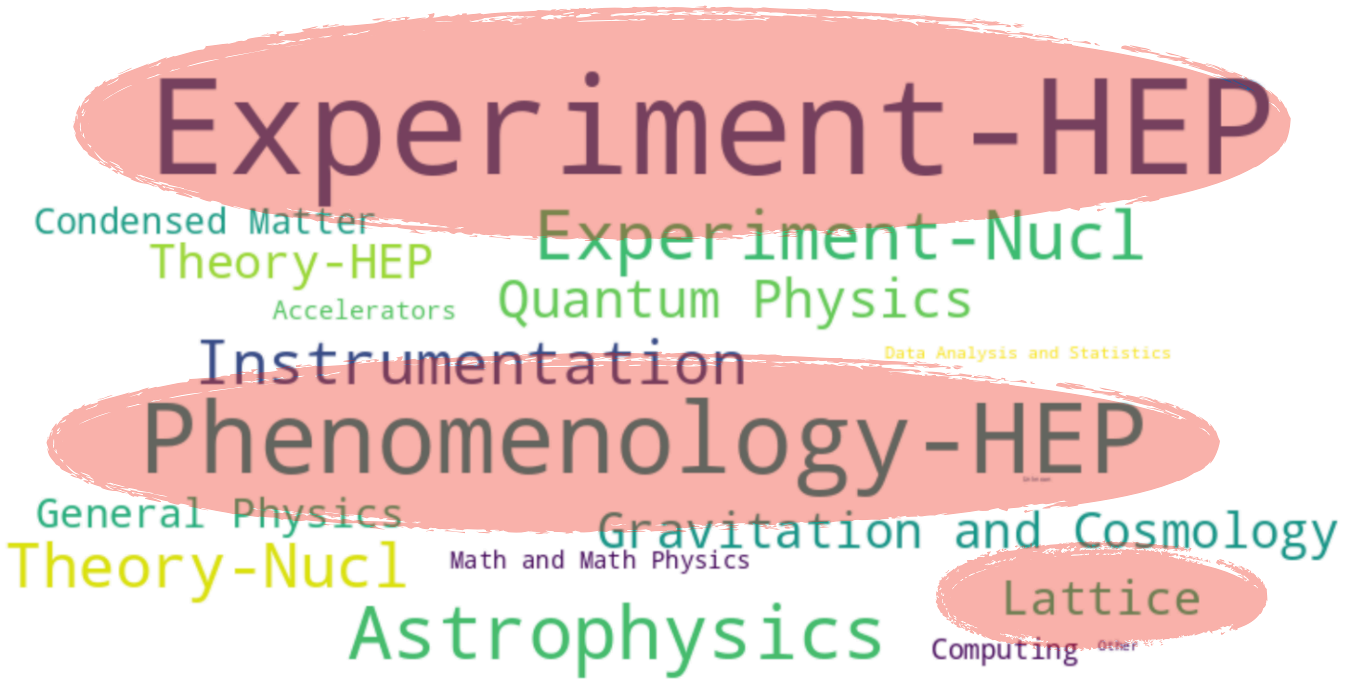
03.04.2025

Heisenberg-Programm

DFG: Heisenberg Programme (project number: 532635001)

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076 DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907 , NSF: PHY-2012289

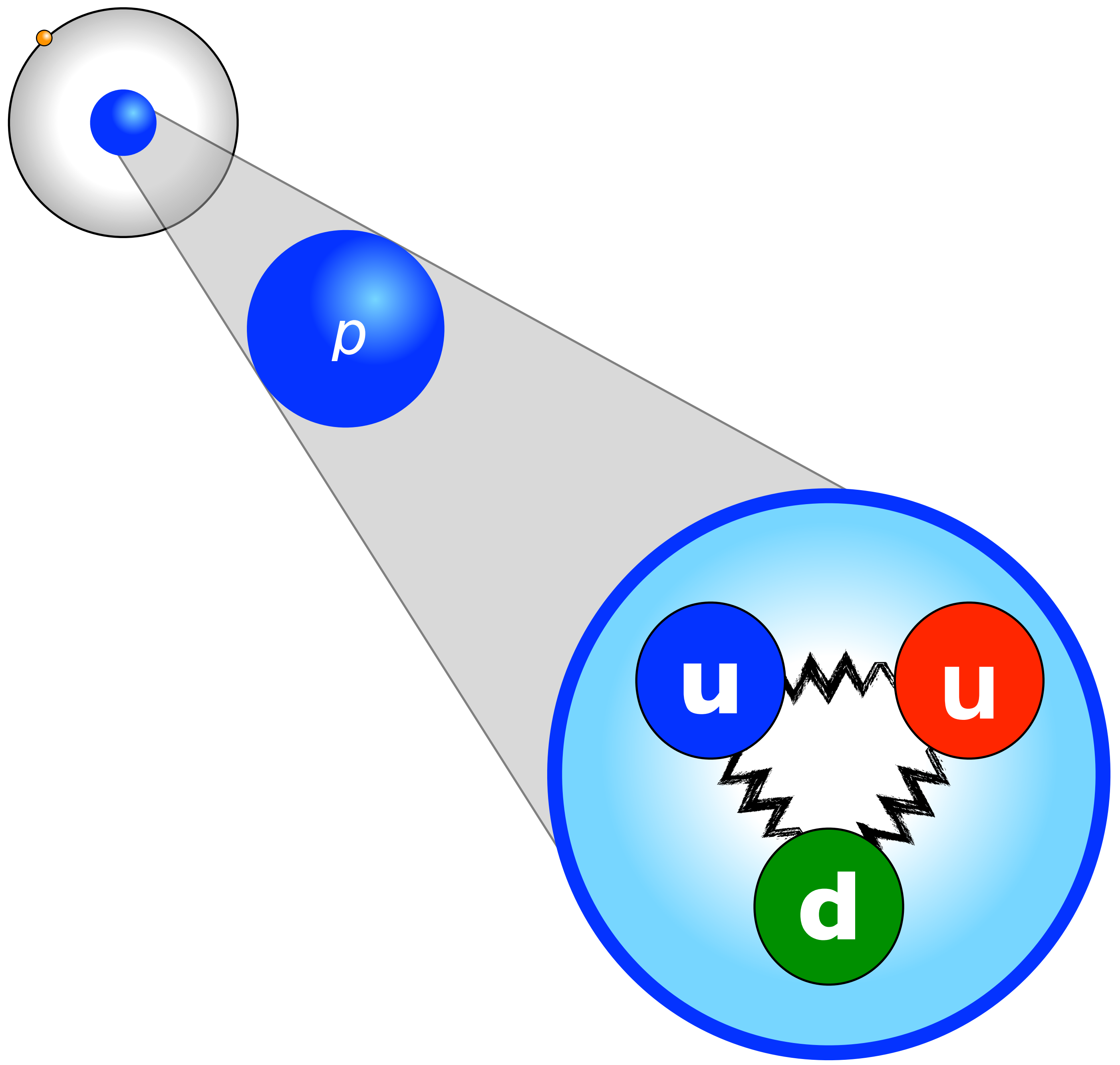




art of the talk **) low-energy

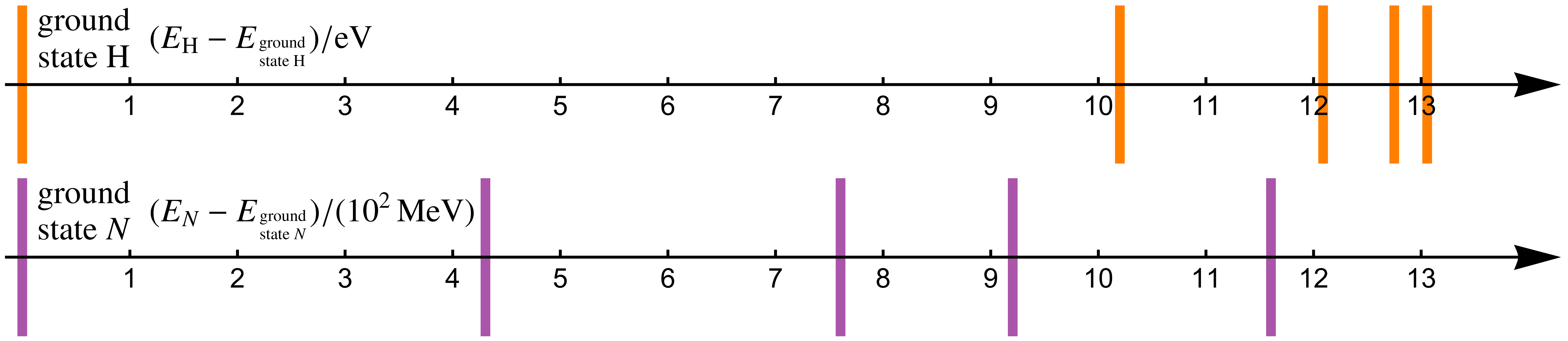
#inspirehep#PKU#2020-2025

BIG PICTURE



Protons/neutrons

- 99% of the mass of visible matter in the universe
- Building blocks: **quarks & gluons (strong force)**
- Part of a large class of particles: **hadrons**



Hydrogen spectrum **(✓)**

Proton spectrum **(?)**

HADRON SPECTRUM

Experimental progress

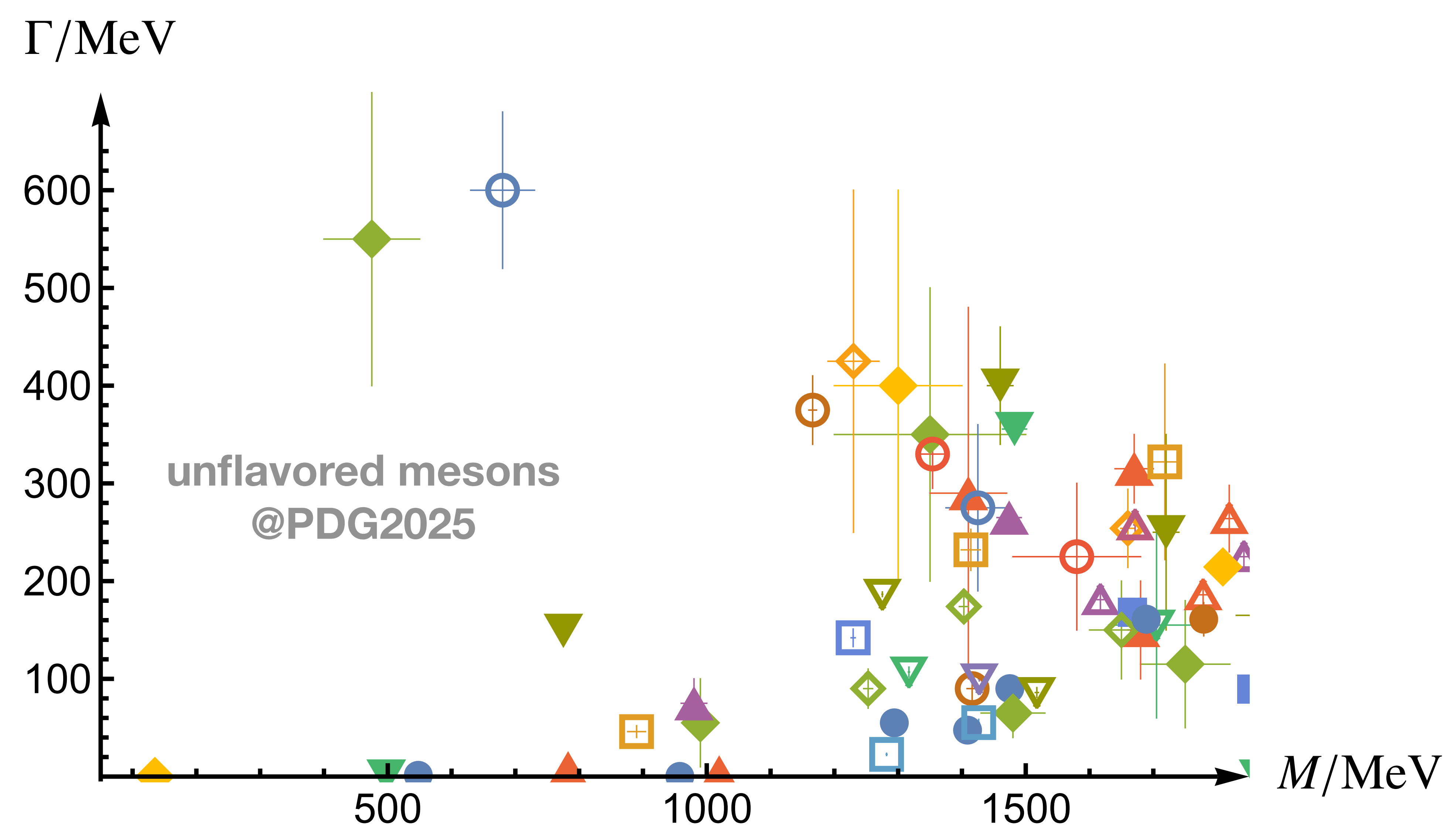
- 70y research ($\Delta(1232), \rho(770), \omega(782), \ldots$)
- ongoing progress, new techniques and experiments
- mostly excited states

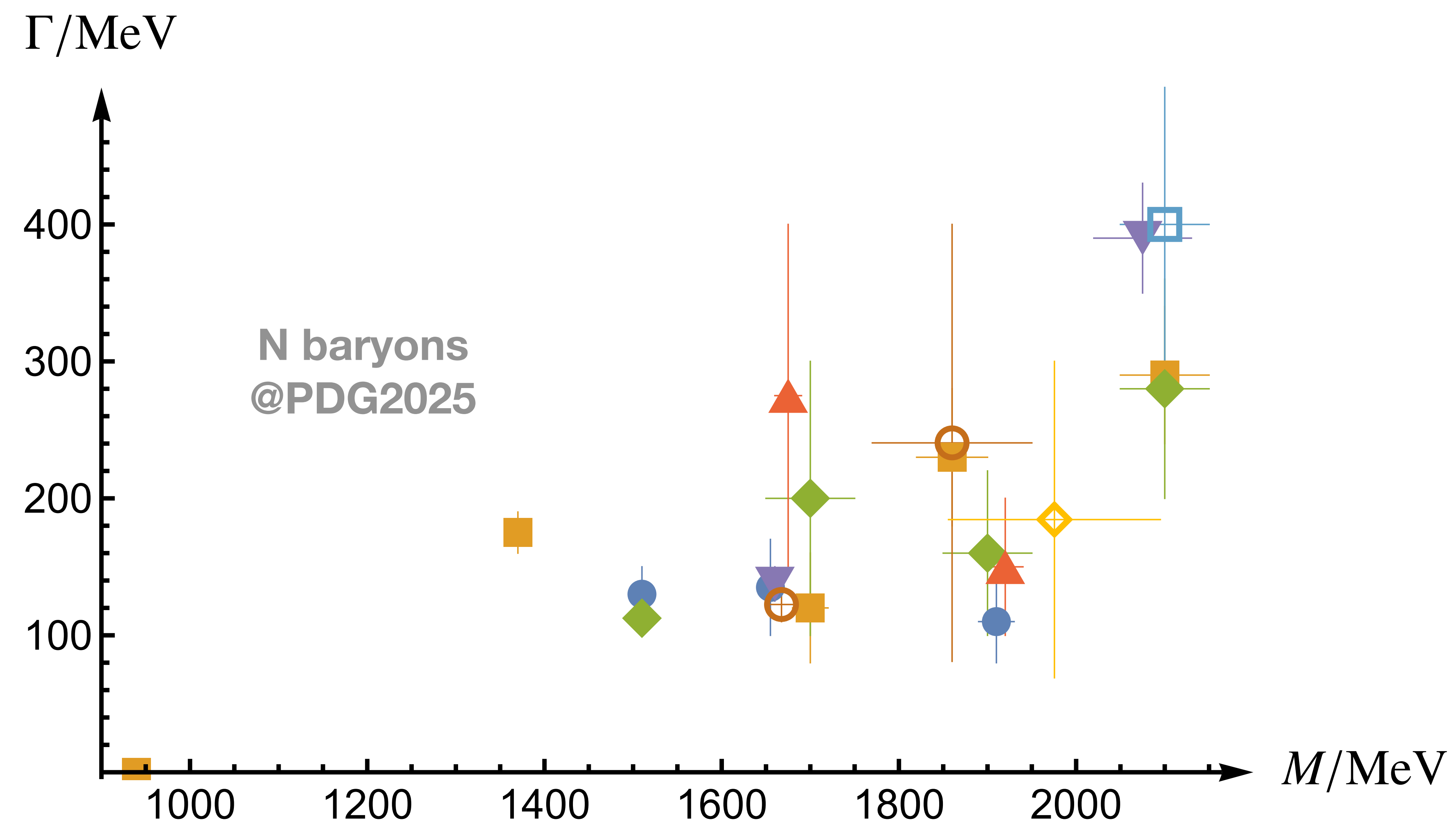
$\approx$ 100 mesons + 50 baryons (****)

Key questions

🦅 “what is the pattern of these states?“

🐸 “how are they formed?”





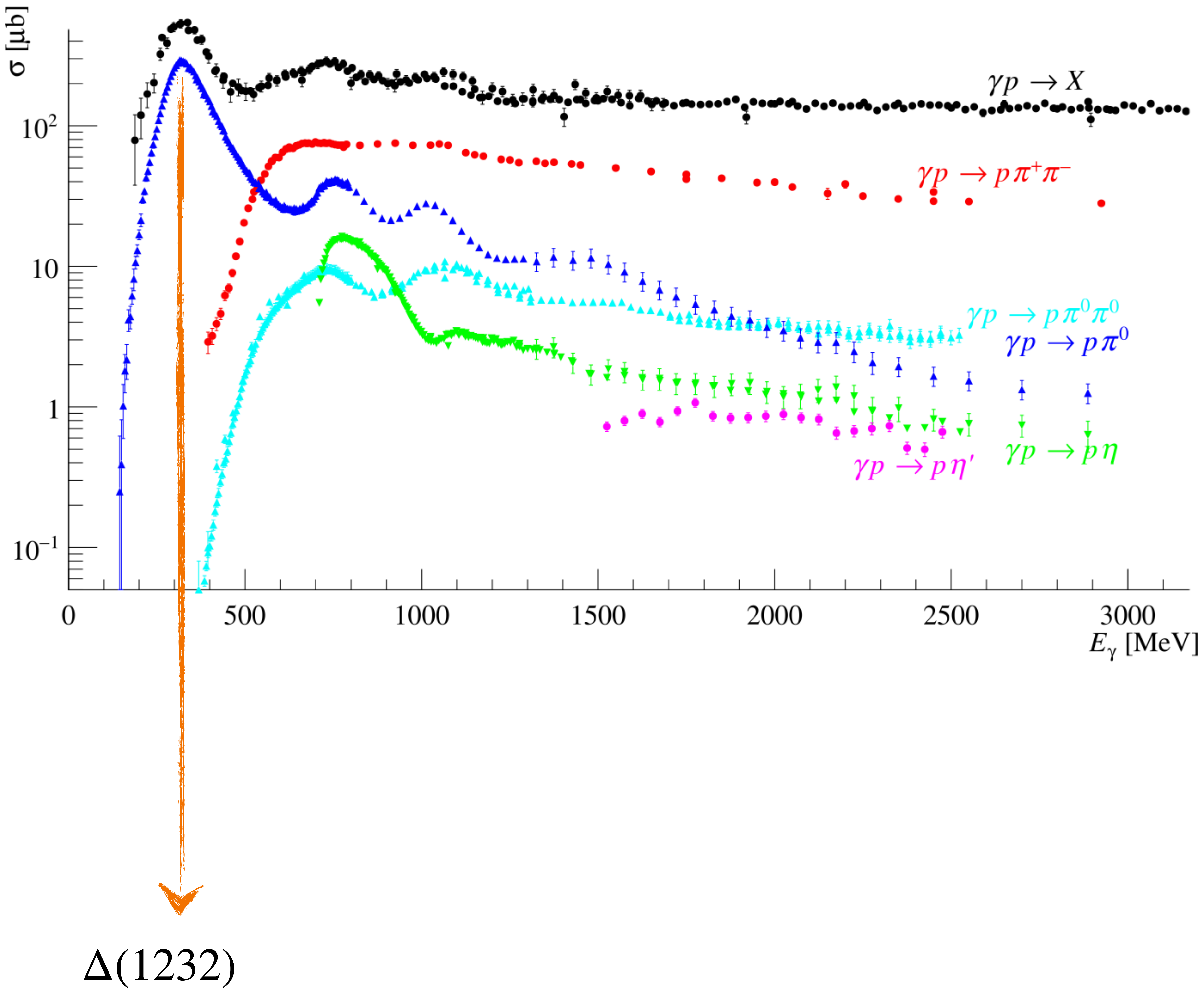
Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6

EXPERIMENTS / RESONANCES

Data: JLAB, ELSA, MAMI CLAS12, GlueX, …
Plot: Thiel, Afzal, Wunderlich, Light Baryon Spectroscopy, Prog. Part. Nucl. Phys. 125 (2022) 103949

Observations

- many available data and ongoing experiments
  BESII, GlueX, LHCb, CLAS…
- resonances:
  - increased interaction rates (bumps)



Anderson/Fermi/…PhysRev.85.934

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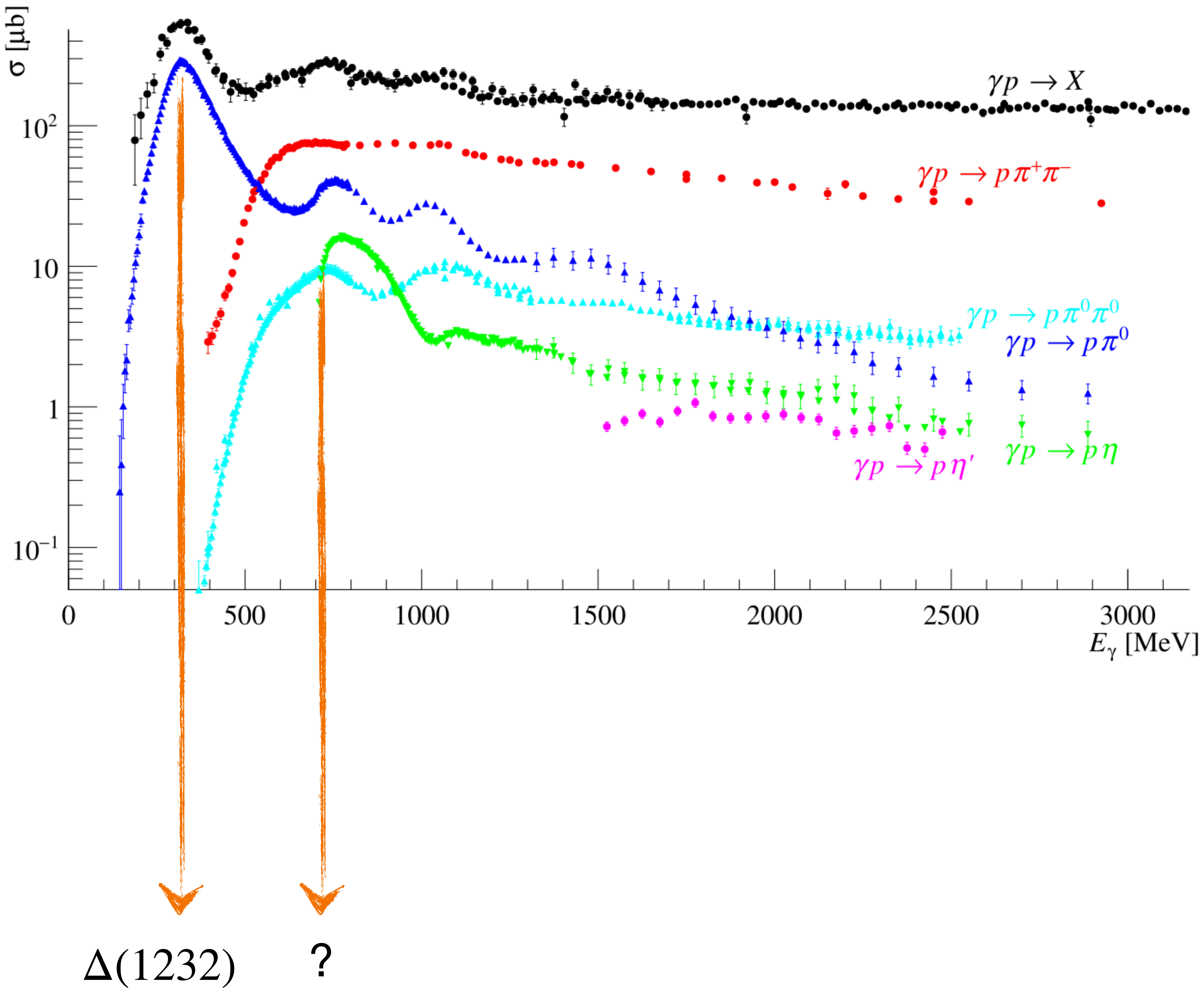

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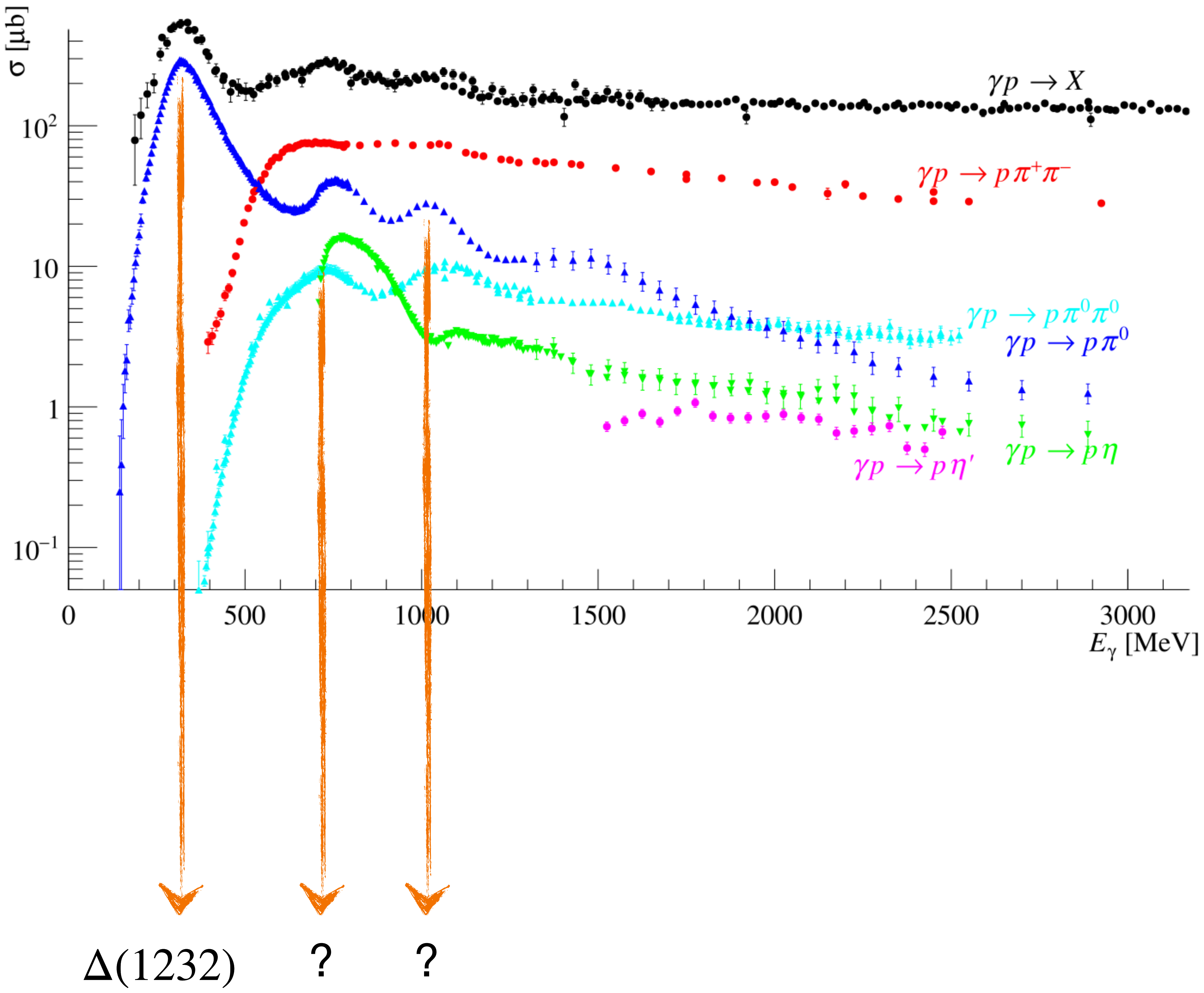

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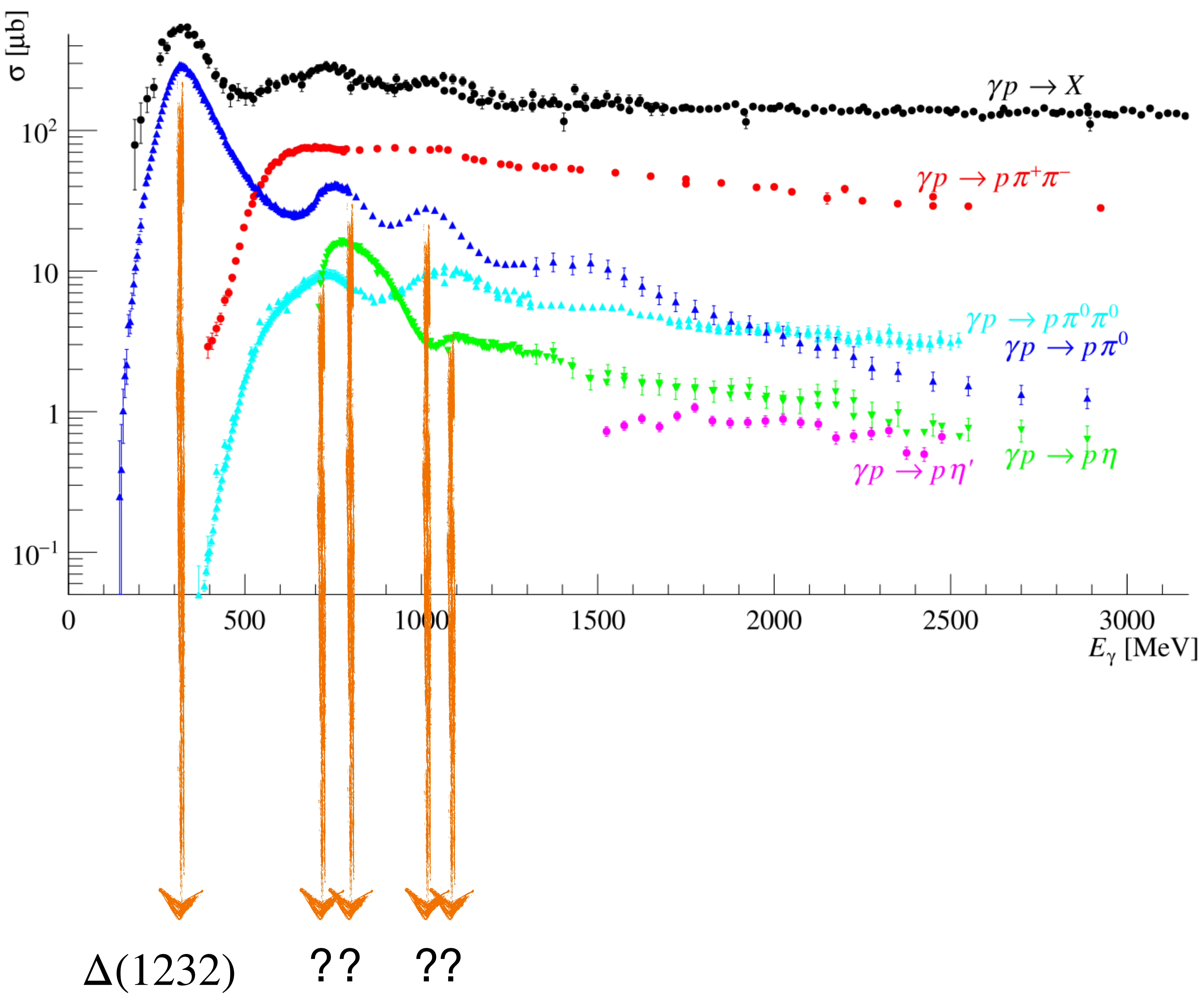

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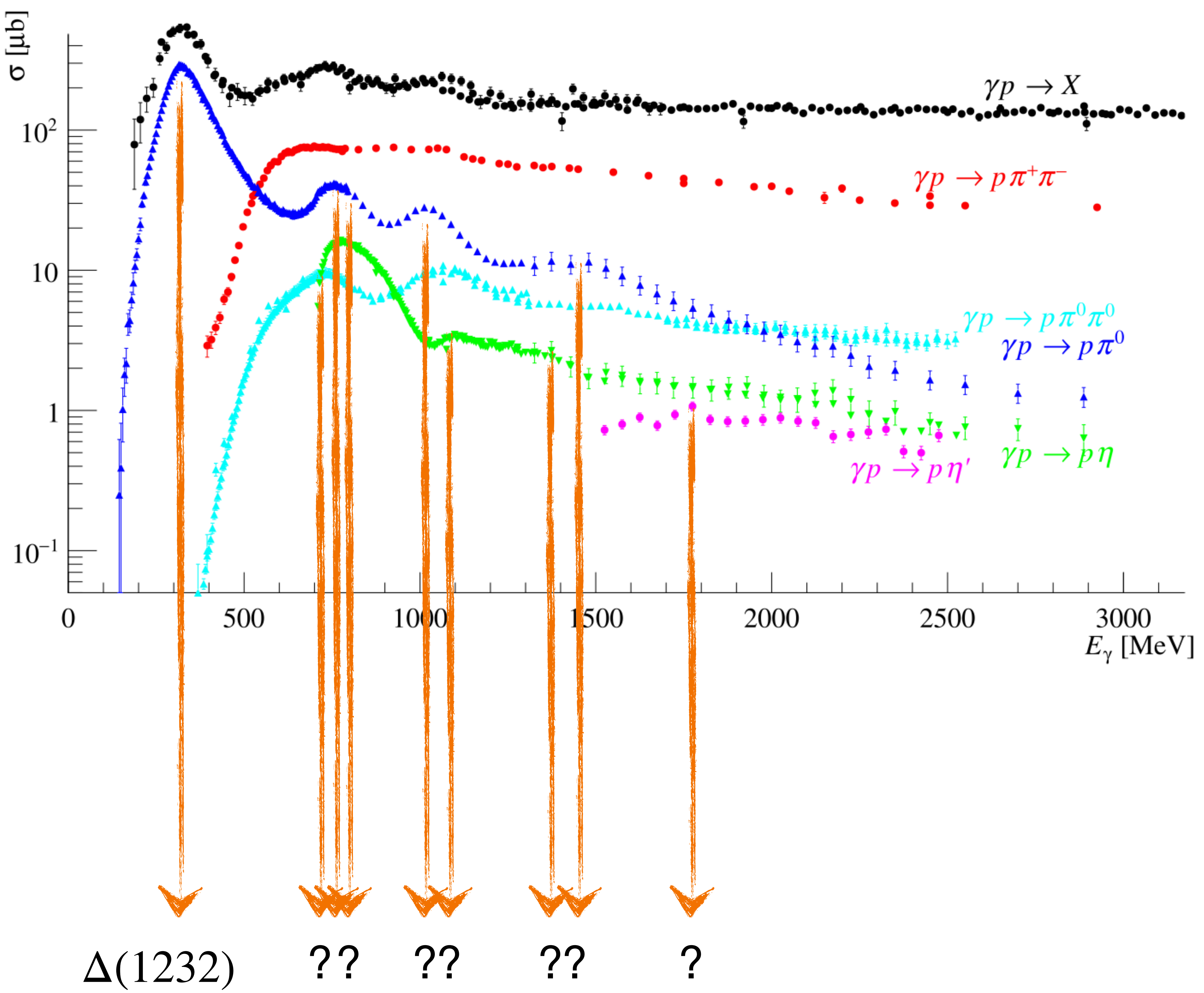

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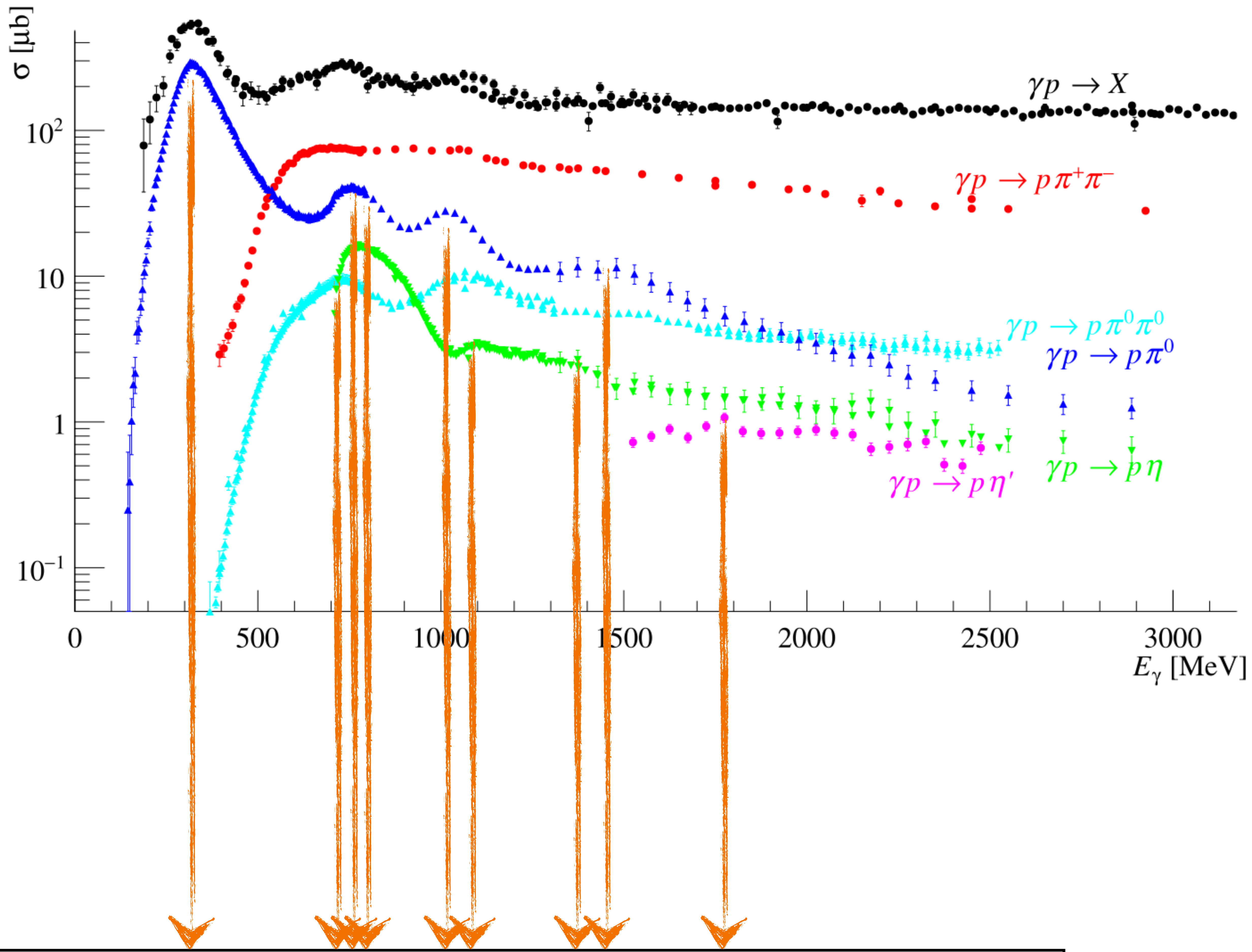


Observations

- many available data and ongoing experiments
  BESII, GlueX, LHCb, CLAS…
- resonances:
  - increased interaction rates (bumps)

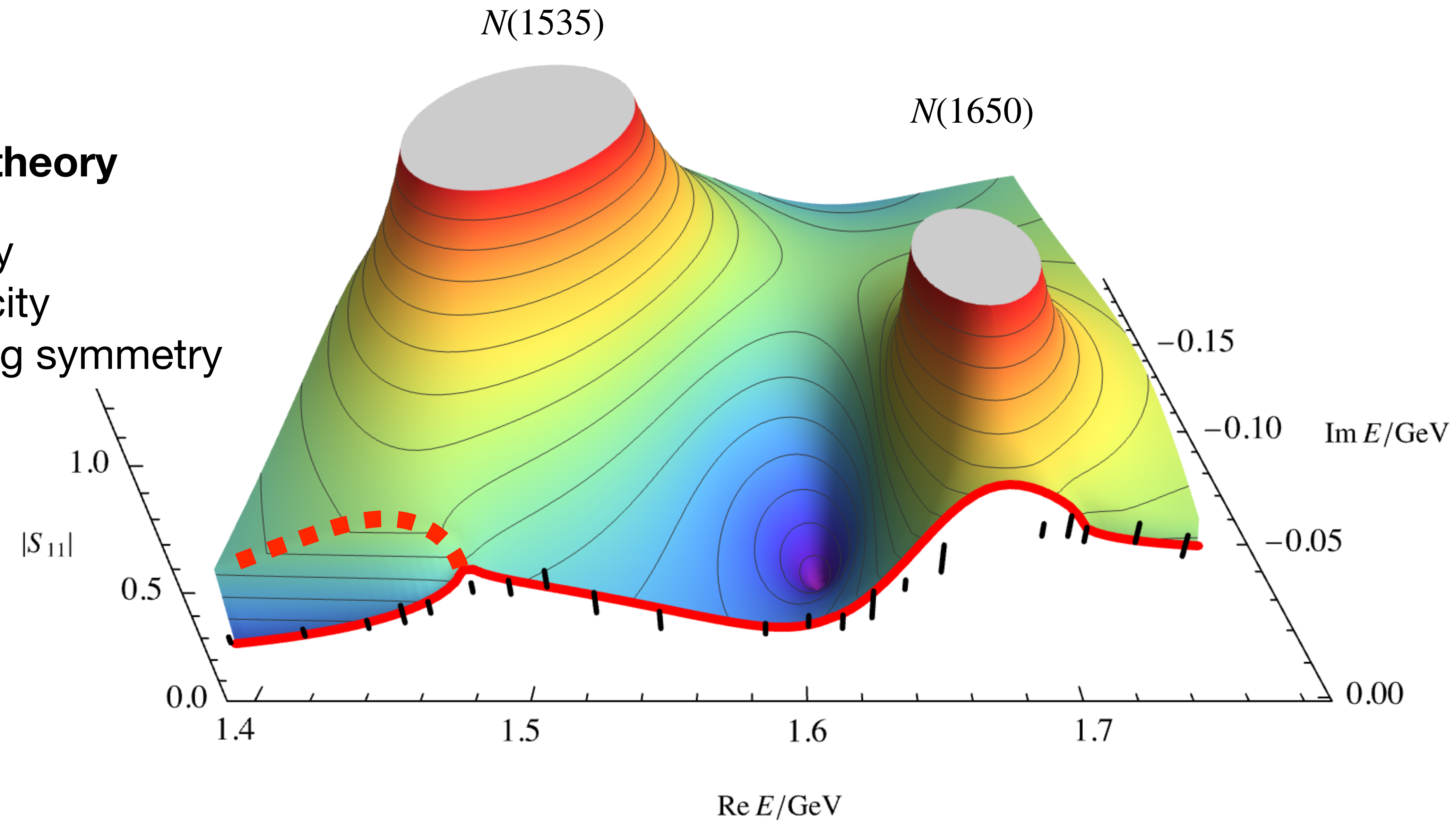
- reaction-type dependence
- overlapping resonances
- kinematical effects (cusps/triangle singularities/…)

P ~~D~~ B G=*"particle bump group"*

TRANSITION AMPLITUDES

S-matrix theory

- Unitarity
- Analyticity
- Crossing symmetry



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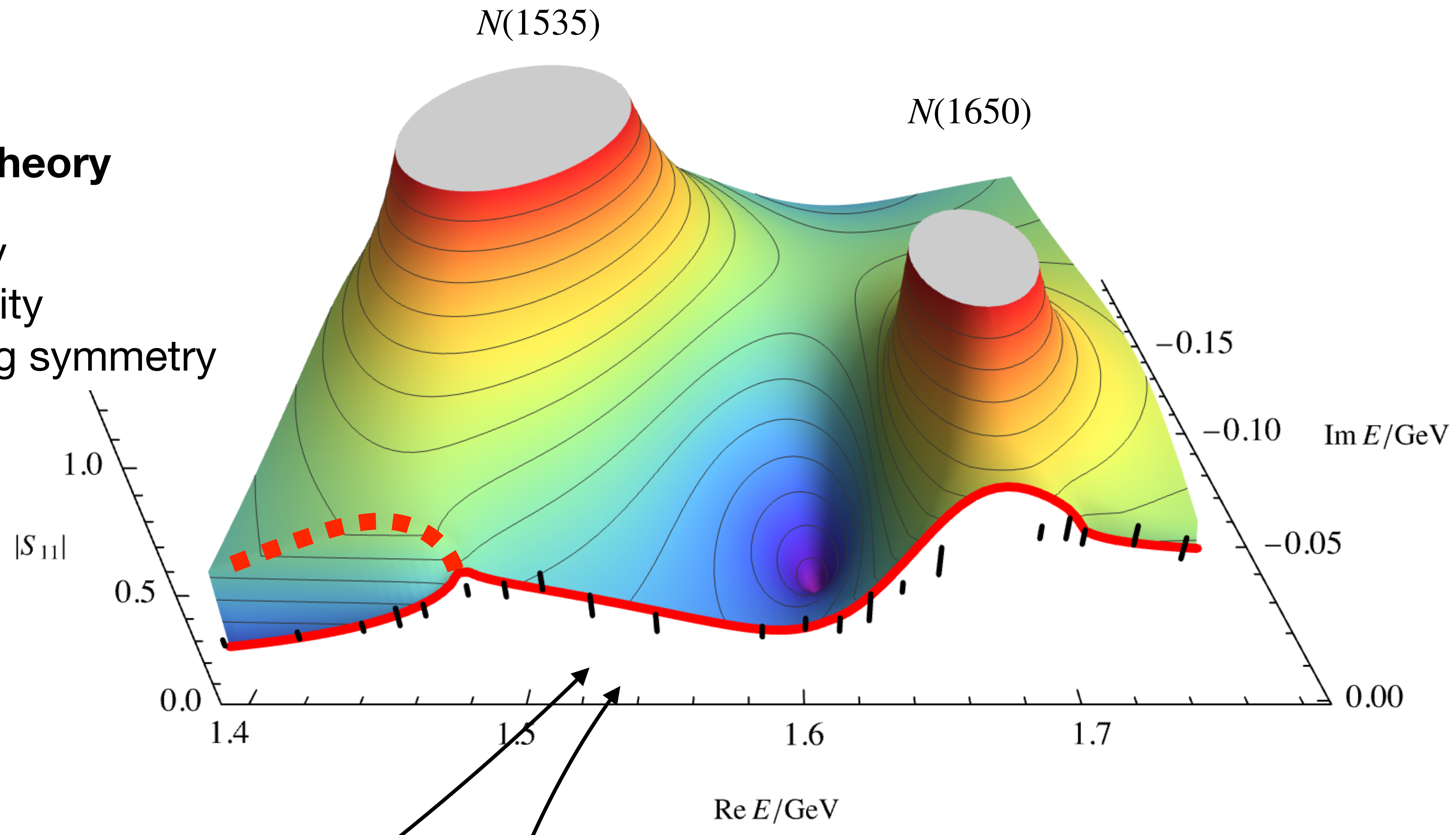
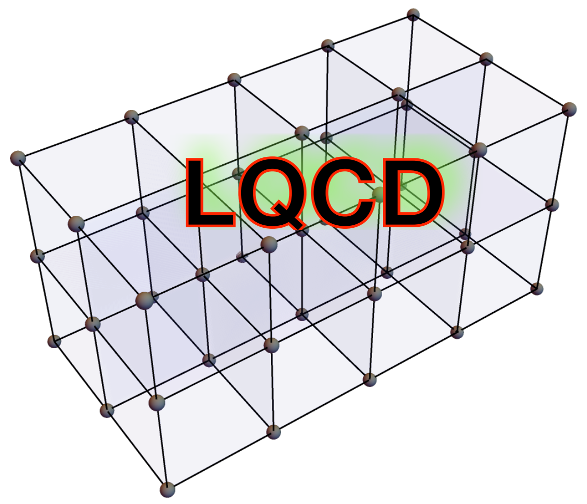
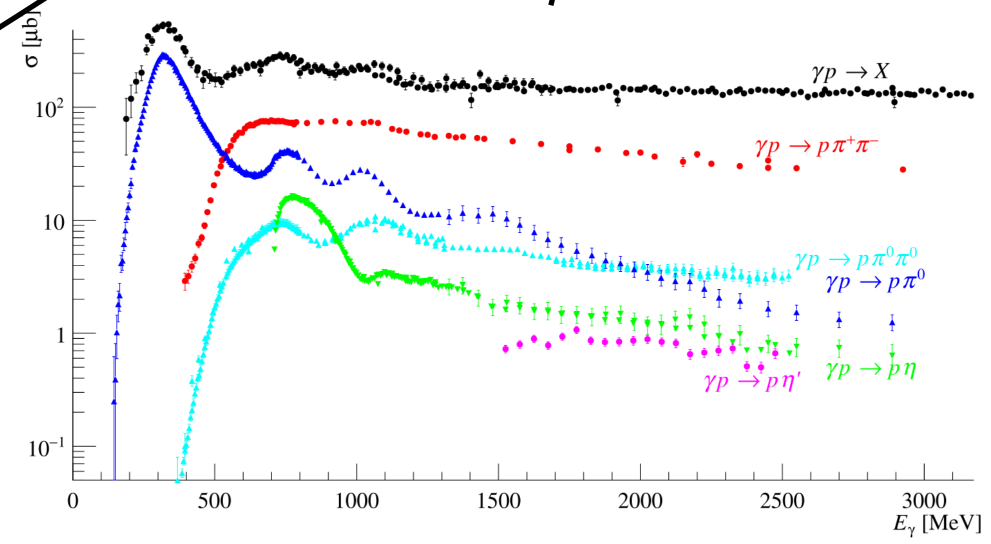


Boundary ($E \in \mathbb{R}$):

- Experiment
- Lattice QCD
- Effective Field Theories





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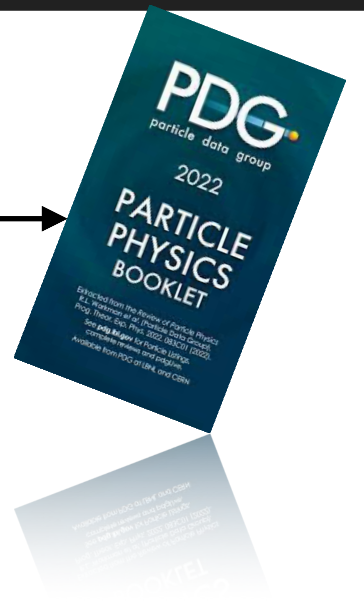


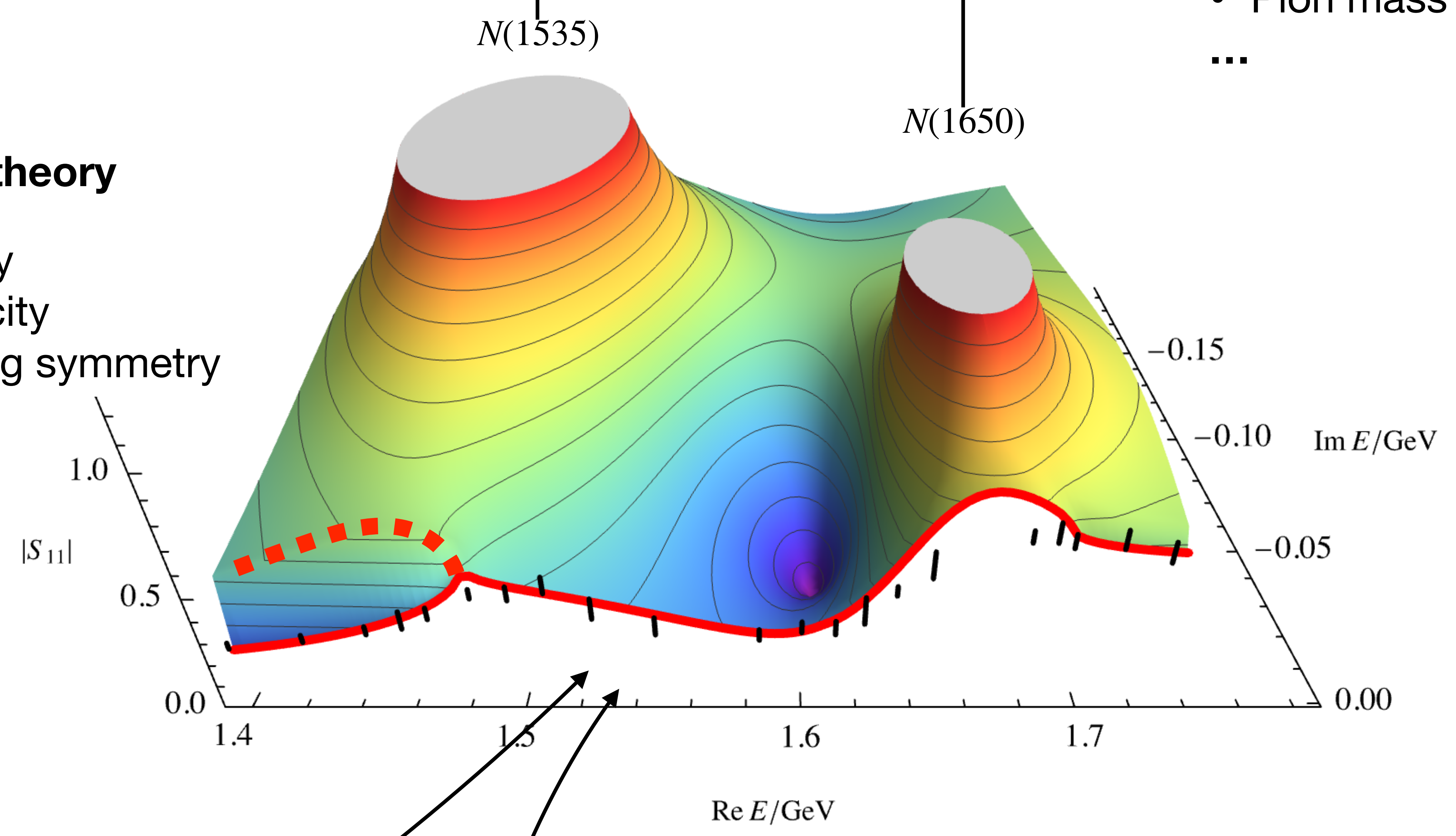
Poles on unphysical Riemann Sheets

- Universal resonance parameter
- Excludes triangle singularities
- Overlapping singularities
- Pion mass dependence

…

S-matrix theory

- Unitarity
- Analyticity
- Crossing symmetry



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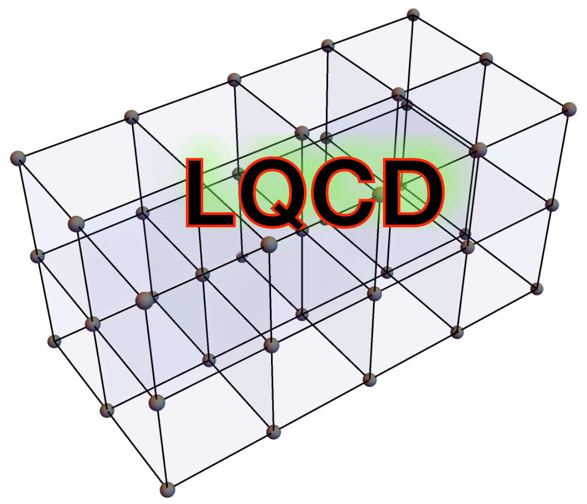

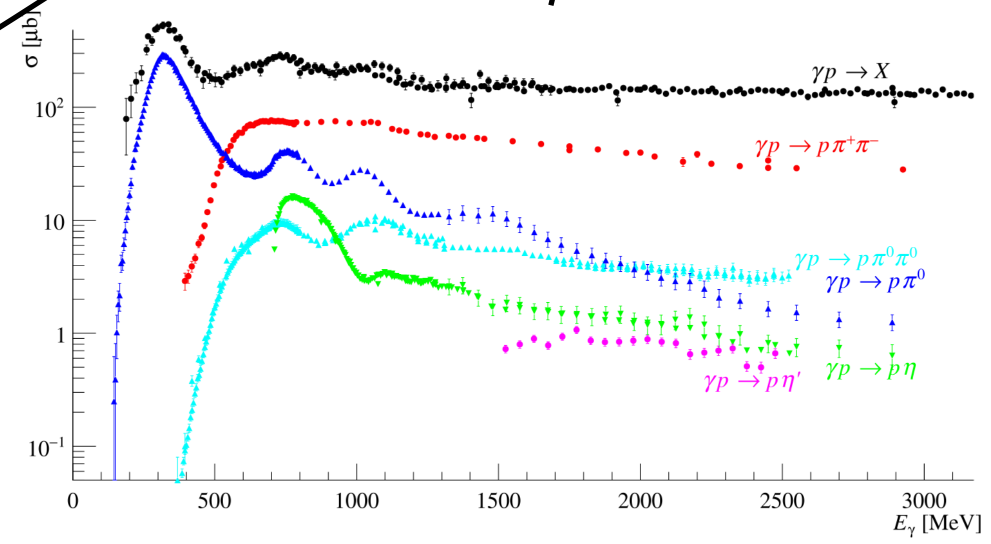

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OUTLINE

1. Motivation

    Observation, Theory, …

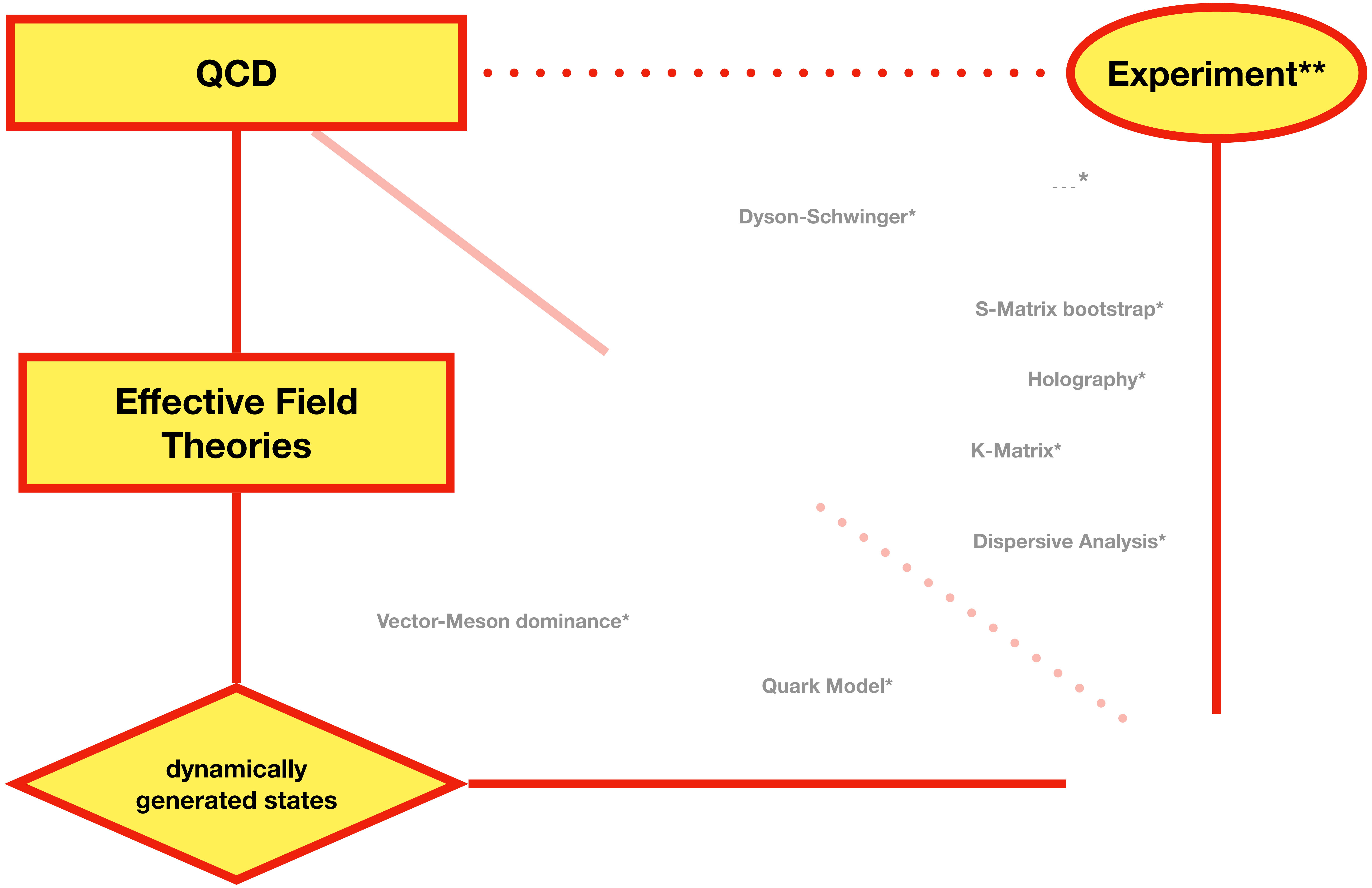
2. **Dynamically Generated Resonances**

    Methodology, Examples, $\Lambda(1405)$, …

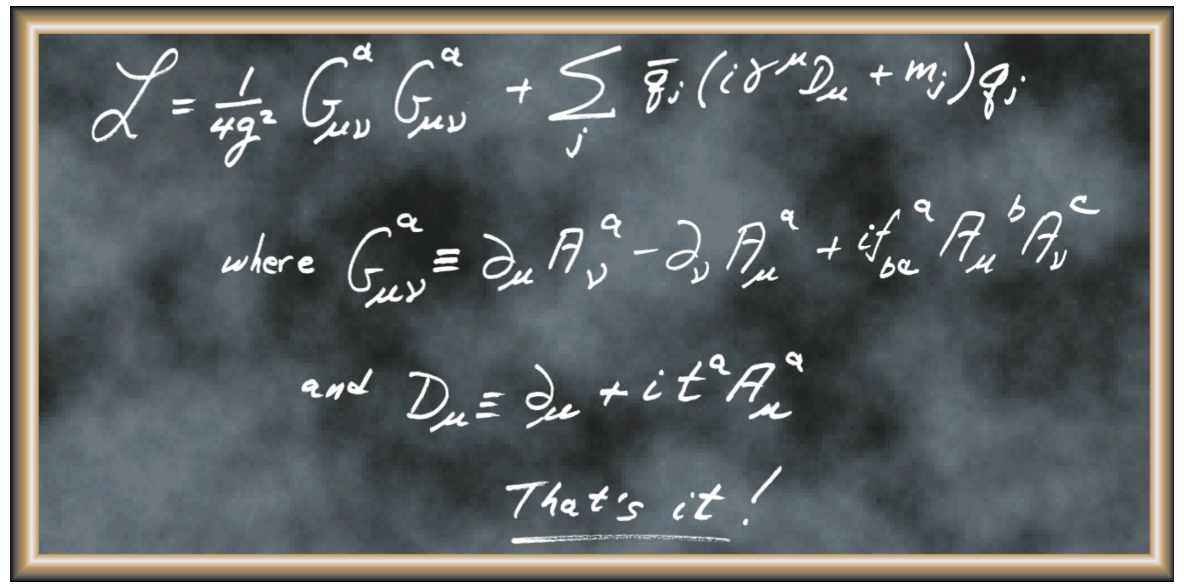
3. Applications to LQCD

    Chiral extrapolations,
    Quantization Conditions…

4. Summary/Outlook



THEORY



http://frankwilczek.com/Wilczek_Easy_Pieces/298_QCD_Made_Simple.pdf

Low-energy regime of QCD = double trouble

- small relative momenta
- non-perturbative energy regime
- need to evaluate <u>infinitely</u> many diagrams

Further approaches: Functional methods, holography, K-matrix, dynamical models, …

Review: Eichmann/Sanchis-Alepuz/Alkofer/Fischer Prog.Part.Nucl.Phys. 91 (2016) 1-100
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Review: Döring/Haidenbauer/Sato/MM PPNP in progress

THEORY

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

That's it!

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Low-energy regime of QCD = double trouble

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$$Z[J] = \int [DU] e^{\int i d^4x\, \mathscr{L}_{\mathrm{eff}}(U,v,a,s,p)}$$

Effective Field Theory (CHPT)

- Effective/Hadronic degrees of freedom
- Infinitely many low-energy constants
- Well-defined power counting
- Benchmark for many low-energy hadronic interactions

Weinberg (1979) Gasser, Leutwyler (1981), …
Reviews: Bernard, Meißner, Ann. Rev. Nucl. Part. Sci. 57, 33 (2007), Scherer, Adv. Nucl. Phys. 27, 277 (2003), …

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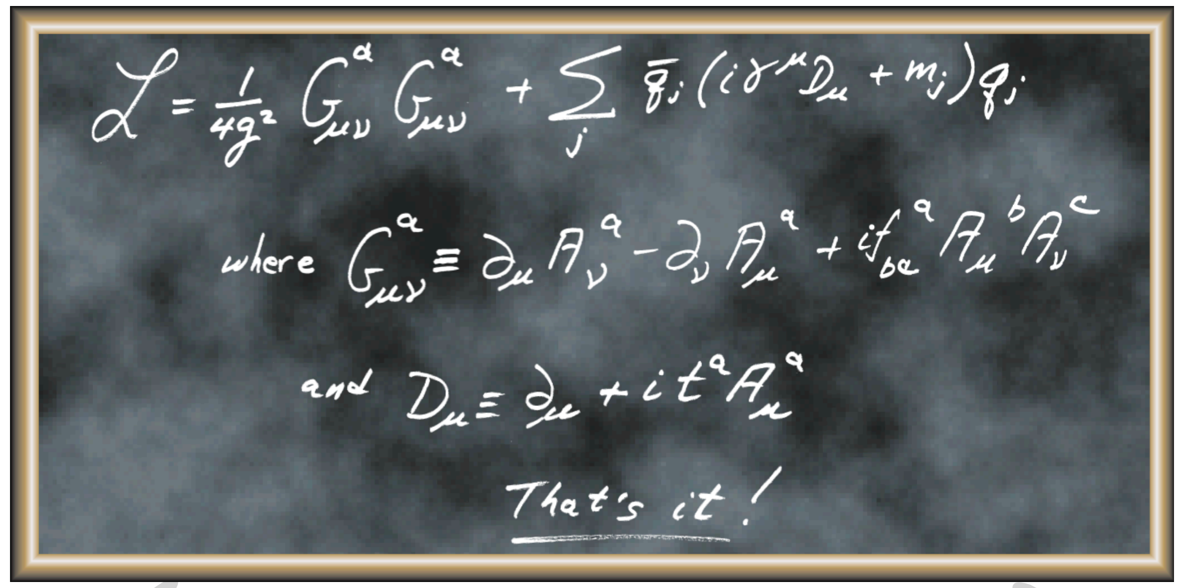

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$$Z[J] = \int [DU] e^{-S_E} \det[M[U]]$$

Lattice Gauge Theory

- QCD degrees of freedom
- discretized (Euclidean) space-time
- finite volume
- unphysical quark mass

Wilson, Phys. Rev. D10 (1974) 2445 , …
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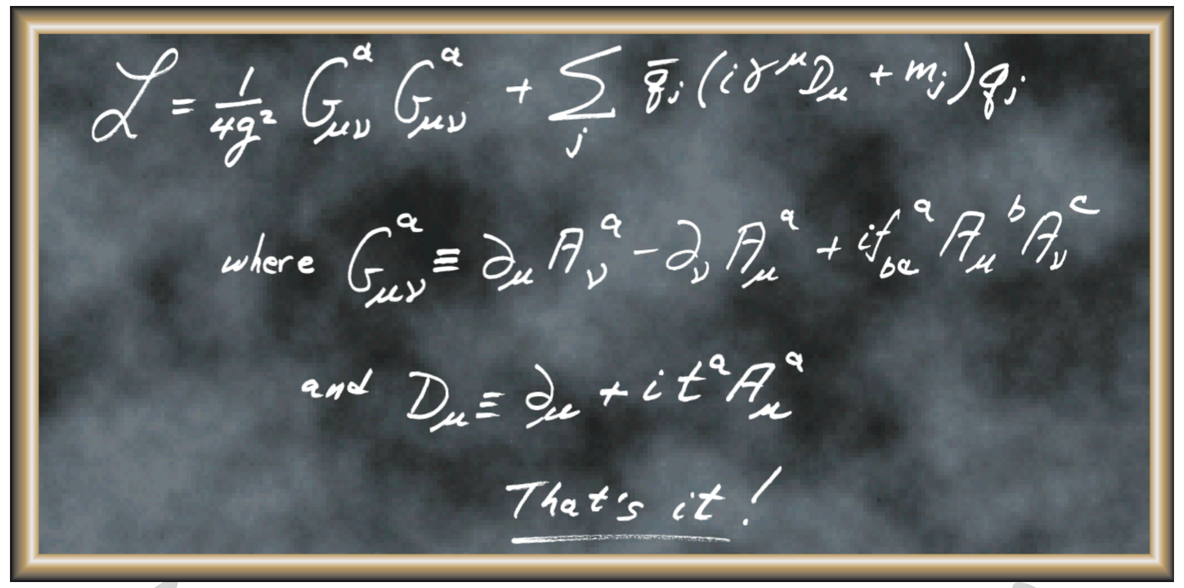

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quark mass dependence

first principle non-perturbative input

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EXAMPLE: BARYON CHPT

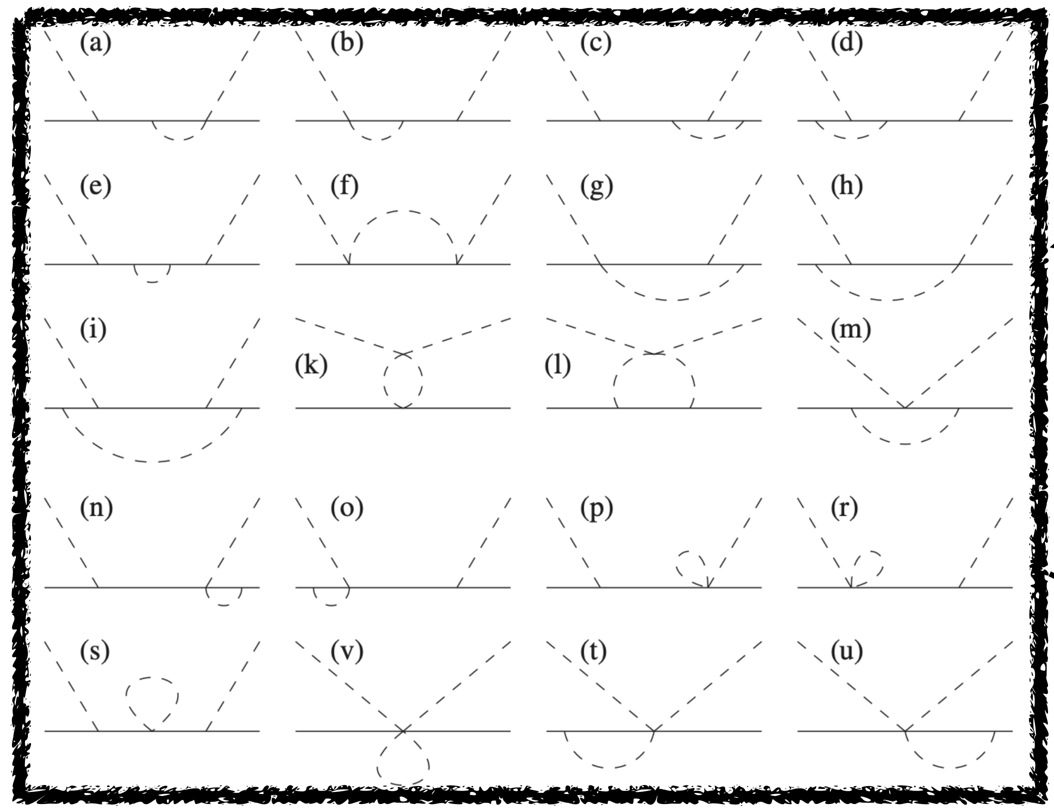
Meson-baryon scattering from CHPT

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

- full SU(3) dynamics near threshold
- agrees with experiment in many cases
- provides predictions for not measured channels

$$\begin{aligned}\mathcal{L}^{(2)}_{\phi B} = {} & b_{D/F}\langle \bar B[\chi_+, B]_\pm\rangle + b_0\langle\bar B B\rangle\langle\chi_+\rangle + b_{1/2}\langle\bar B[u_\mu,[u^\mu,B]_\mp]\rangle + b_3\langle\bar B\{u_\mu,\{u^\mu,B\}\}\rangle + b_4\langle\bar BB\rangle\langle u_\mu u^\mu\rangle \\ & + i\sigma^{\mu\nu}(b_{5/6}\langle\bar B[[u_\mu,u_\nu],B]_\mp\rangle + b_7\langle\bar B u_\mu\rangle\langle u_\nu B\rangle) + \frac{ib_{8/9}}{2m_0}(\langle\bar B\gamma^\mu[u_\mu,[u_\nu,[D^\nu,B]]_\mp]\rangle + \langle\bar B\gamma^\mu[D_\nu,[u^\nu,[u_\mu,B]]_\mp]\rangle) \\ & + \frac{ib_{10}}{2m_0}(\langle\bar B\gamma^\mu\{u_\mu,\{u_\nu,[D^\nu,B]\}\}\rangle + \langle\bar B\gamma^\mu[D_\nu,\{u^\nu,\{u_\mu,B\}\}]\rangle) + \frac{ib_{11}}{2m_0}(2\langle\bar B\gamma^\mu[D_\nu,B]\rangle\langle u_\mu u^\nu\rangle \\ & + \langle\bar B\gamma^\mu B\rangle\langle[D_\nu,u_\mu]u^\nu + u_\mu[D_\nu,u^\nu]\rangle), \end{aligned} \tag{8}$$

$$u_\mu = -i\frac{\partial_\mu\phi}{F} + \mathcal{O}(\phi^3)$$



Channel =	$\mathcal{O}(q^1)$	$+\mathcal{O}(q^2)$	$+\mathcal{O}(q^3)_{\rm HB}$	$\Sigma_{\rm HB}$
$a^{(3/2)}_{\pi N}$ =	-0.12	$+0.05^{+0.02}_{-0.03}$	$-0.06^{+0.00}_{+0.00}$	$-0.13^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi N}$ =	$+0.21$	$+0.05^{+0.02}_{-0.03}$	$+0.00^{+0.00}_{+0.00}$	$+0.26^{+0.03}_{-0.03}$
$a^{(3/2)}_{\pi\Xi}$ =	-0.12	$+0.04^{+0.03}_{-0.03}$	$-0.09^{+0.00}_{+0.00}$	$-0.17^{+0.03}_{-0.03}$
$a^{(1/2)}_{\pi\Xi}$ =	$+0.23$	$+0.04^{+0.03}_{-0.03}$	$-0.03^{+0.00}_{+0.00}$	$+0.23^{+0.03}_{-0.03}$
$a^{(2)}_{\pi\Sigma}$ =	-0.24	$+0.07^{+0.01}_{-0.01}$	$-0.07^{+0.00}_{+0.00}$	$-0.24^{+0.01}_{-0.01}$

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

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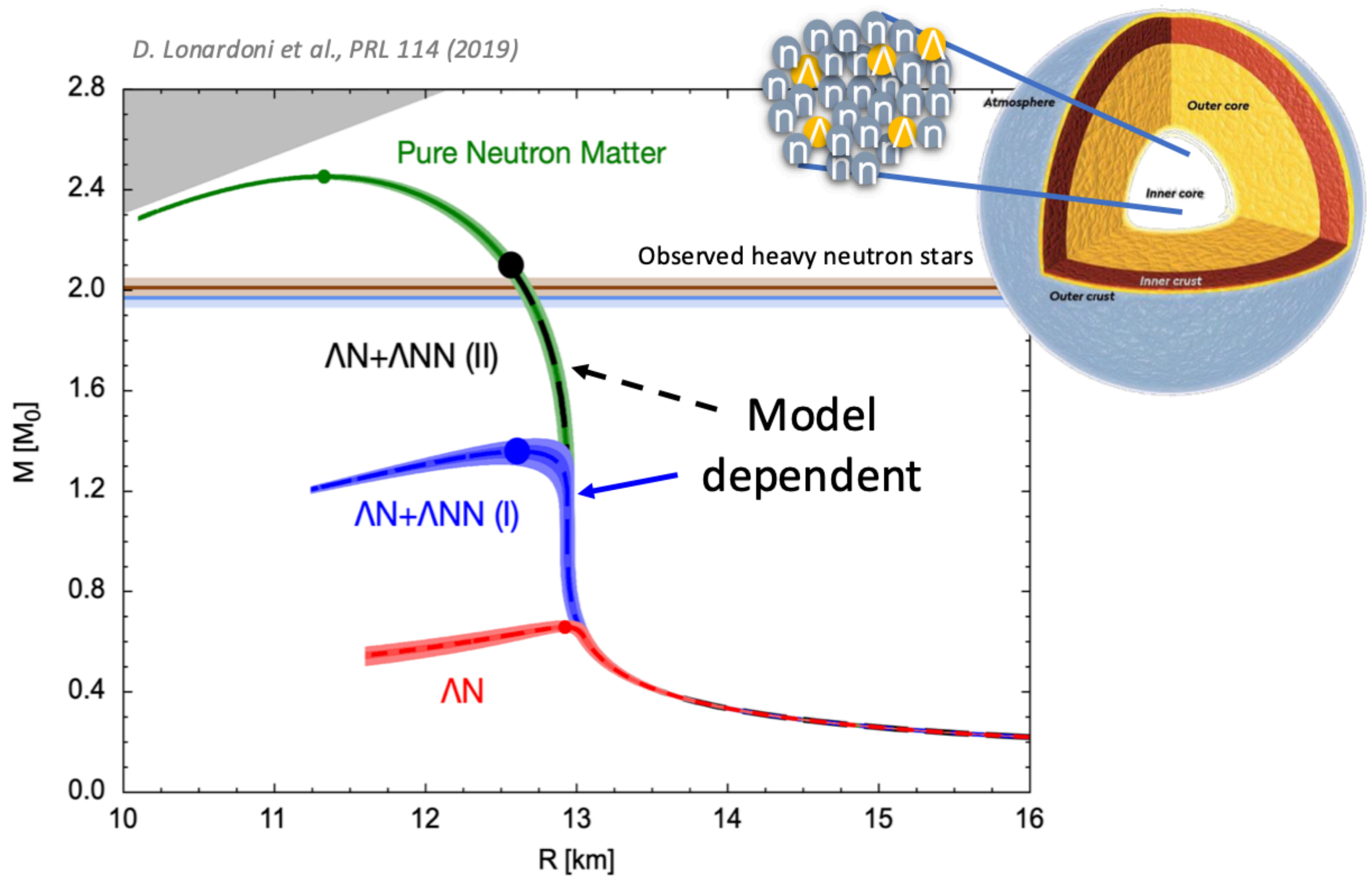
- **Fails for resonant (strangeness) channel**
  - Kaon mass is large → convergence
  - Relevant thresholds are widely separated → convergence
  - Resonance just below $\bar{K}N$ threshold → non-perturbative

$$a^{I=0}_{\bar{K}N} = \Big((+0.53)_{\rm LO} + (+0.97)_{\rm NLO} + (-0.40+0.22i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,,$$
$$a^{I=1}_{\bar{K}N} = \Big((+0.20)_{\rm LO} + (+0.22)_{\rm NLO} + (-0.26+0.18i)_{\rm NNLO} + \ldots\Big)\ \text{fm}\,.$$

MM/P.C.Bruns/Ulf-G. Meißner/B.Kubis Phys.Rev.D 80 (2009) 094006

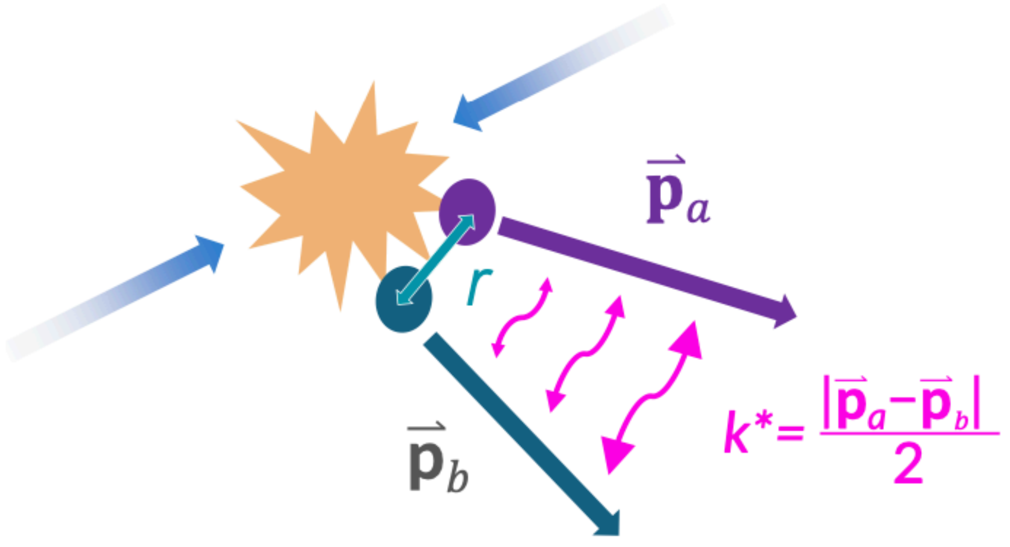
$\bar{K}N$ INTERACTION

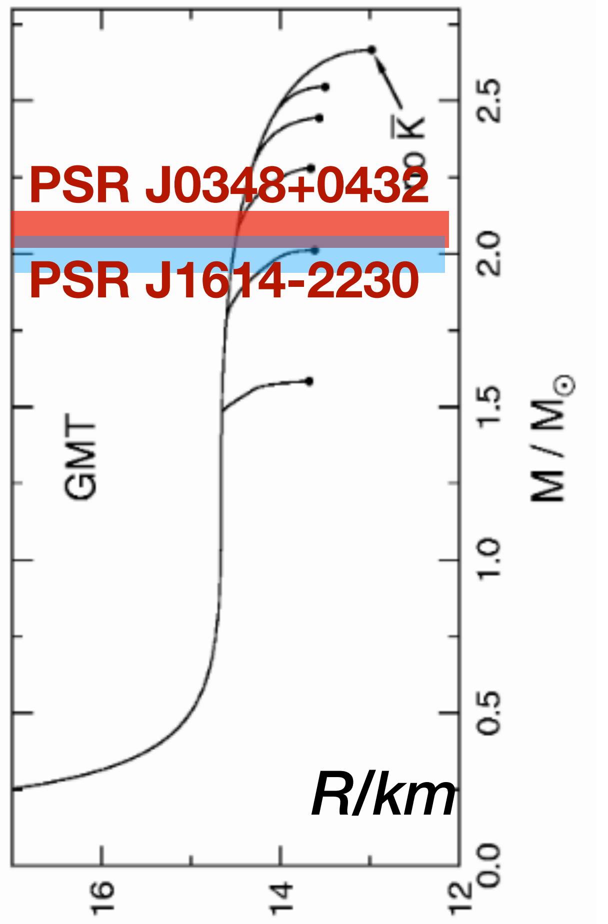


Overarching impact

- Test of our understanding of QCD
  Modern/Upcoming experiments: CLAS12, Klong, SIS100
- Kaonic hydrogen/deuterium energy shift
  DAPHNE/DEAR…
- $\bar{K}NN$ & $\bar{K}NNN$ bound states (JPARC/…)
  Review: Gal/Hungerford/Millener (2016); Iwasaki et al. Phys.Rev.C 110 (2024) 1, 014002, …
- K^- in medium
  Mareš et al. Acta Phys. Polon. B 51, 129 (2020), Hrtánkova et al. Phys.Lett. B 785, 90 (2018), …
  Cassing/Tolos/Bratkovskaya/Ramos *Nucl.Phys.A* 727 (2003) 59-94

  >> K^--condensate in NS >> Equation of State





Pal/Greiner…,
Nucl. Phys. A 674, 553 (2000)

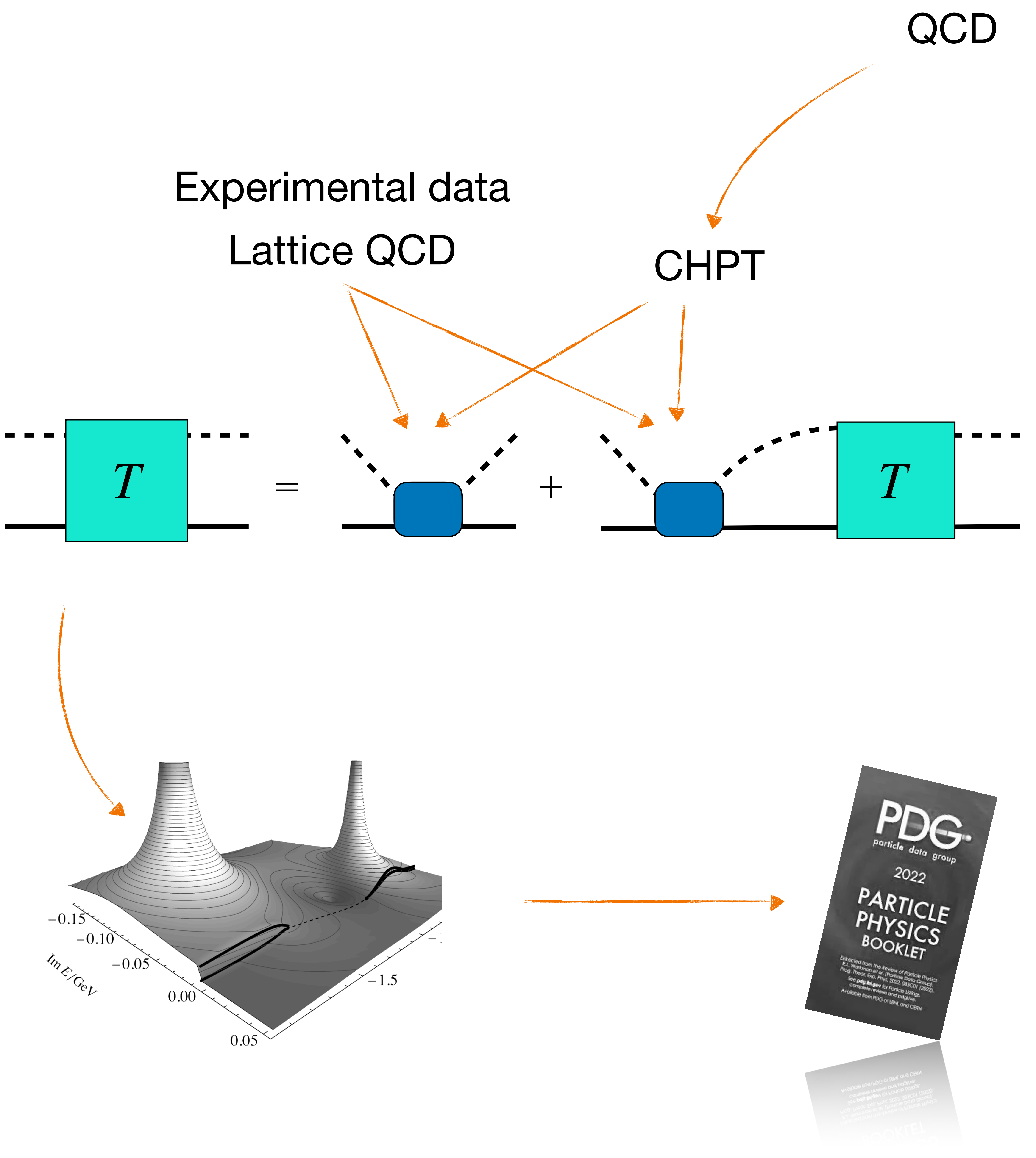
- Femtoscopy/Correlations
  Michael Annan Lisa et al, Ann.Rev.Nucl.Part.Sci. 55 (2005) 357-402, L. Fabbietti et al., ARNPS 71 (2021), 377-402

UNITARISATION

Extension to resonant channels/higher energies — Chiral Unitary Approach

- **Good**
  - Non-perturbative scheme
  - Record complex pole-positions (*II Riemann Sheet*)
  - Often works: $N(1535), N(1650), \Lambda(1405), \Lambda(1380), \ldots$

Kaiser/Siegel/Weise Phys.Lett.B 362 (1995)
Lutz/Soyeur Nucl.Phys.A 773 (2006); MM et al. Phys.Lett.B 697 (2011); …



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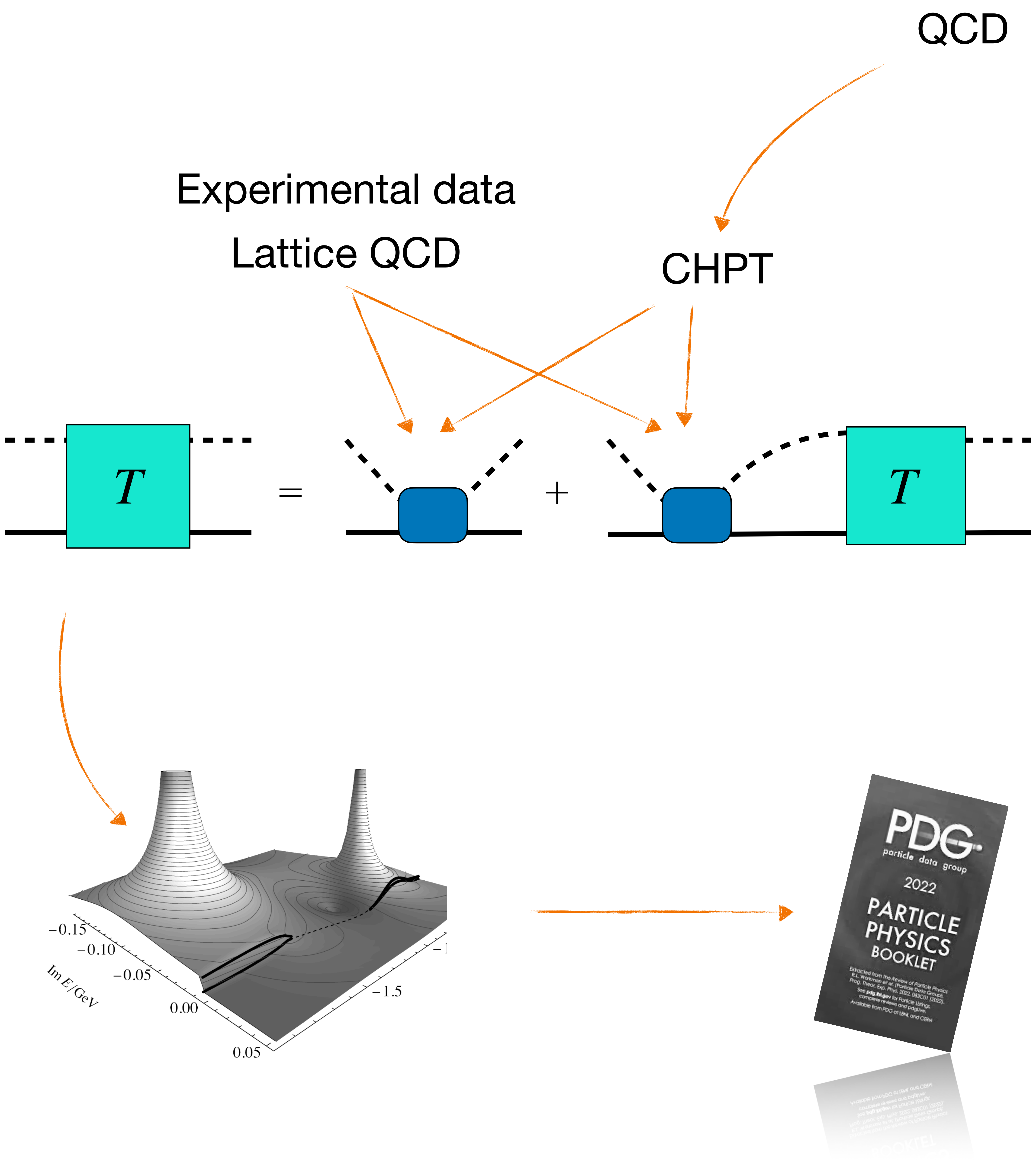
Attention (model dependence)

Review: MM, Eur.Phys.J.ST 230 (2021) 6, 1593-1607

- Renormalisation
- Crossing symmetry
- Power counting
- Choice of the interaction kernel

only perturbatively



STATUS: $\Lambda(1405)\ldots\Lambda(1380)$

"A curious case of a strangeness resonance" *

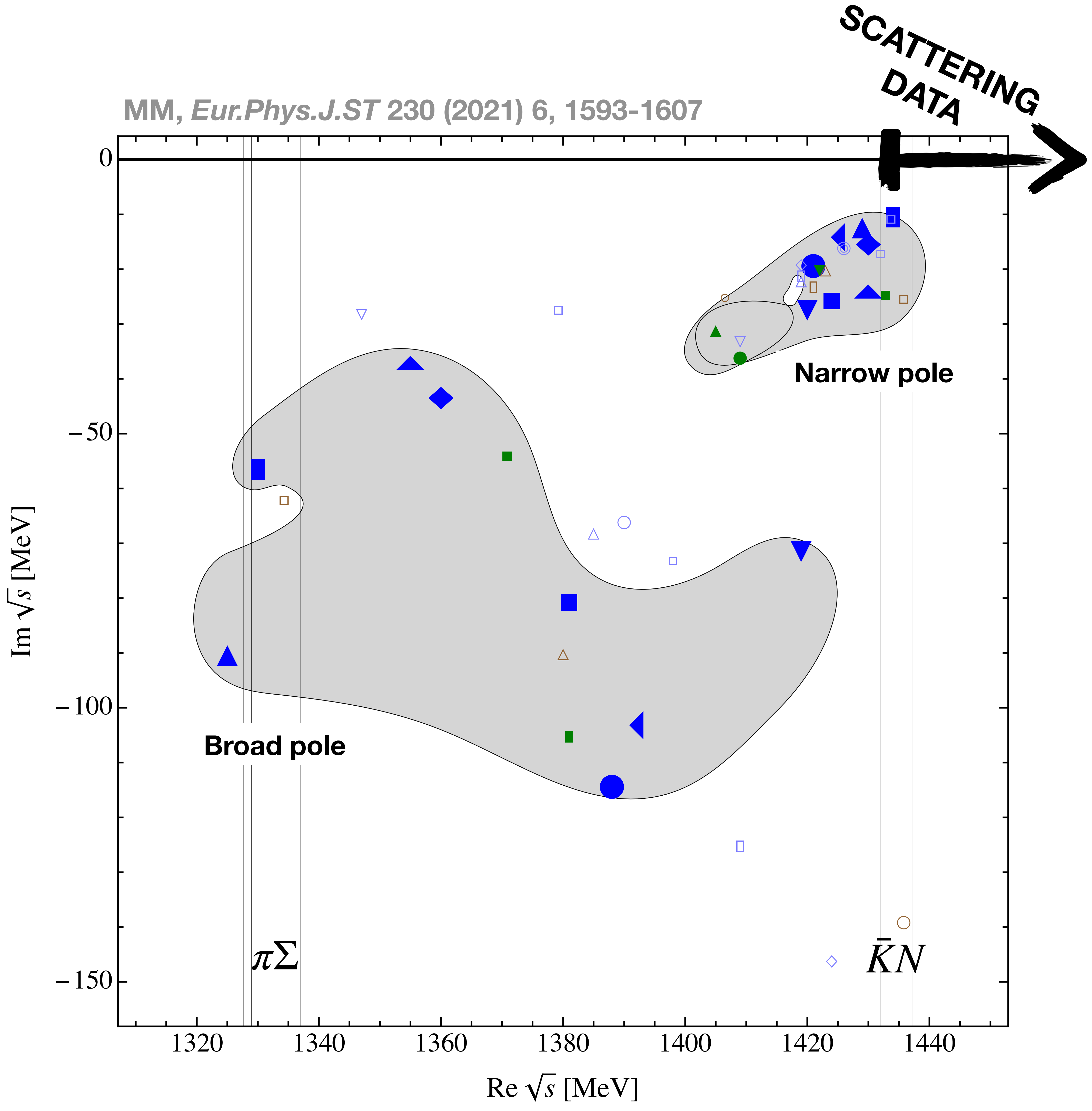
MM, *Eur.Phys.J.ST* 230 (2021) 6, 1593-1607

- Sub-($\bar{K}N$)-threshold $\Lambda(1405)$ resonance
- second state $\Lambda(1380)$ predicted from UCHPT
  - no direct experimental verification
    - indirectly through photoproduction experiments

      [CLAS] Moriya et al. Phys.Rev.Lett. 112 (2014) 8
      MM/Meißner *Eur.Phys.J.A* 51 (2015) 3, 30
    - confirmed by many critical tests & LQCD

      Bulava et al. [BaSc] *Phys.Rev.Lett.* 132 (2024) 5, 051901

MM, *Eur.Phys.J.ST* 230 (2021) 6, 1593-1607



Models:
Ikeda/Weise/Feijoo/MM/Meißner/Ramos/Hyodo/…

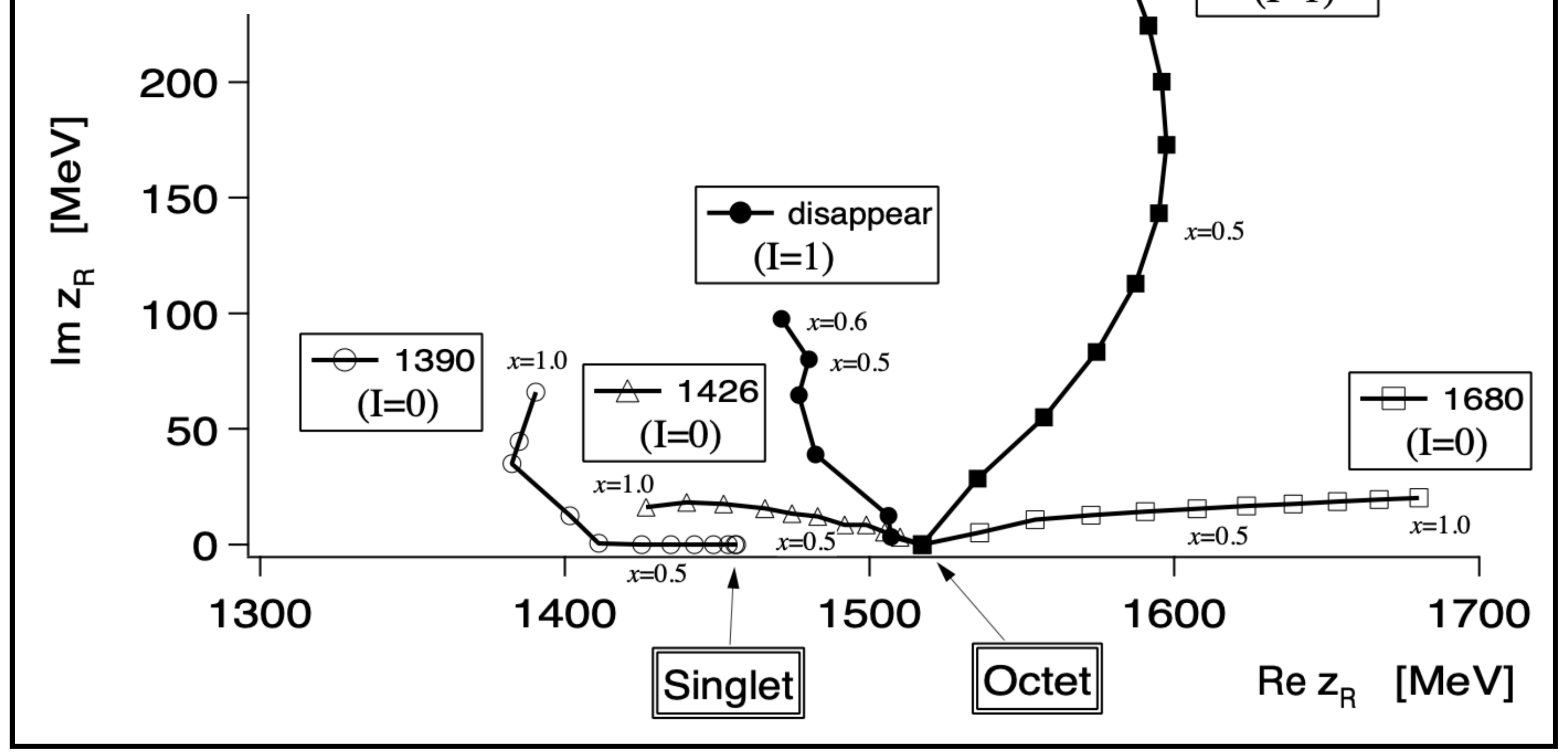
QUARK MASS DEPENDENCE

CHPT encodes quark mass dependence

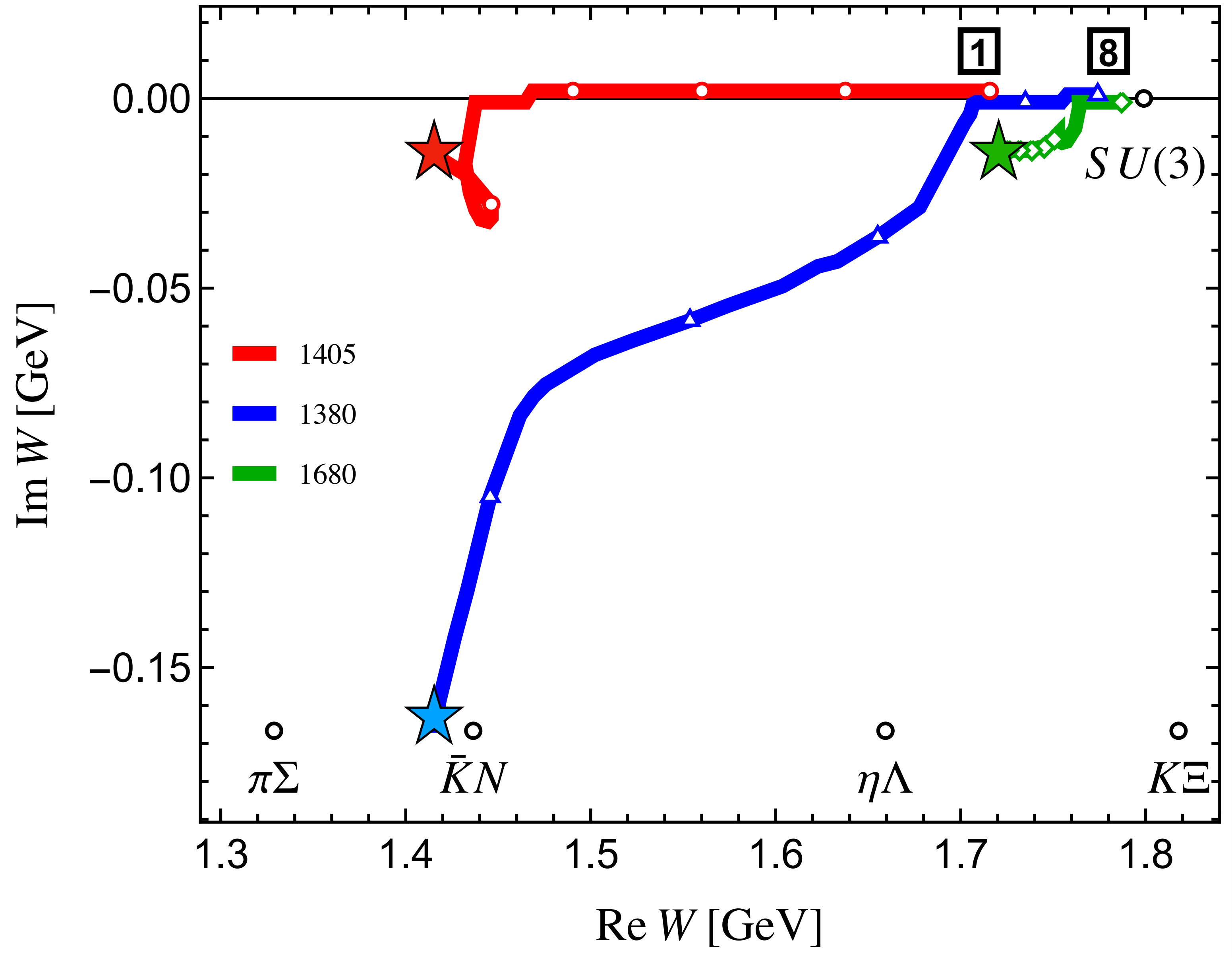
- SU(3) limit provides a simpler resonance structure
  Jido et al. Nucl.Phys.A 725 (2003); Garcia-Recio/Lutz/Nieves Phys.Lett.B 582 (2004) 49-54;
  - 1 singlet + 2 octet poles
  - LO/NLO UCHPT *pole-"tracks"* differ
    Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)
    Zhuang/Molina/Geng/Lu 2405.07686
- Resonance ⇜⇝ virtual bound state ⇜⇝ bound state

(?) Lattice QCD (later…)

LO UCHPT Jido et al. Nucl.Phys.A 725 (2003)



NLO UCHPT Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)



OUTLINE

1. Motivation

    Observation,Theory, …

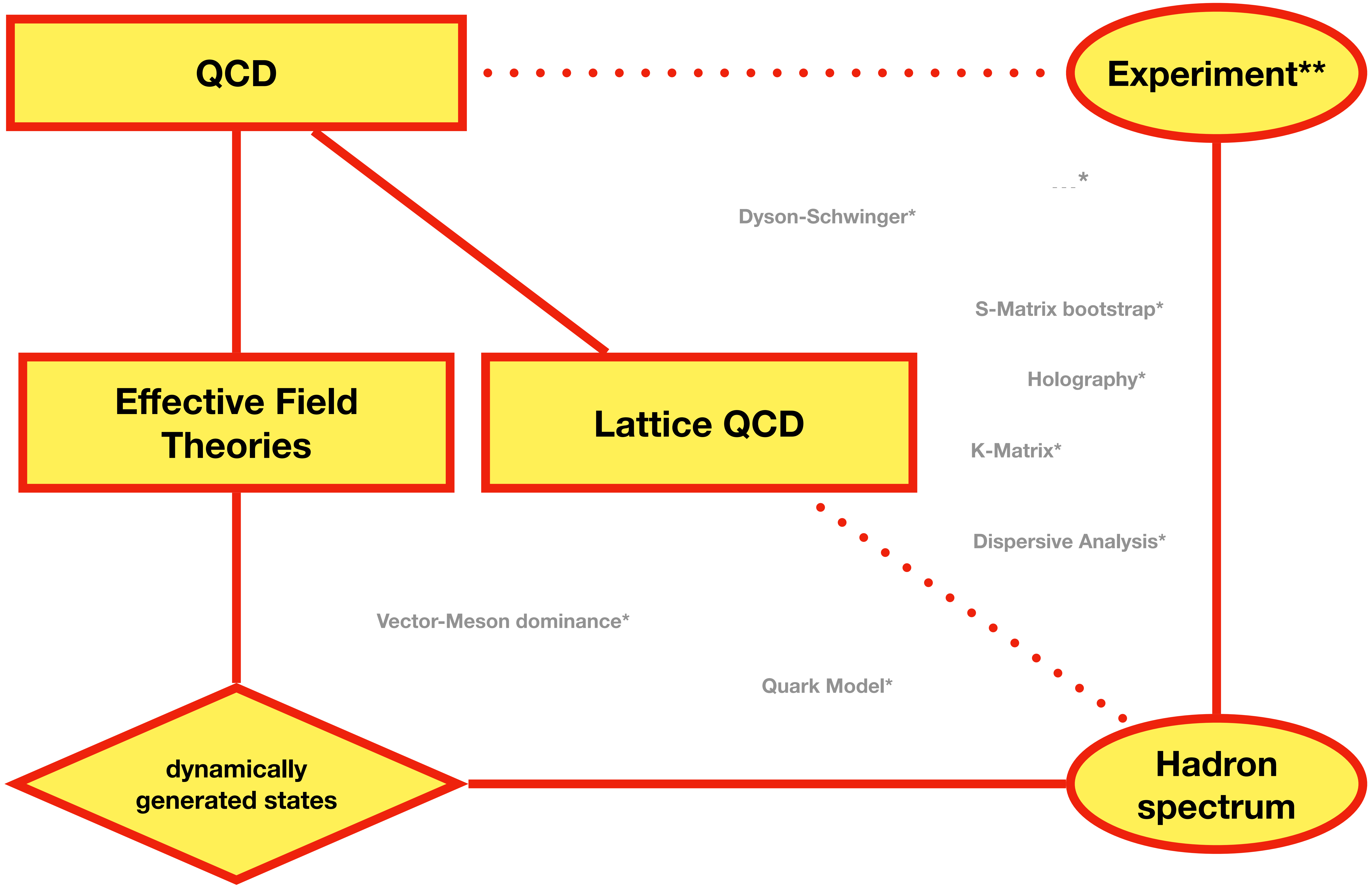
2. Dynamically Generated Resonances

    Methodology, Examples, $\Lambda(1405)$, …

3. **Applications with/to LQCD**

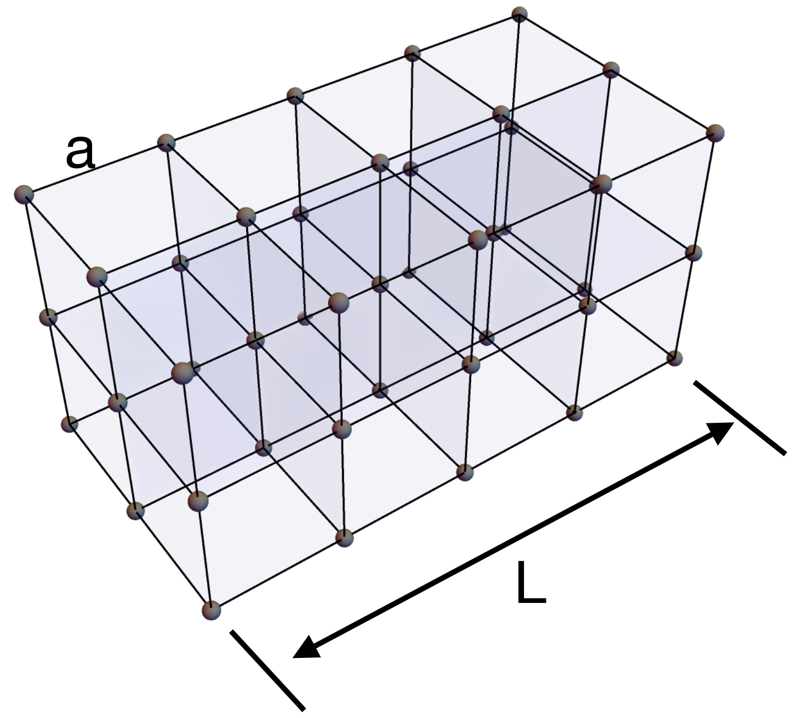
    Chiral extrapolations,
    Quantization Conditions…

4. Summary/Outlook



*) not part of the talk **) low-energy

LATTICE QCD (SPECTR.)



Discretization of space-time

Euclidean space-time

Boundary conditions

Gauge and fermion degrees of freedom

links

plaquettes

Nielsen–Ninomiya theorem

Fermion doublers

Lattice QCD action

Construction of the action

Wick's theorem

measure of integration in the path integral.

Generating functional

Hybrid Monte-Carlo simulation

The transcription of the operators used to probe the physics

Operator construction

Correlation functions

Generalized eigenvalue problem

scale setting

Energy eigenvalues ($\mathbb{R}$)

K. Wilson, Phys. Rev. D10 (1974) 2445 , …
Introduction to lattice QCD: Course Rajan Gupta hep-lat/9807028 [hep-lat] ….

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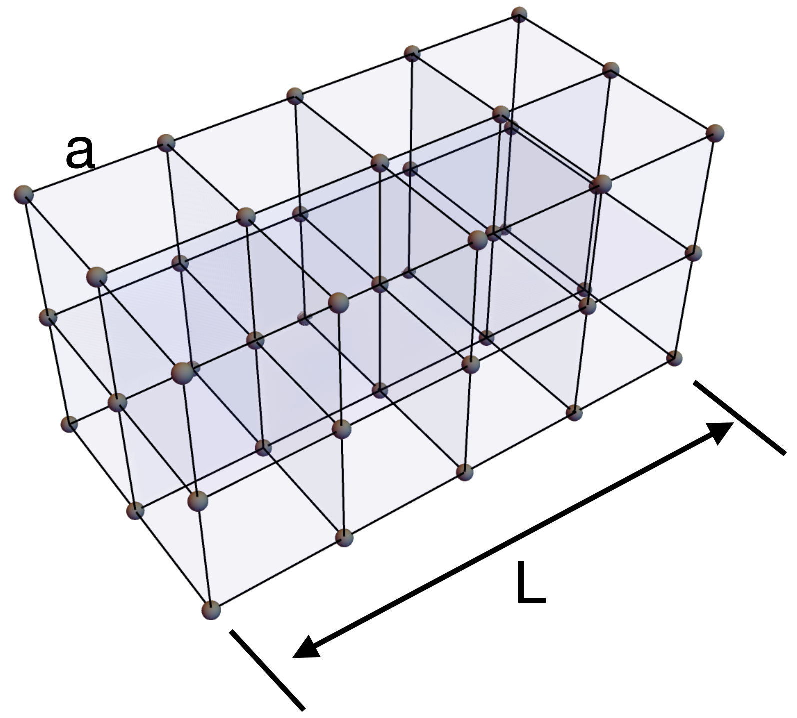

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Roadblocks

- discretized (Euclidean) space-time — continuum extrapolation
- unphysical quark mass — extrapolations tools from CHPT
- finite volume — quantization conditions needed

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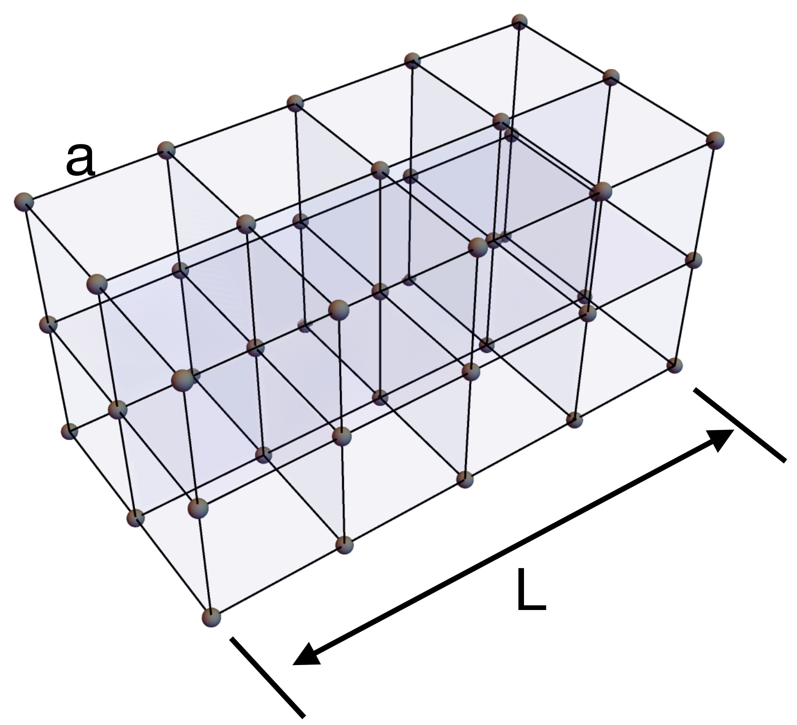

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Advantages

- QCD degrees of freedom (first principles)
- Experimentally inaccessible scenarios:
  - → Unconventional quantum numbers (later…)
  - → Three-body scattering/… (later…)
  - → Chiral trajectory (later …)

K. Wilson, Phys. Rev. D10 (1974) 2445 , …
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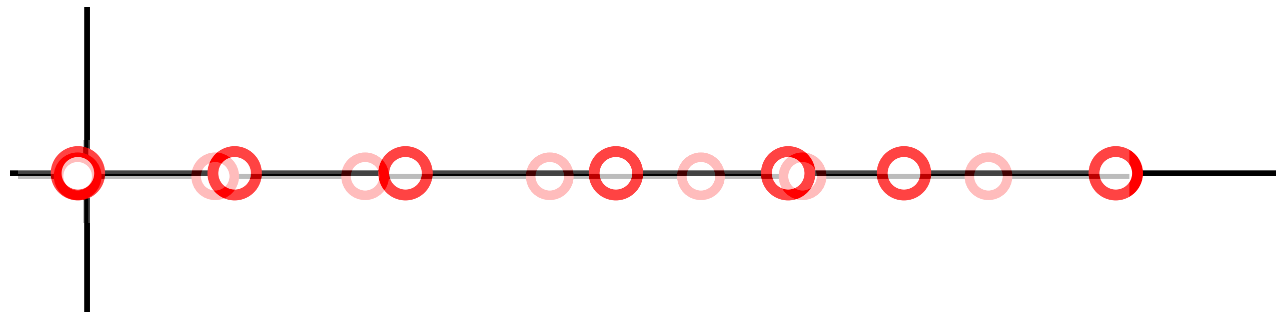
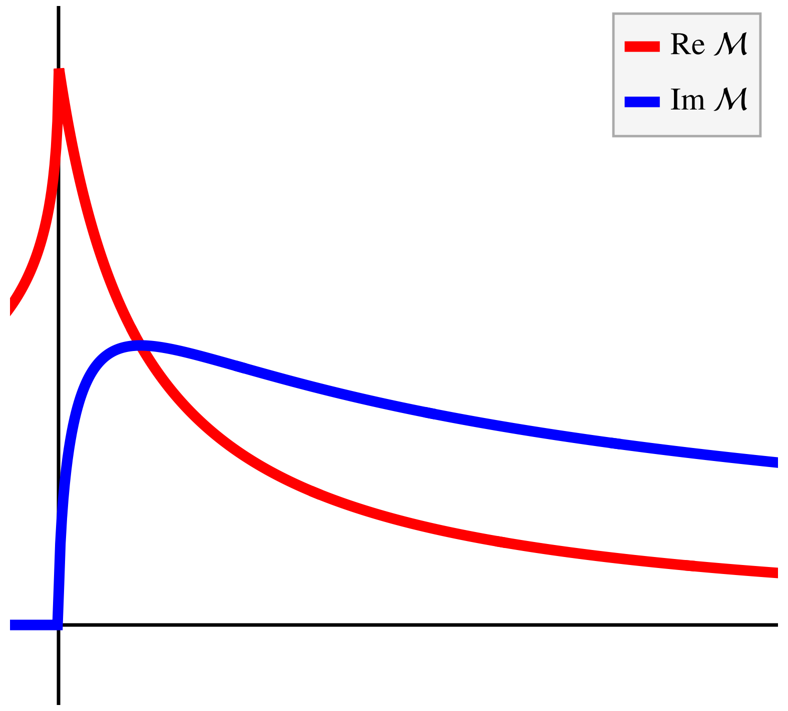
QUANTIZATION CONDITIONS



- Finite volume calculations: no direct access to scattering quantities
- Real-valued energy eigenvalues
  - Shifted from free energies — physical information
  - Relation to observables = **Quantization condition**

Finite volume QFT (Lattice QCD)

continuum QFT



Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
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Review: MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

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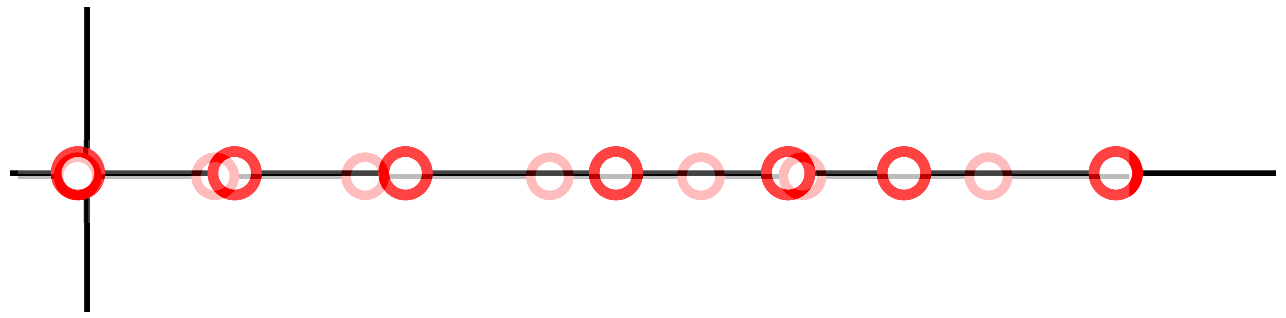

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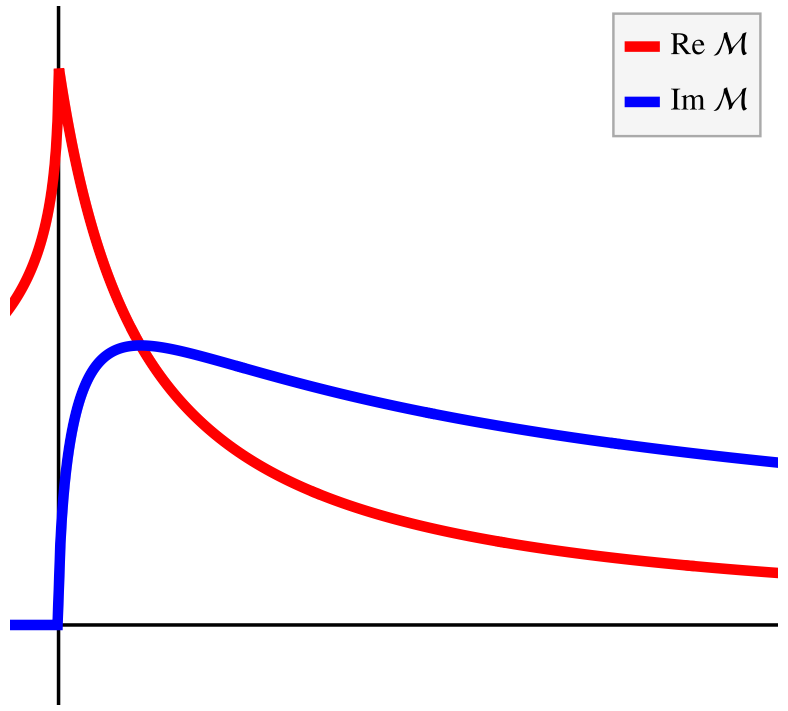
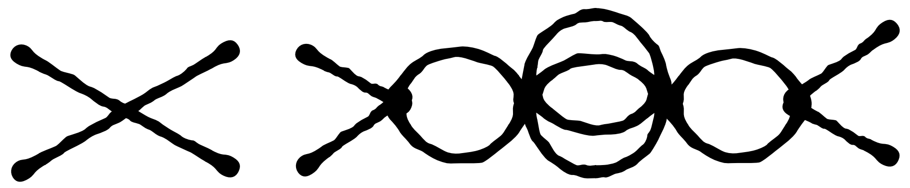
- **one-way of thinking:**
  - off-shell configurations decay exponentially $\sim e^{-ML}$
  - … use S-matrix Unitarity to identify on-shell configurations





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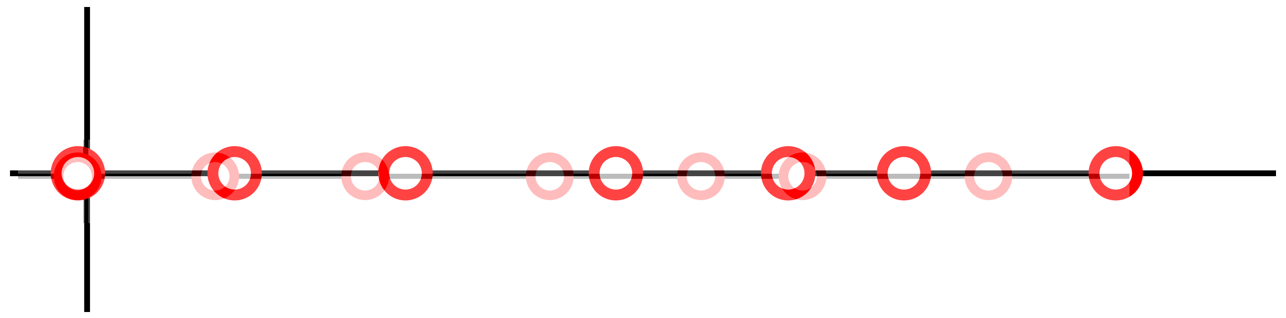

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$$M_\infty^{-1} = \tilde{K}^{-1} - \int \frac{d^3l}{(2\pi)^3}\frac{1}{2E_l(s-4E_l^2+i\epsilon)}$$

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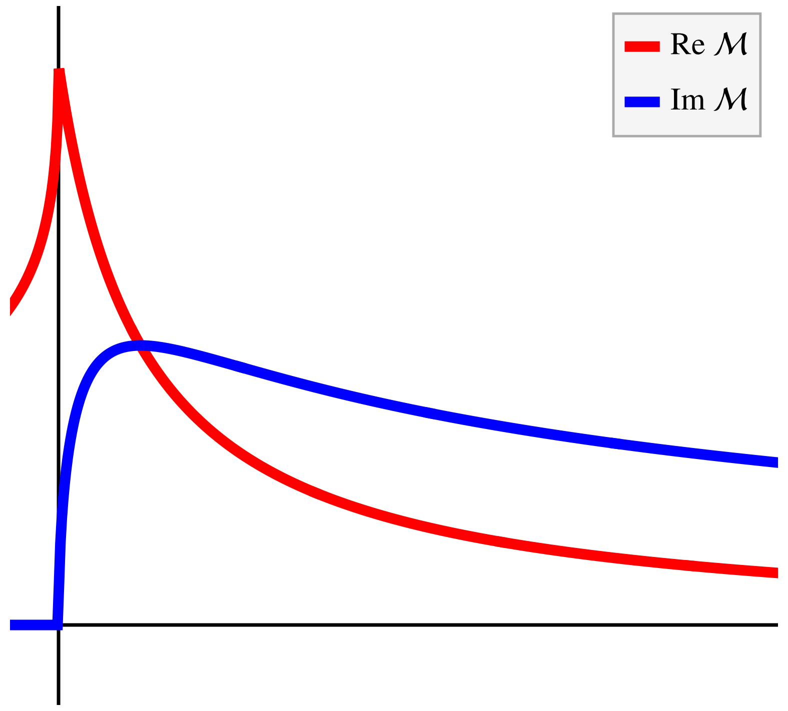

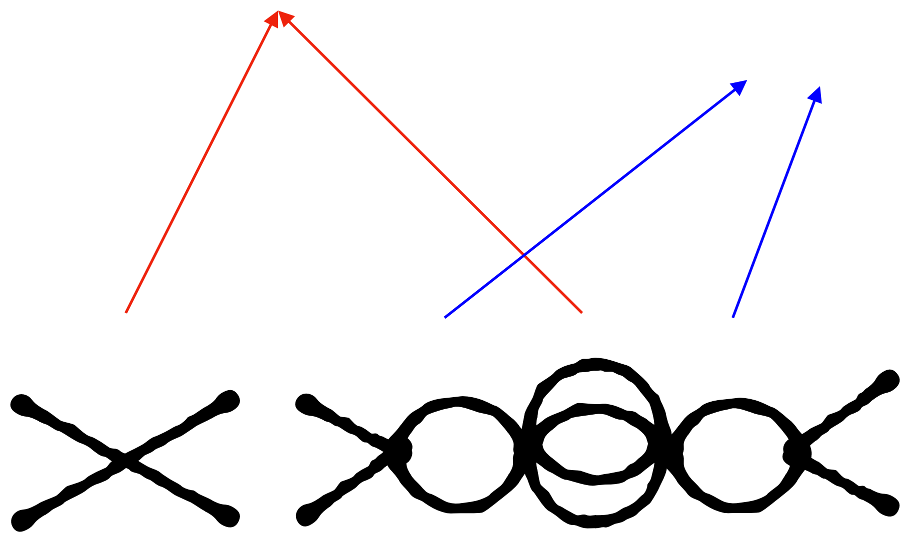

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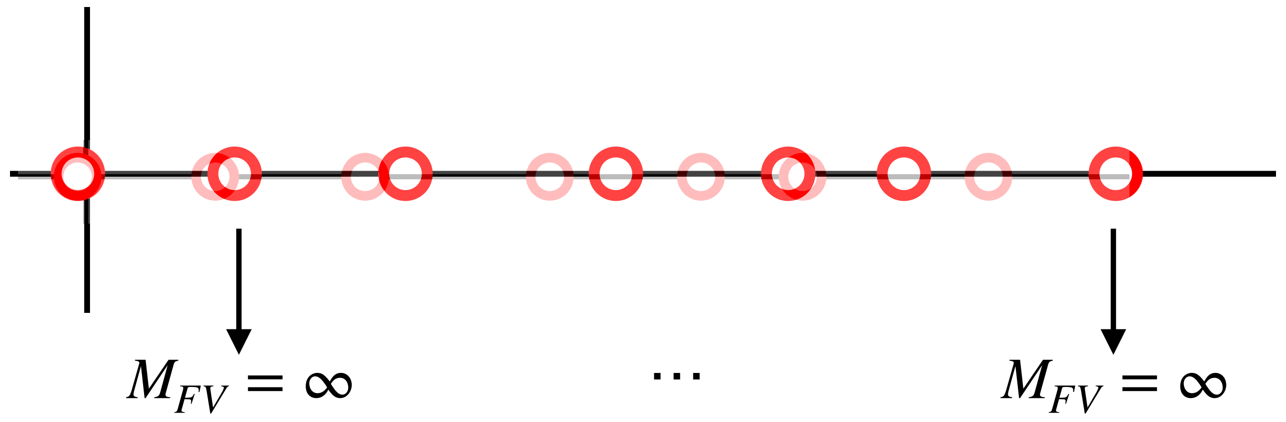

$Z_{00} + \mathcal{O}(e^{-ML})$

Lüscher Zeta-function

$$M_{FV}^{-1} = p\cot\delta - \left(\frac{1}{L^3}\sum_{\vec p}\ldots - Re\int_{\vec l}\ldots\right)$$

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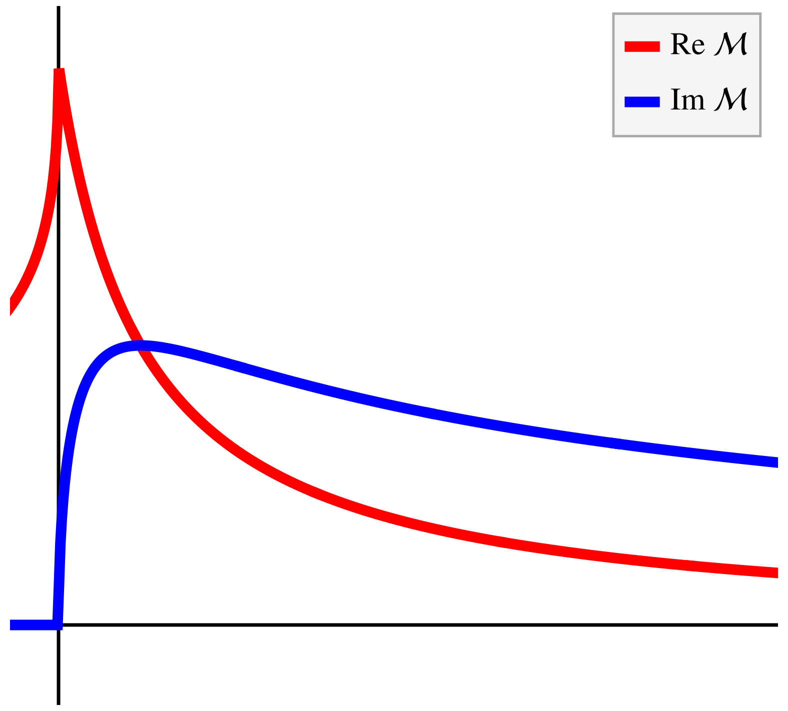

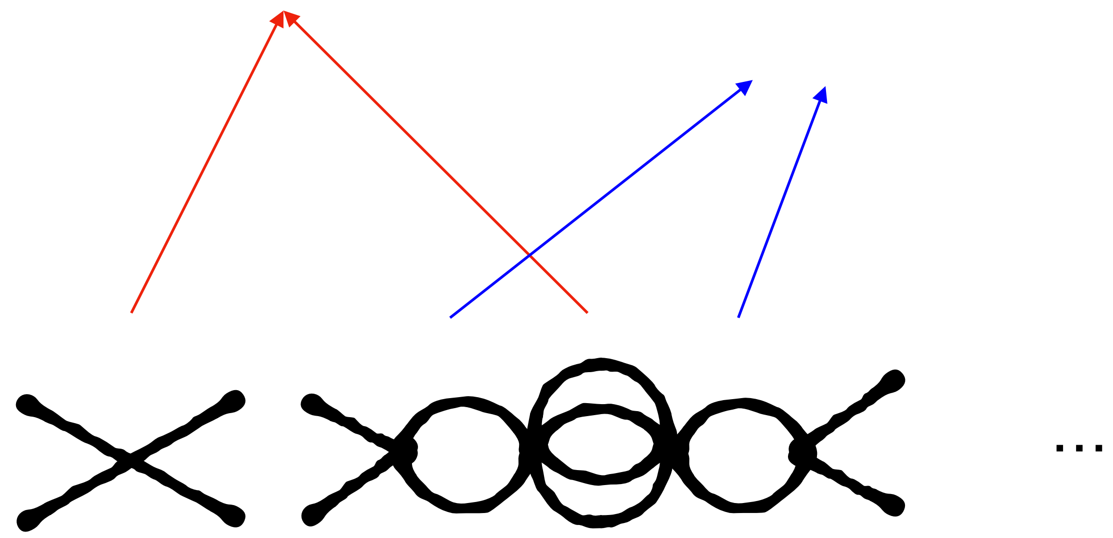

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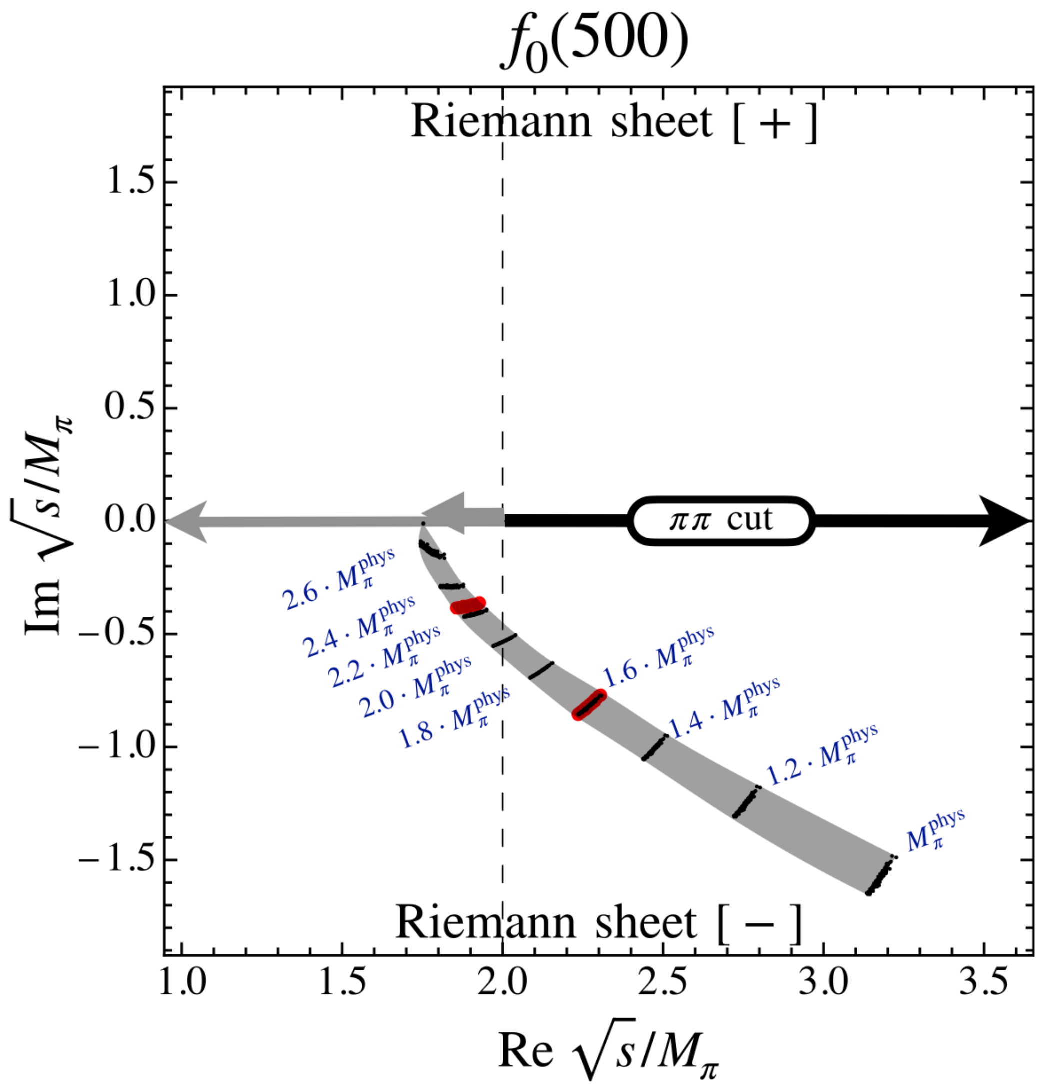
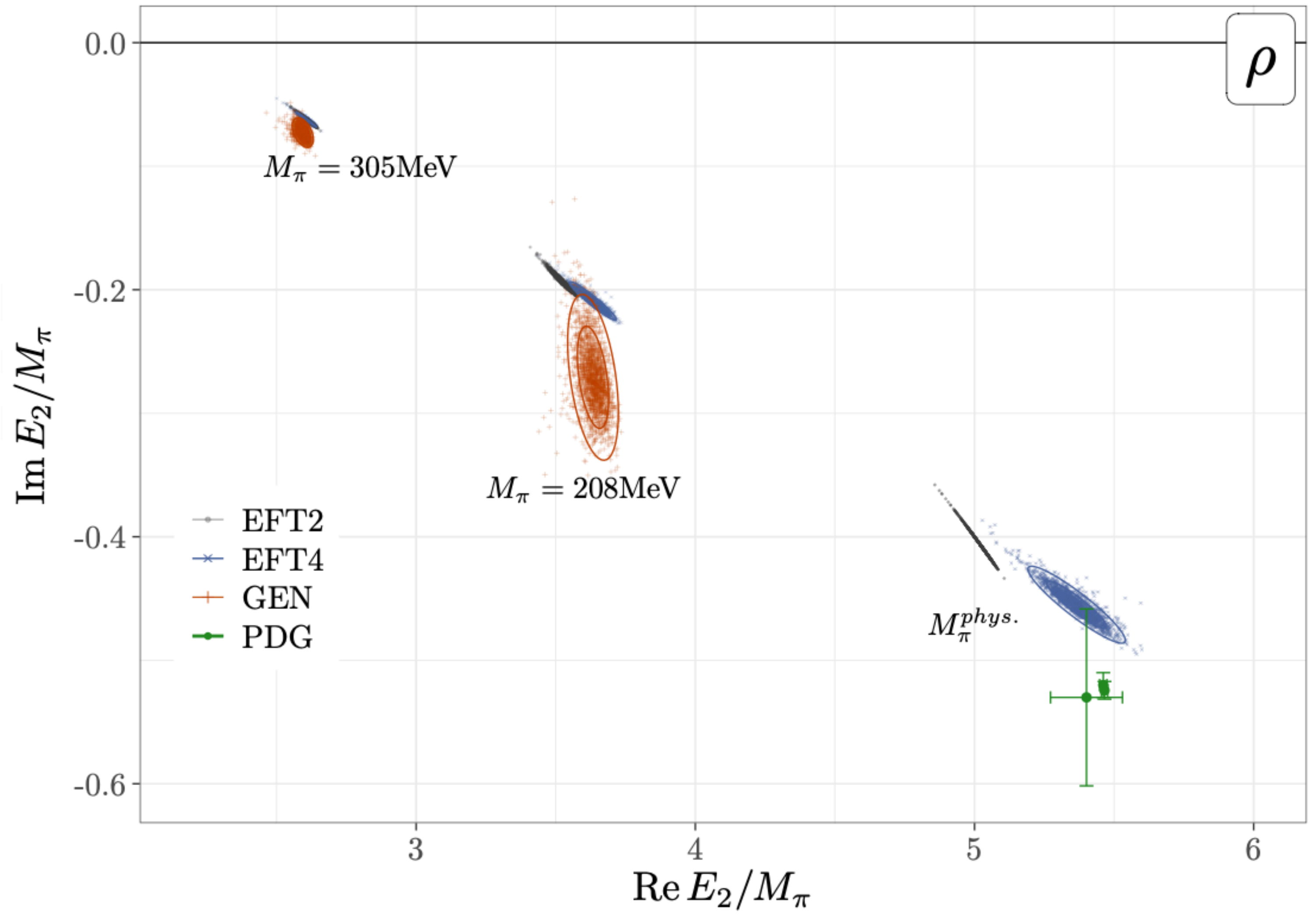
APPLICATION I

Two pion system

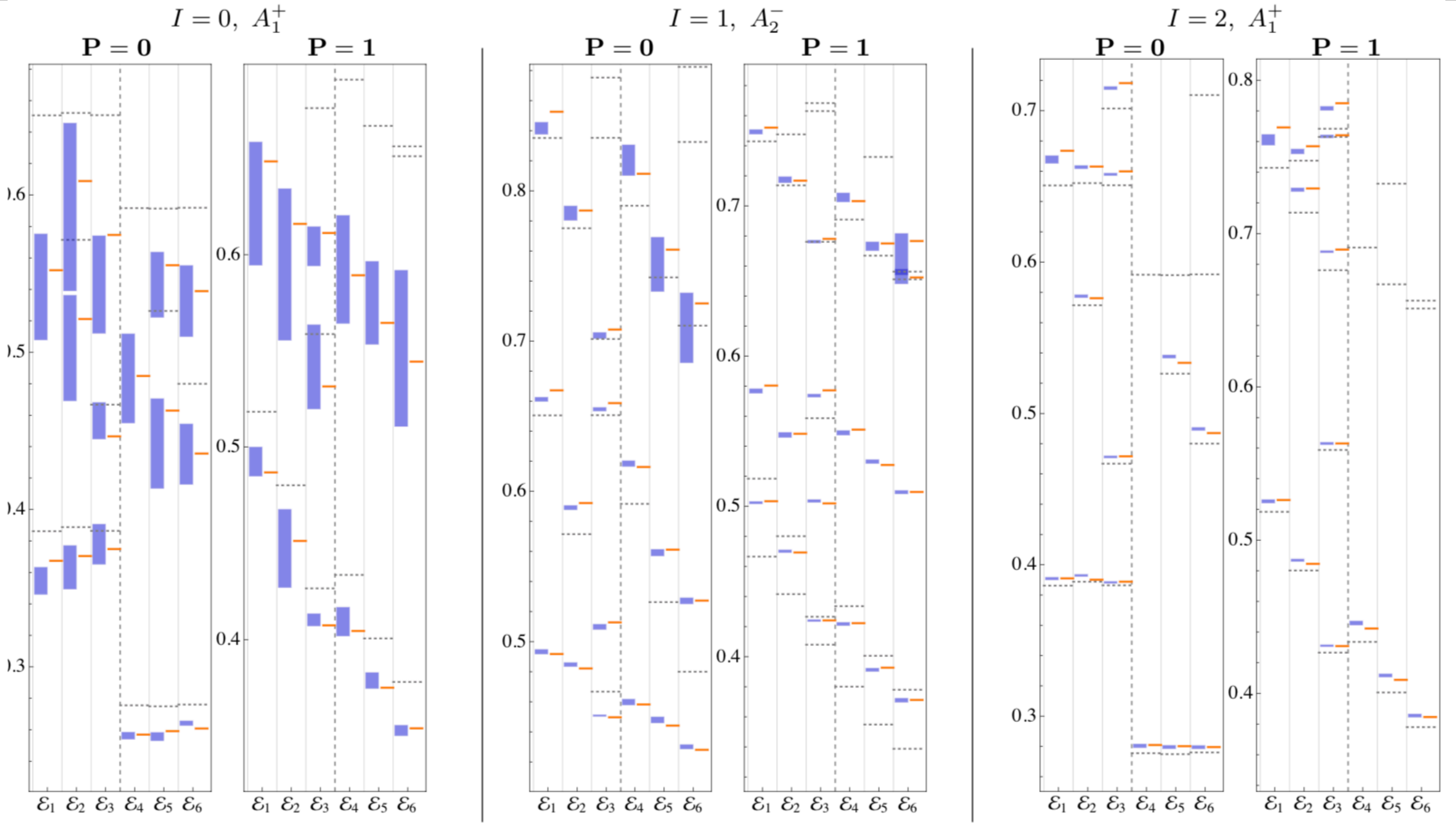
- simplest 2-hadron system
- many LQCD results
  NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;….
- simultaneous description of all $\pi\pi$ interaction channels through CHPT — UCHPT
  GWQCD: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)

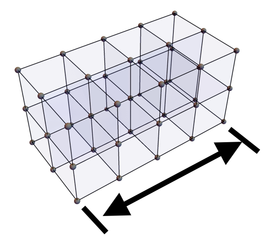


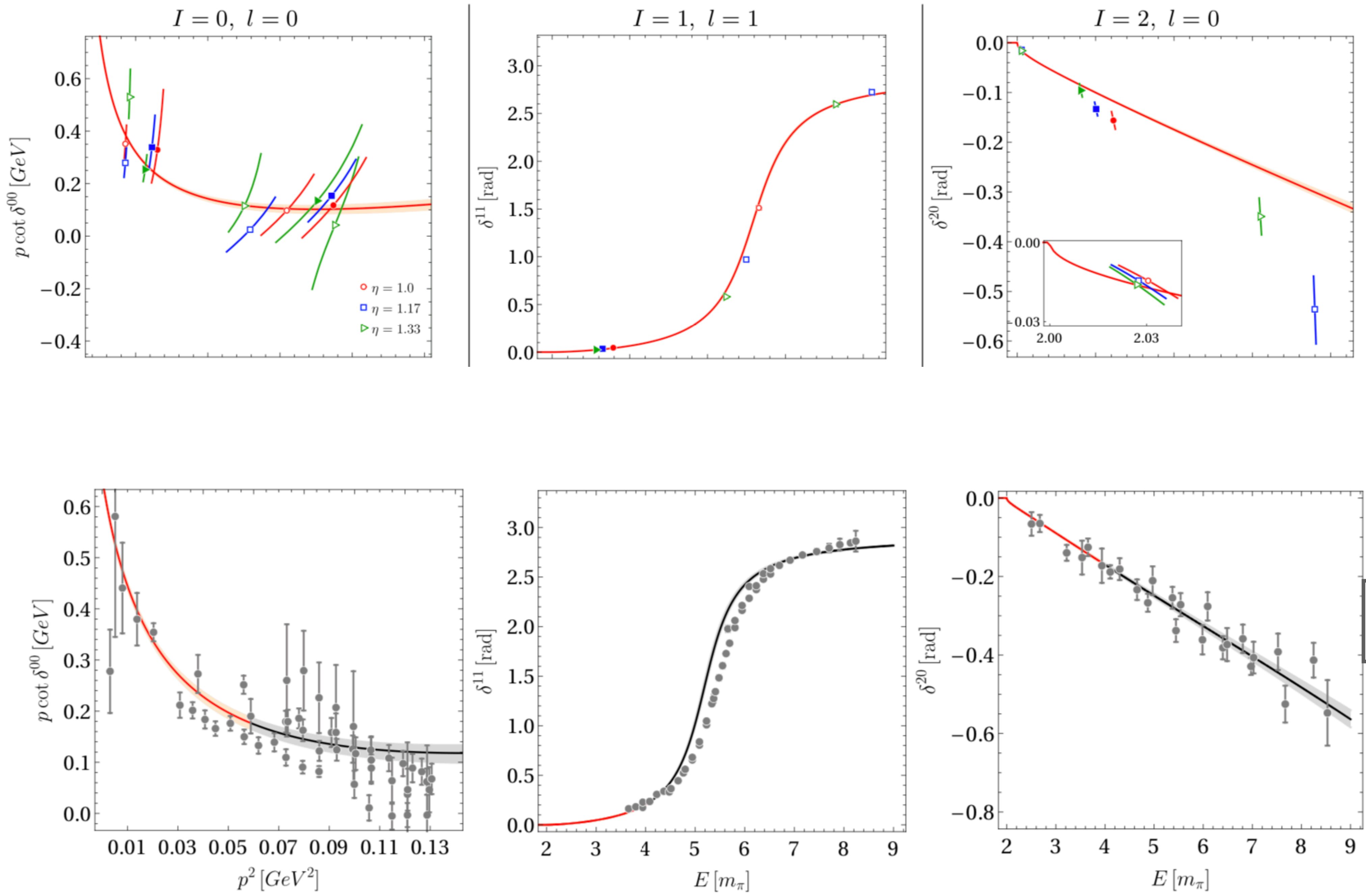
MM/Urbach/Meißner *Phys.Rept.* 1001 (2023) 1-66



Yan/MM/… Phys.Rev.Lett. 133 (2024) 21





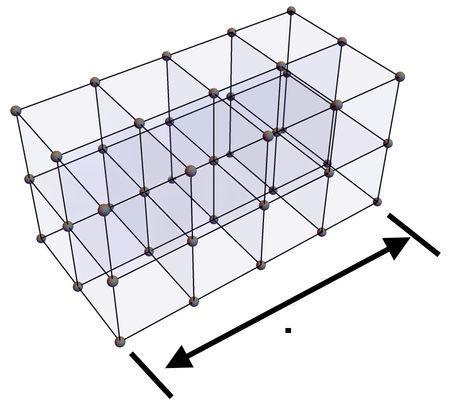


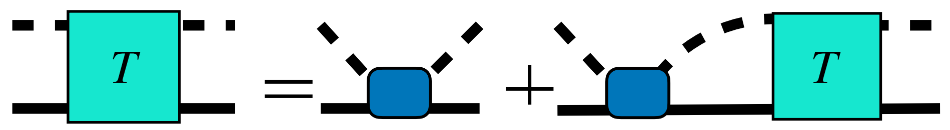
Quantization Condition

Chiral extrapolation

[GWQCD] MM/Culver/… Phys.Rev.D100(11)(2019) 114514

APPLICATION II





Meson-baryon systems ($\bar{K}N/\pi\Sigma/\pi\Lambda/K\Xi$)

- Available Lattice spectrum

[BaSc] Bulava et al. Phys.Rev.Lett. 132 (2024) 5; 2307.13471

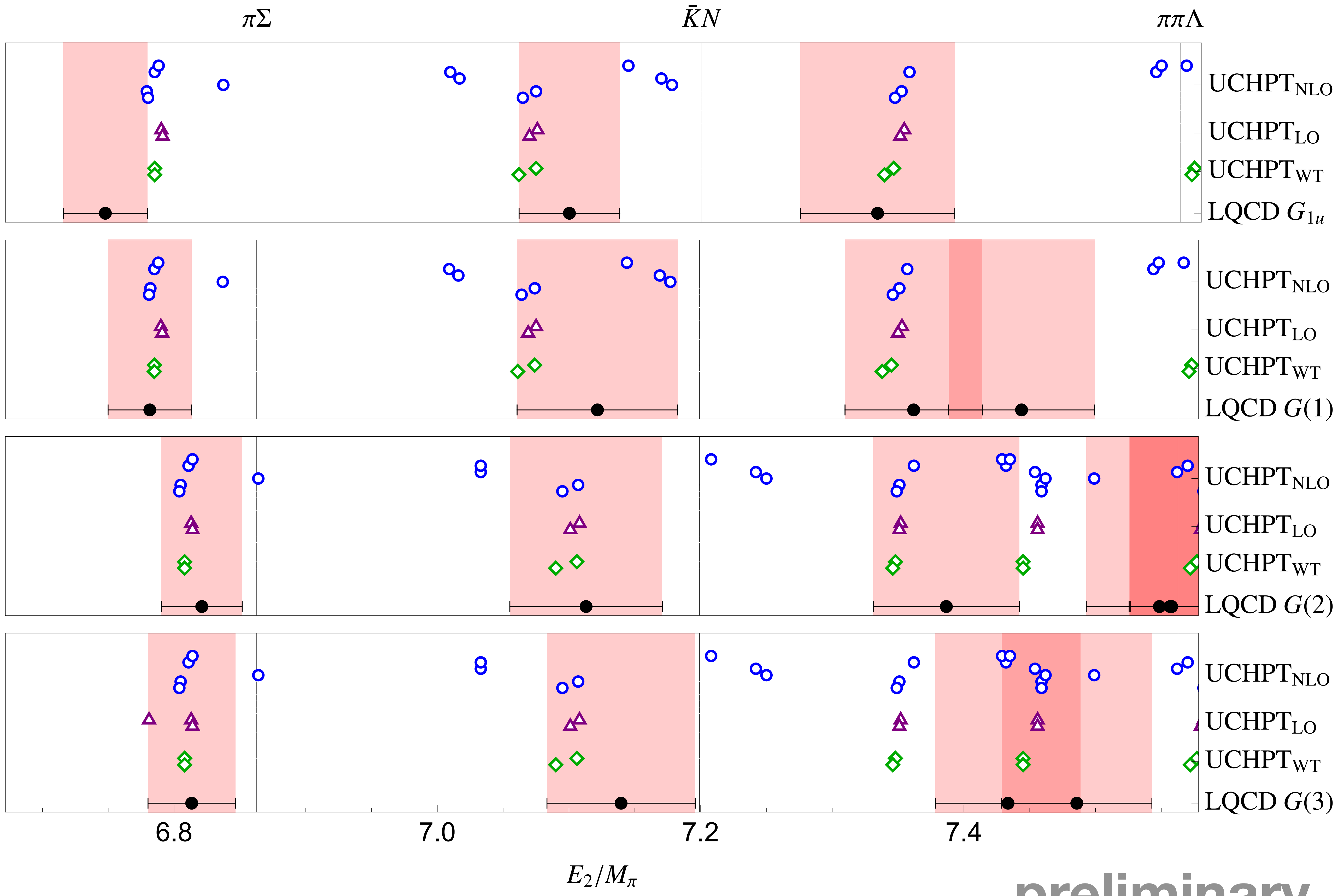
$M_\pi \approx 200\,\mathrm{MeV}$ $M_K \approx 487\,\mathrm{MeV}$

$M_\pi L = 4.181(16)$ $a = 0.0633(4)(6)\,\mathrm{fm}$

- **Compare to UCHPT**
  - Unified analysis LQCD+UCHPT+EXPERIMENT

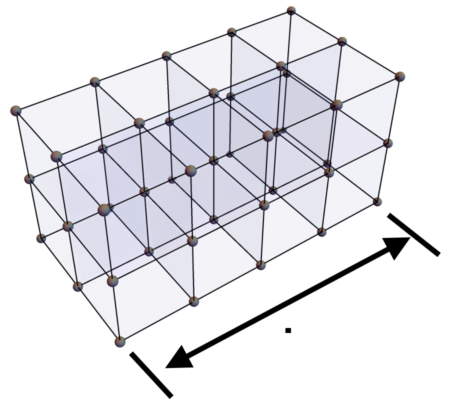
… mostly ok, but not always

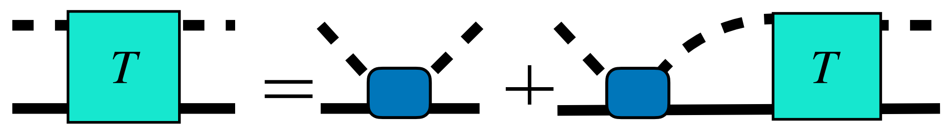
… ongoing work



preliminary

APPLICATION II





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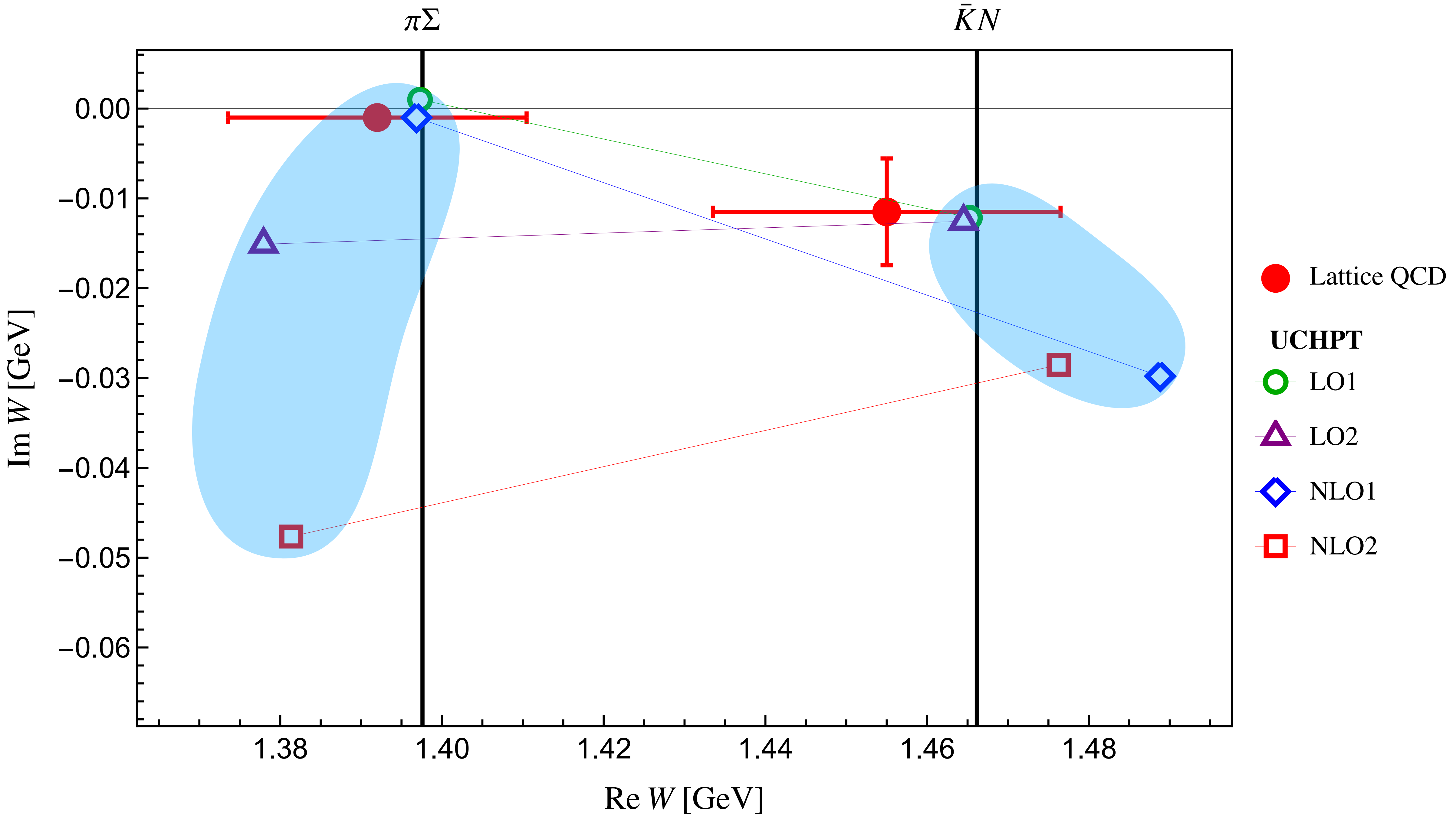
  $M_\pi L = 4.181(16) \quad a = 0.0633(4)(6)\,\mathrm{fm}$

- pole positions from available UCHPT approaches



Guo/Kamyia/MM/Meißner Phys.Lett.B 846 (2023)

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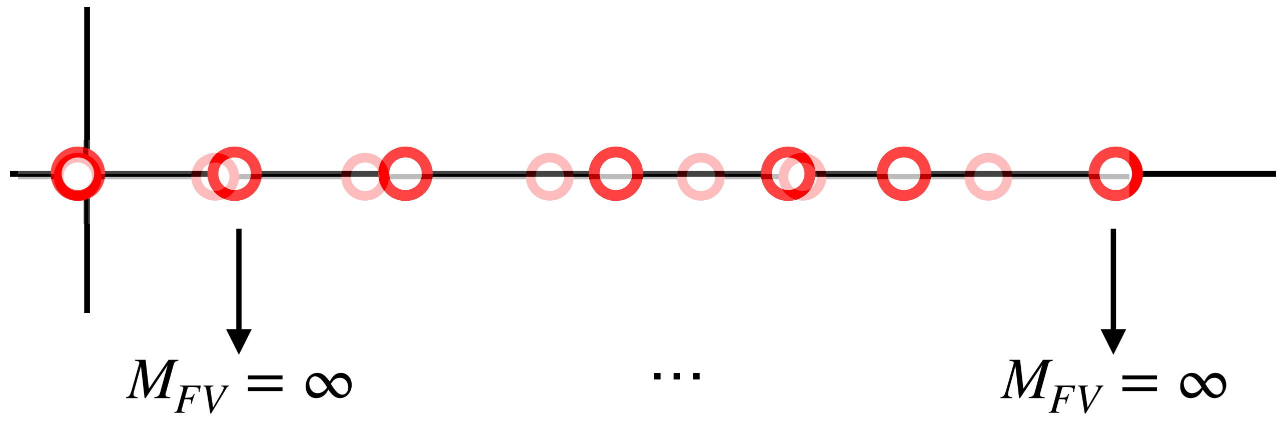

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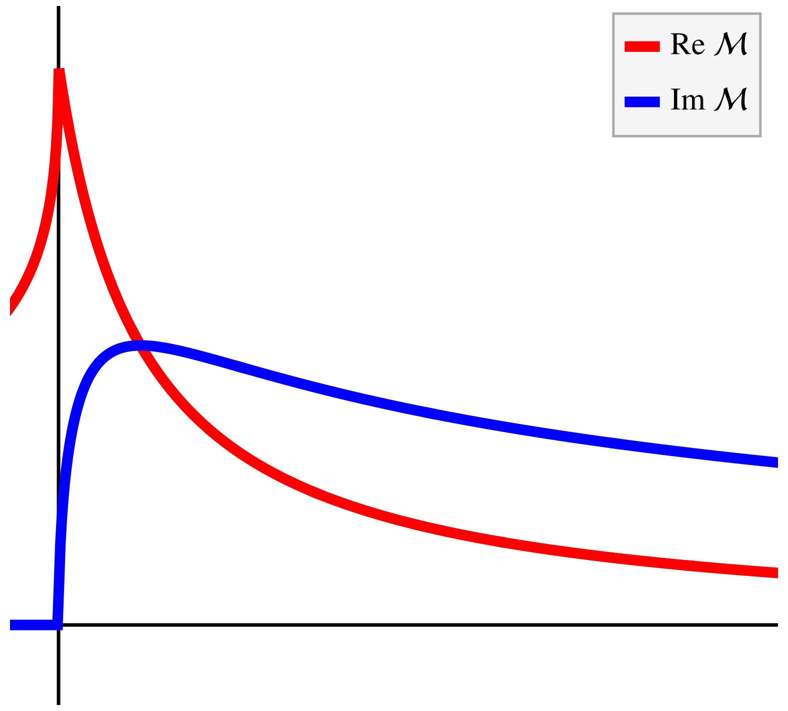
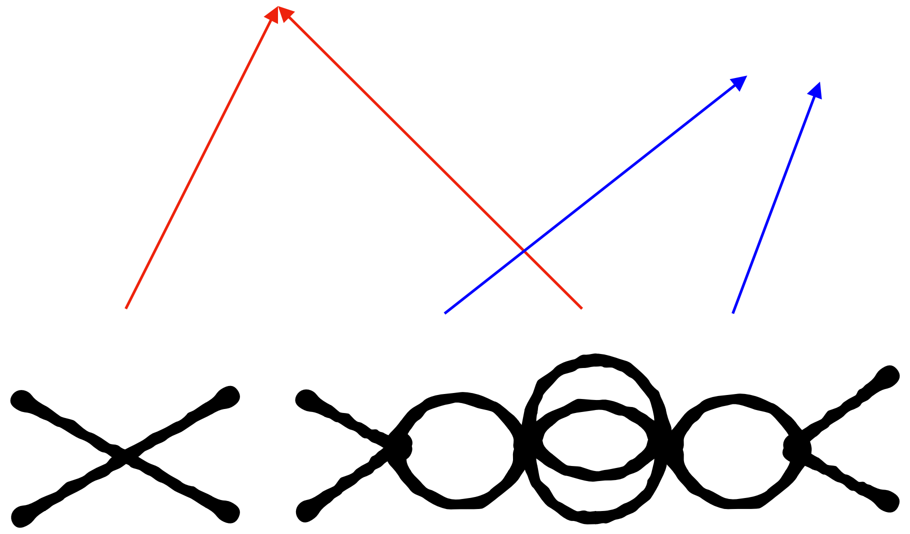
➡ Unitarity $$M_{\infty}^{-1} = \tilde{K}^{-1} - \int \frac{d^3 l}{(2\pi)^3}\frac{1}{2E_l(s - 4E_l^2 + i\epsilon)}$$





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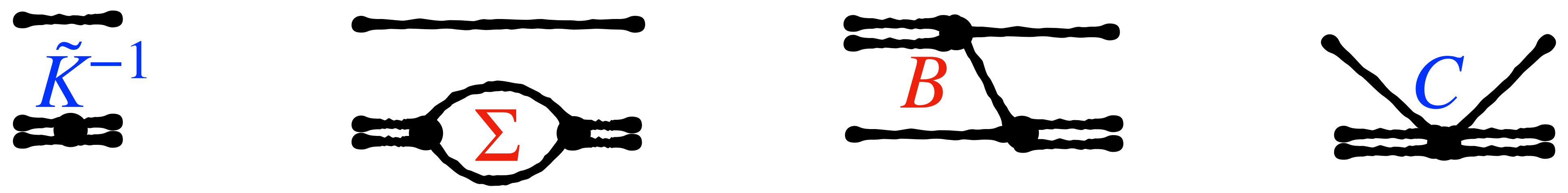
Review: MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY

$\sqrt{s} < 4$ ... more combinatorial possibilities



Generalization to 3-body states — Finite Volume Unitarity (FVU) approach

- 3-body unitarity accounts for all on-shell states
- genuine determinant condition

- Alternatives: RFT, NREFT (equivalence in different regimes shown)
  RFT(Hansen/Sharpe 2014) NREFT(Rusetsky/Hammer/Pang 2017)
  Jackura et al. *Phys.Rev.D* 100 (2019) 3, 034508, Garofalo et al. *JHEP* 02 (2023) 252

FVU Finite Volume Unitarity

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]_{\ell}^{\Lambda} \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

Many new applications

- proof of concepts and spin-less repulsive systems
  MM/Doring Phys.Rev.Lett. 122 (2019) 6, Fischer et al. Eur.Phys.J.C 81 (2021) 5, Blanton, Lopez, Hansen, Briceno, …
- systems with left-hand cut
  Hansen et al. *JHEP* 06 (2024) 051, Dawid et al. *JHEP* 01 (2025) 060, Rusetsky, …
- 3-body resonant systems (later …)
  MM/Culver Phys.Rev.Lett. 127 (2021) 22
  Yan et al. Phys.Rev.Lett. 133 (2024) 21

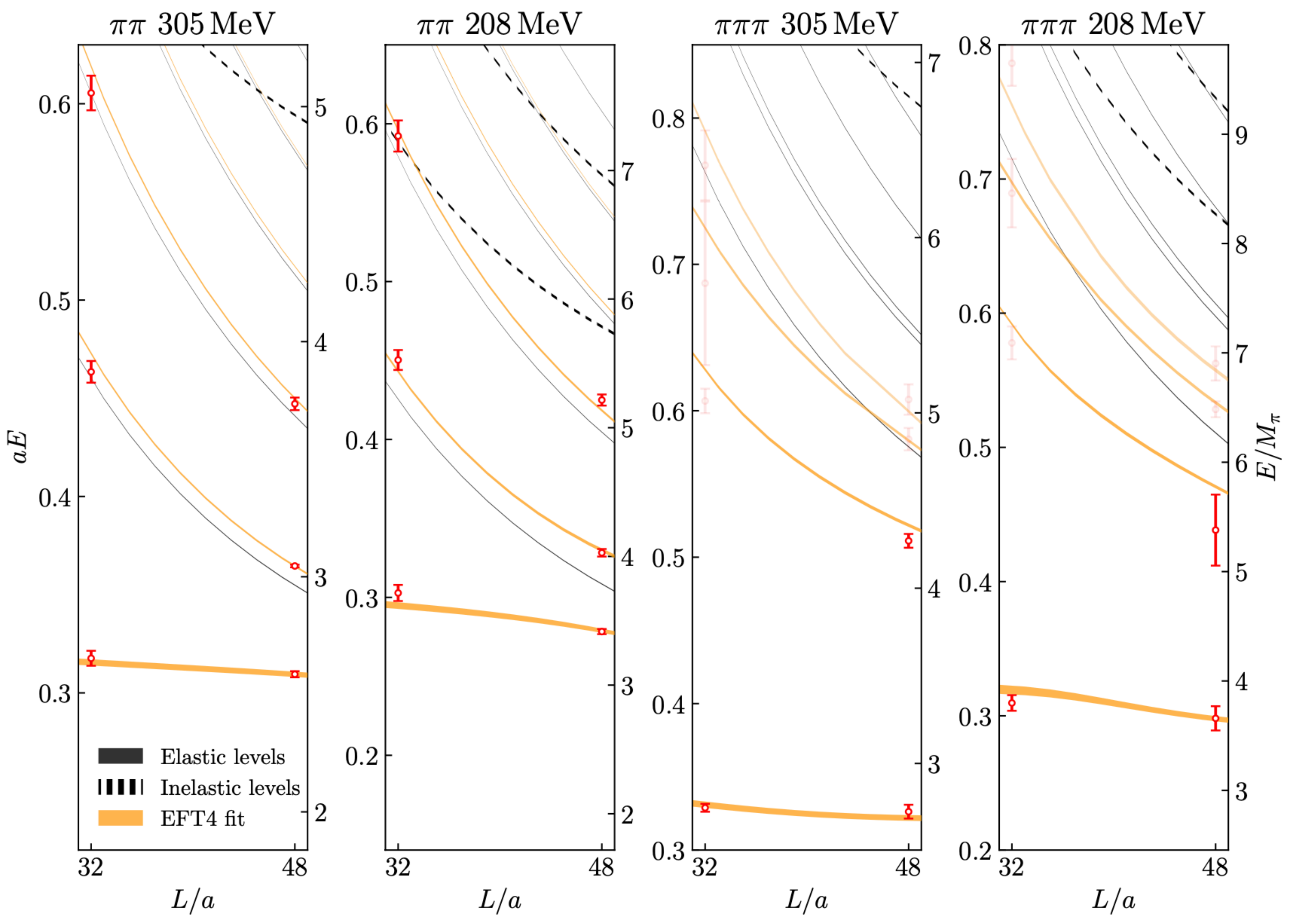
IVU Infinite Volume Unitarity

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{\left(B + C\right)}{2E_l} \frac{1}{\tilde{K}^{-1} - \Sigma} T^c$$

APPLICATION III $\omega \to \pi\pi\pi$

Lattice QCD

- Nf = 2 + 1 Clover fermions
- 2/3 particle operators
- 2 pion masses ($\approx 210, 305$ MeV) 2 volumes ($L^3 = 32^3, 48^3$)



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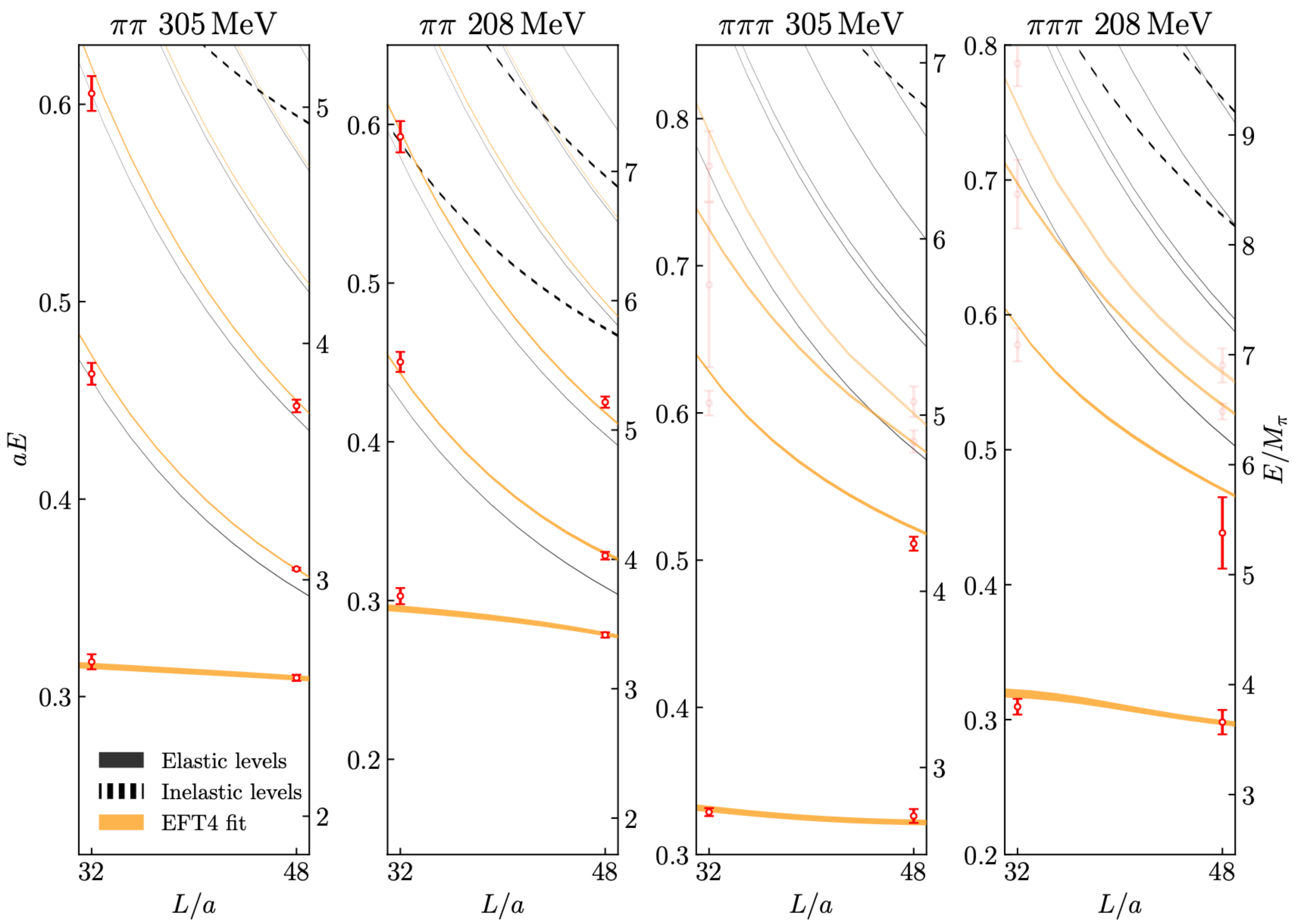
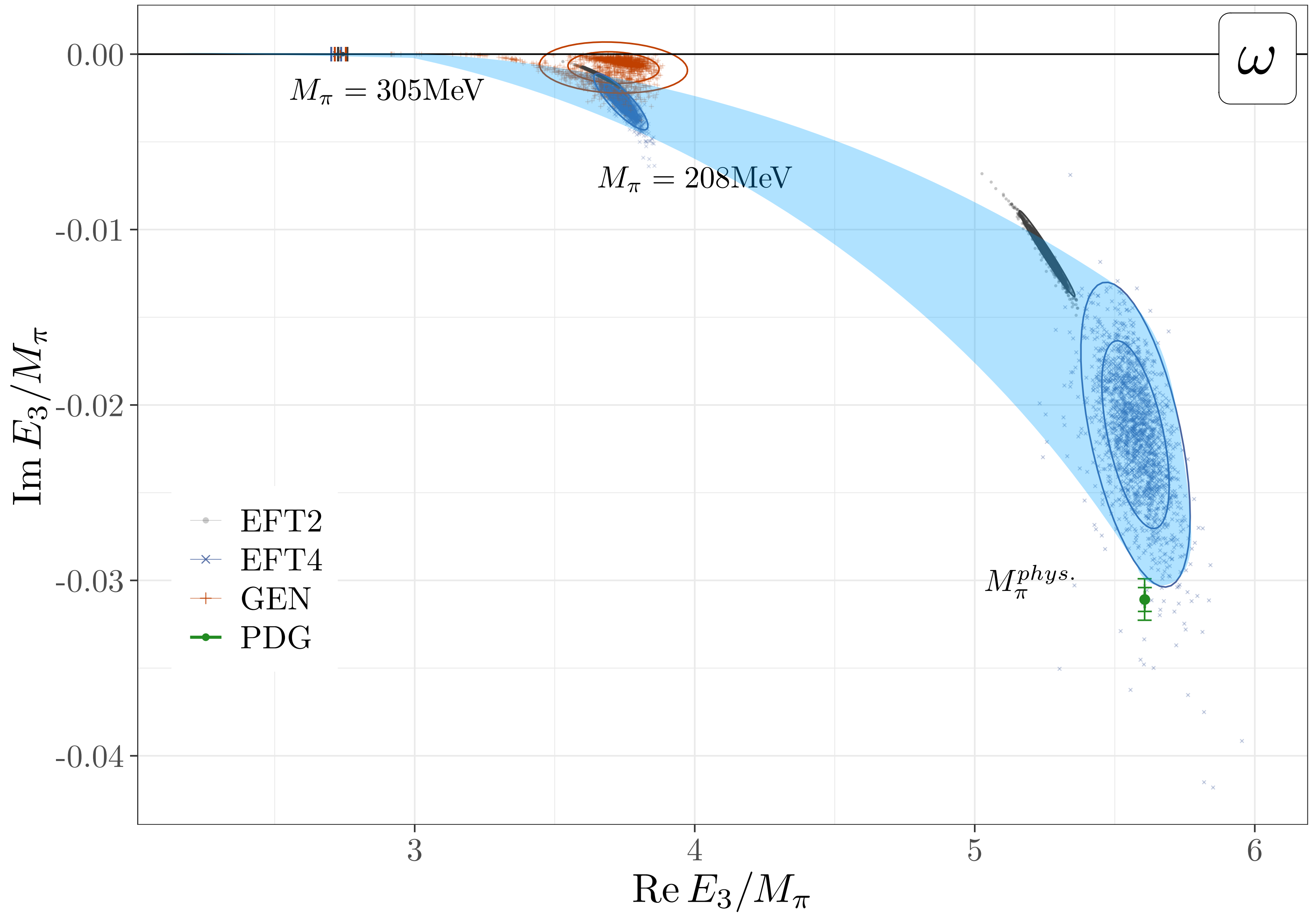


FVU

$$\det\left[2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1}-\Sigma^L\right)-B-C\right]^\Lambda \equiv 0$$

Result

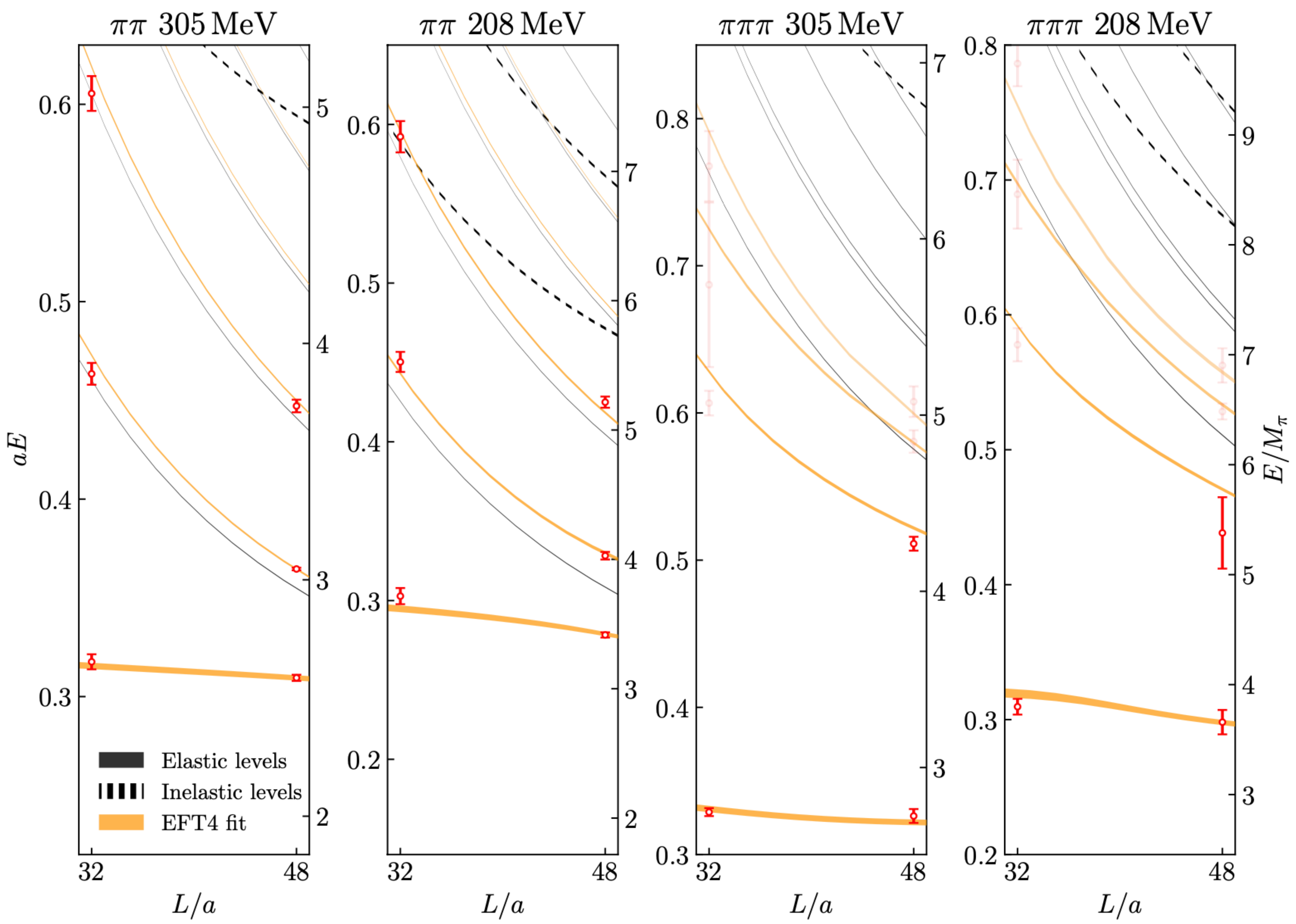
- Various EFT based ansatzes
- $\omega(782)$ becomes abound state at ~300 MeV
- at the physical point very close to the EXP value



APPLICATION III $\omega \to \pi\pi\pi$

Lattice QCD

- Nf = 2 + 1 Clover fermions
- 2/3 particle operators
- 2 pion masses ($\approx 210, 305$ MeV) 2 volumes ($L^3 = 32^3, 48^3$)



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Result

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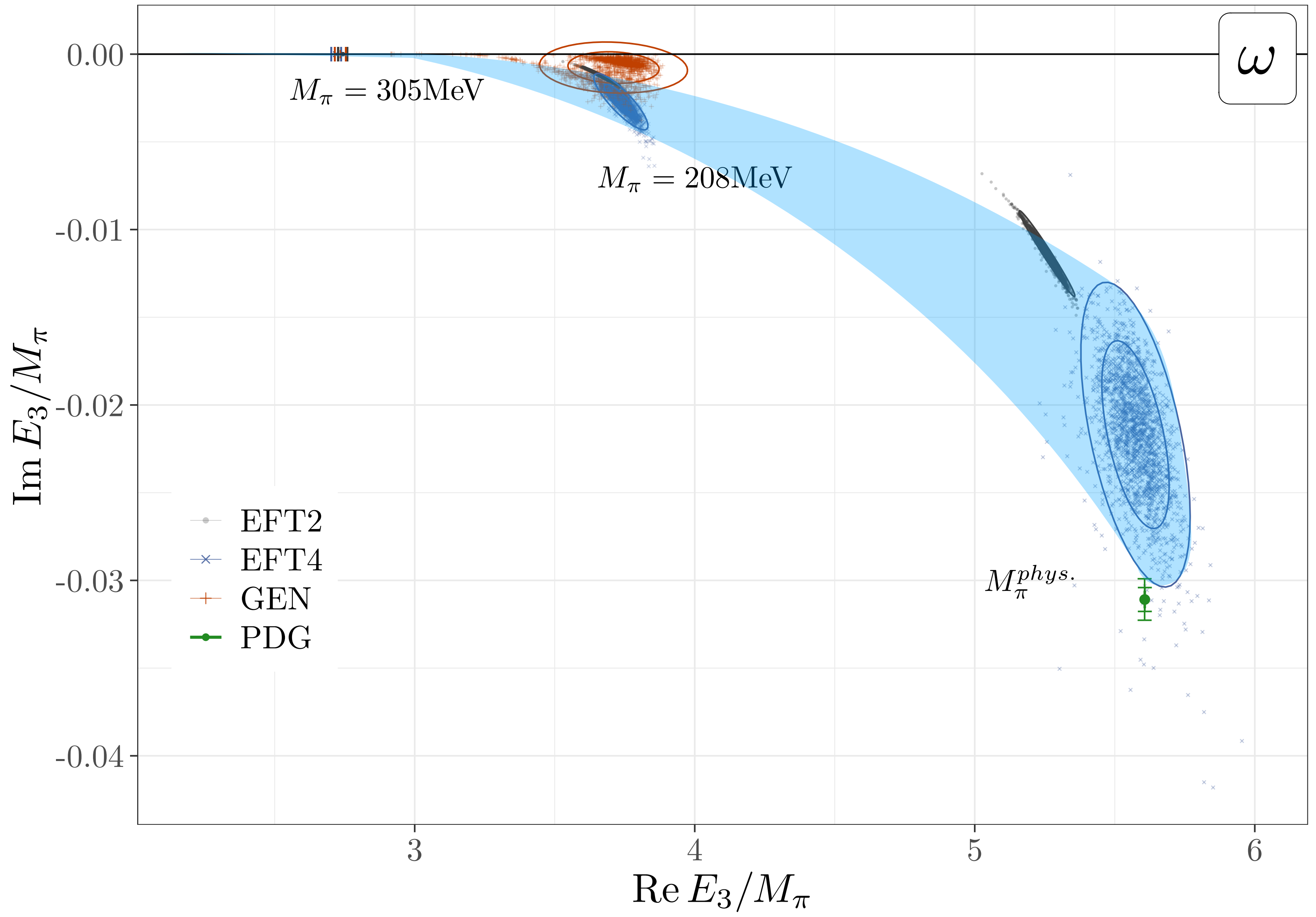
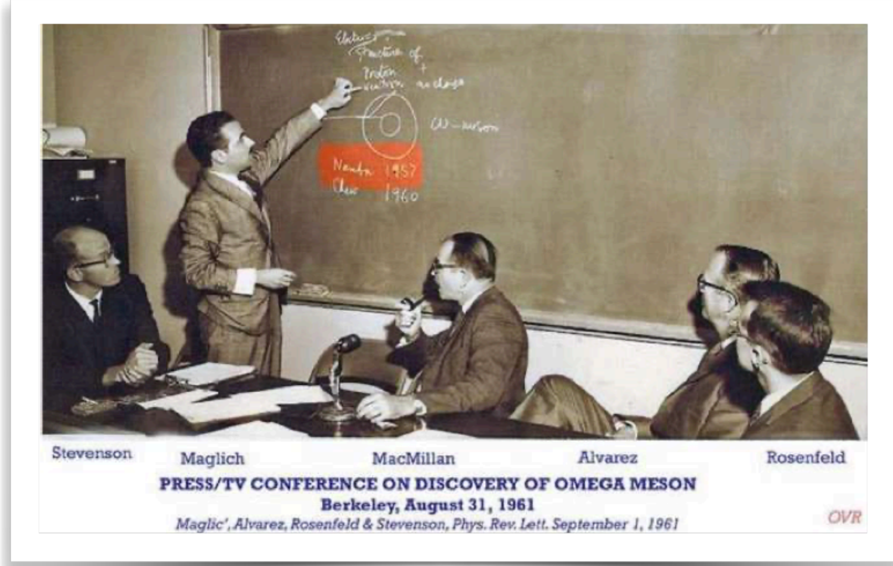
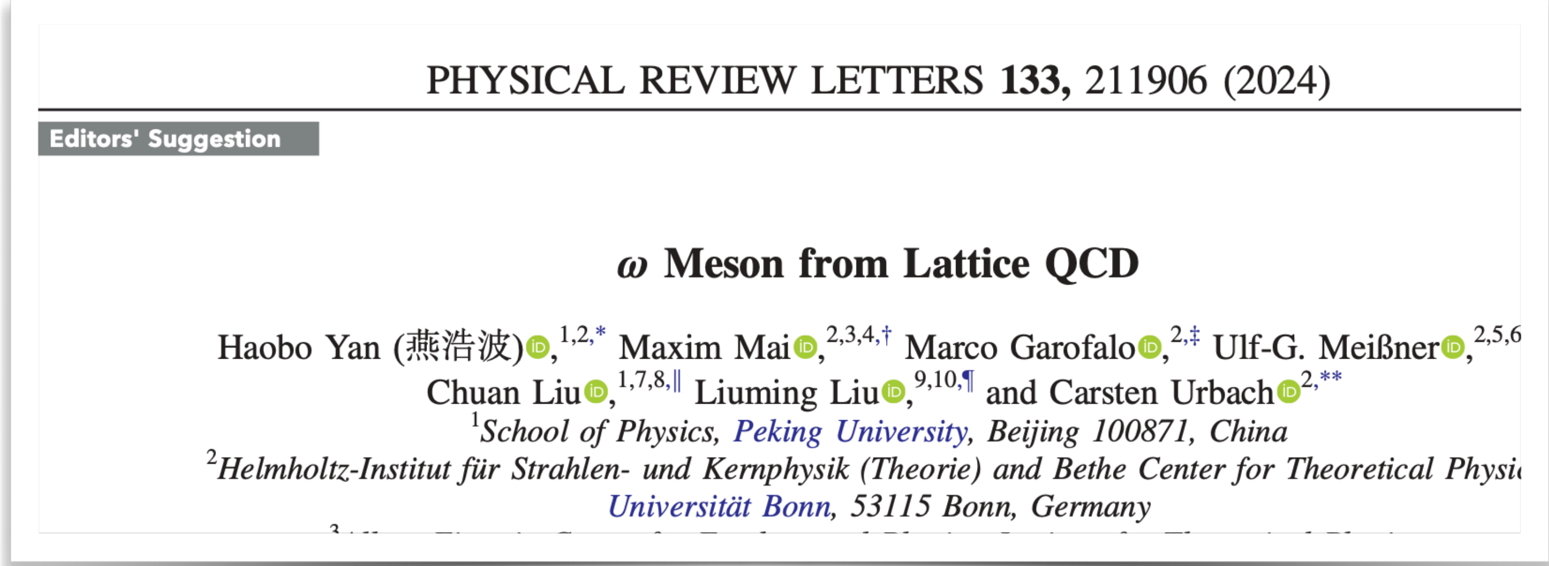




63 years →


PHYSICAL REVIEW LETTERS **133,** 211906 (2024)

Editors' Suggestion

ω Meson from Lattice QCD

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Universität Bonn, 53115 Bonn, Germany

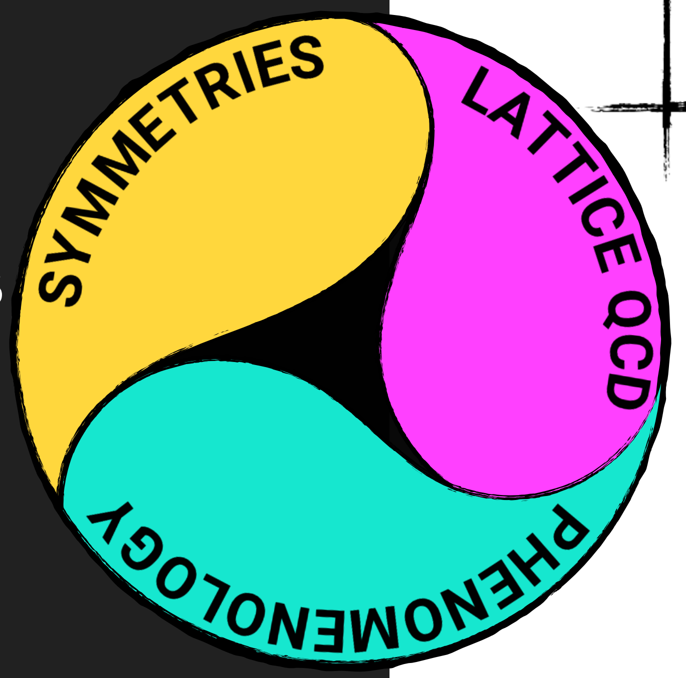
TAKAWAYS

Effective Field Theories

- quark-mass dependence
- analytical tools
- dynamically generated resonances

S-matrix

- Mathematical constraints on transitions
- Universal resonance parameter



Lattice QCD:

- ab-initio calculations
- universal tool for physical und unphysical scenarios
- many new advances and results

UCHPT models

- [x] $f_0(500), \rho(770), \ldots$ well established quark-mass dependence
- [x] Two-pole structure: $\Lambda(1405), \Lambda(1380)$ **discovered**

Novel FVU 3b Quantization Condition

- [x] pilot results on $3\pi(I=3{,}2..), a_1(1260), \phi^4, \ldots$
- [x] **Re-discovered $\omega(782)$ from QCD** — pole and chiral trajectories

Outlook

- [ ] $N(1440)$, $DD\pi$, … spin-exotics
- [ ] Triangles/Strangeness — $a_1(1420)$ … *first steps: hys.Rev.D* 110 (2024), *JHEP* 10 (2024) 246
- [ ] UCHPT + LQCD $\Lambda(1405), \Lambda(1380)$ ongoing …
- [ ] is there something for the in-medium calculations?
- [ ] …